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A Novel Analytical Method for Finger Position Regions on Grasped Object

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Abstract

In this paper, an analytical approach is proposed for obtaining finger position regions of an object with multi-fingered hand. At first, a method to obtain which combination of the object edges is possible to be used for grasping, is given. Then, Graspable Finger Position Region on a combination of edges is defined where the object can be held successfully. It is shown that the region is bounded by plural boundary hyperplanes. With the combining these boundary hyperplanes, two propositions for exactly obtaining the Graspable Finger Position Region by using analytical method, are proposed. Lastly, numerical examples are performed to show the effectiveness of the proposed approach.

1 Introduction

The grasp of an object by a robot hand is a primitive but very important subtask in automatic systems. Advanced applications sometimes require multiple robotic fingers to perform a task coordinately. If a multifingered robot hand is used to grasp an object, synthesizing successful grasps is required. Therefore, determination of the graspable finger position regions of an object is a fundamental and important issue.

In literatures, finger position and finger force on an object were treated as variables simultaneously. The problem of solving the graspable finger position regions of an object was dealt with as a nonlinear problem. The finger positions were computed by the method of scanning every sample points selected on the object, which is complicated and the computing load is large.

About the finger forces, the methods for constructing of the force-closure and the form-closure were proposed in [1]~[4]. About finger position regions for multifingered hand, many attempts have been made by analytical method. Omata proposed an algorithm for approximately computing the positions of fingertips with maintaining equilibrium when a polyhedral object is grasped [5]. But the object grasp regions has not been exactly solved by analytical method yet.

The purpose of this paper is to exactly determine the graspable finger position regions of a given object. In this paper, we distinguish candidates from combinations of the object edges touched by fingertips. Then, we propose an analytical method to solve the problem of the finger position regions for grasping the object.

A brief preview of this paper is as follows: Section 2 explains how to select graspable candidates from all of the combinations of the object edges by using the force equilibrium. For a selected candidate, Section 3 analyzes that the regions of graspable finger position are bounded by boundary hyperplanes induced by using the moment equilibrium and length bounds of object edges. Two propositions for obtaining the exect regions by using analytical method are proposed. Section 4 gives an algorithm for the fingertip forces corresponding to a finger position vector of the finger position regions obtained. In Section 5 numerical examples are performed to show the effectiveness of the proposed approach. Section 6 summarizes our contributions. The main feature of our proposed approach is that the finger position regions on grasped object are determined by using analytical method, and the obtained solution set of finger position regions is exact.

2 Graspable Edge Candidates

2.1 Force and Moment Equilibrium

The following discussion is performed in planar motion and based on two assumptions: (1) The object is a 2D polygon with definite geometric shape, (2) The fingertips of a hand touch the object by point contact with Coulomb friction and the contact point of fingertips on each edge is not more than one.

With respect to a coordinate frame Σ_o as shown in Fig.1, $\boldsymbol{f}_i \in \boldsymbol{R}^2$ is the applied force of ith finger, and the direction of $\boldsymbol{f}_i$ is toward the object. $\boldsymbol{e}_{i1} \in \boldsymbol{R}^2$ and $\boldsymbol{e}_{i2} \in \boldsymbol{R}^2$ are the edge vectors (toward the object) of the friction cone. k_{i1} and k_{i2} denote the magnitudes of the force $\boldsymbol{f}_i$ in $\boldsymbol{e}_{i1}$ and $\boldsymbol{e}_{i2}$ respectively. $\boldsymbol{f}_i$ can be described by the form

$$\boldsymbol{f}_i = k_{i1}\boldsymbol{e}_{i1} + k_{i2}\boldsymbol{e}_{i2}, \quad k_{i1}, k_{i2} \geq 0. \tag{1}$$

For a successful grasp with n fingers, the force equilibrium and moment equilibrium conditions

$$\sum_{i=1}^{n} \boldsymbol{f}_i = \sum_{i=1}^{n} (k_{i1}\boldsymbol{e}_{i1} + k_{i2}\boldsymbol{e}_{i2}) = \boldsymbol{0} \in \boldsymbol{R}^2, \ k_{i1}, \ k_{i2} \geq 0, \tag{2}$$

$$\sum_{i=1}^{n} \boldsymbol{r}_i \times \boldsymbol{f}_i = \sum_{i=1}^{n} \boldsymbol{r}_i \times (k_{i1}\boldsymbol{e}_{i1} + k_{i2}\boldsymbol{e}_{i2}) = 0 \in \boldsymbol{R}^1, \ k_{i1}, \ k_{i2} \geq 0. \tag{3}$$

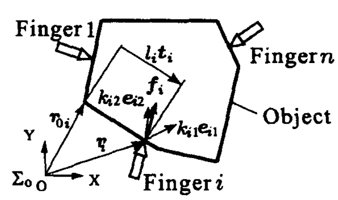

Figure 1: Fingertip forces and fingertip positions

must be satisfied, where $r_i \in R^2$ is the position vector of ith finger.

2.2 Selecting Edge Candidates

We will select candidates for the successful grasp from all combinations of the object edges by using the force equilibrium condition. The force equilibrium condition of eq.(2) for n fingers can be rewritten as

$$E_1 k = 0 \in R^2, \quad k \geq 0, \tag{4}$$

$$E_1 \triangleq [e_{11} \ e_{12} \ e_{21} \ e_{22} \ \cdots \ e_{n1} \ e_{n2}] \in R^{2\times 2n}, \tag{5}$$

$$k \triangleq [k_{11} \ k_{12} \ k_{21} \ k_{22} \ \cdots \ k_{n1} \ k_{n2}]^T \in R^{2n}. \tag{6}$$

In order to select the edge candidates, we have to solve the k in eq.(4). The solution set of k is convex polyhedral cone [6] and can be given as

$$k = H_1 \alpha = [h_{11}, \ h_{12}, \ \cdots, \ h_{1m}]\alpha, \quad \alpha \geq 0, \tag{7}$$

$$\alpha \triangleq \left[\alpha_1 \quad \alpha_2 \quad \cdots \quad \alpha_m \right]^T \in R^m, \tag{8}$$

where h_{1j}, $j = 1, 2, \cdots, m$ are span vectors of the polyhedral cone. α is a coefficient vector representing the component of k in h_{1j}, $j = 1, 2, \cdots, m$. If k does not exist, the combination of edges can be excluded. If k exists, the combination of edges will be one of graspable candidates. Consequently, the finger position regions for grasping are computed only for the selected candidates by using the moment equilibrium condition.

3 Graspable Finger Position Regions

3.1 Boundary Hyperplans of Finger Position Region

The fingertip position vector on ith edges can be described as

$$r_i = r_{0i} + l_i t_i, \quad i = 1, 2, \cdots, n, \tag{9}$$

where $r_{0i} \in R^2$ is a vertex position vector of ith edge, $t_i \in R^2$ the direction vector of the edge, and l_i the position variable (see Fig.1). When the length of ith edge is L_i, the bound of l_i is $0 < l_i < L_i$

For n edges touched by n fingers, let

$$l \triangleq [l_1 \ \ l_2 \ \ \cdots \ \ l_n]^T \in R^n \tag{10}$$

refer to a *Finger Position Vector*, whose bounds are

$$0 \leq l \leq L, \tag{11}$$

$$L \triangleq [L_1 \ \ L_2 \ \ \cdots \ \ L_n]^T \in R^n. \tag{12}$$

In this paper, the permissible region of l is called as *Graspable Finger Position Region* (GFPR hereafter) that meets the force equilibrium, the moment equilibrium and the edge length bounds for the stable grasp of a object.

Substituting eqs.(7) and (9) into eq.(3), the equation of variables l and α can be obtained as

$$(l^T A + b)k = (l^T A + b)H_1 \alpha = 0, \quad \alpha \geq 0, \tag{13}$$

where A and b are denoted as

$$A \triangleq \begin{bmatrix} [t_1 \otimes] & 0_{1\times 2} & \cdots & 0_{1\times 2} \\ 0_{1\times 2} & [t_2 \otimes] & \cdots & 0_{1\times 2} \\ \vdots & \vdots & \ddots & \vdots \\ 0_{1\times 2} & 0_{1\times 2} & \cdots & [t_n \otimes] \end{bmatrix} E_2 \in R^{n\times 2n}, \tag{14}$$

$$E_2 \triangleq \begin{bmatrix} e_{11} & e_{12} & 0_{2\times 1} & 0_{2\times 1} & \cdots & 0_{2\times 1} & 0_{2\times 1} \\ 0_{2\times 1} & 0_{2\times 1} & e_{21} & e_{22} & \cdots & 0_{2\times 1} & 0_{2\times 1} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0_{2\times 1} & 0_{2\times 1} & 0_{2\times 1} & 0_{2\times 1} & \cdots & e_{n1} & e_{n2} \end{bmatrix} \in R^{2n\times 2n}, \tag{15}$$

$$\begin{aligned} b &\triangleq \left[\ b_1 \ \ b_2 \ \cdots \ b_m \ \right] \\ &= \left[\ [r_{01}\otimes] \ \ [r_{02}\otimes] \ \cdots \ [r_{0n}\otimes] \ \right] E_2 \in R^{1\times 2n}, \end{aligned} \tag{16}$$

$$[t_i \otimes] = [-t_{iy} \ \ t_{ix}] \in R^{1\times 2}, \tag{17}$$

$$[r_{0i} \otimes] = [-r_{0iy} \ \ r_{0ix}] \in R^{1\times 2}. \tag{18}$$

Eq.(13) shows a nonlinear problem with respect to the variables l and α. To solve the problem linearly, we introduce boundary hyperplanes of l corresponding to the span vectors of polyhedral cone of eq.(7). Then, we derive an algorithm to determine the GFPRs using the boundary hyperplanes.

Eq.(13) represents a hyperplane of l for a given α. From eq.(7), the solution set of k is a convex polyhedral expressed by m span vectors h_{1j}, $j = 1, 2, \cdots, m$. For one span vector h_{1j} where $\alpha_j = 1$, $\alpha_s = 0$, $s = 1, 2, \cdots, m$, $s \neq j$, we can obtain one hyperplane P_j. For m span vectors of k, we can obtain m hyperplanes of l as

$$P_j = \{l | (l^T A + b)h_{1j} = 0\}, \quad j = 1, 2, \cdots, m. \tag{19}$$

Each P_j for span vector h_{1j} is a boundary hyperplane of the GFPRs and divides the space R^n into two hemispaces

$$P_j^+ = \{l | (l^T A + b)h_{1j} \geq 0\}, \quad j = 1, 2, \cdots, m, \tag{20}$$

$$P_j^- = \{l | (l^T A + b)h_{1j} \leq 0\}, \quad j = 1, 2, \cdots, m. \tag{21}$$

3.2 Graspable Finger Position Regions Formed by Two Hyperplanes

According to eq.(13), the graspable finger position vector l corresponding two span vectors h_{1q} and h_{1r} of k exists in the following set

$$W_{qr} = \{l | (l^T A + b)h_{1q}\alpha_q + (l^T A + b)h_{1r}\alpha_r = 0, \ \ \alpha_q, \alpha_r \geq 0\} \tag{22}$$

(see Fig.2). Eq.(22) shows that W_{qr} is the linear combination of $(l^T A + b)h_{1q}$ and $(l^T A + b)h_{1r}$ depending on α_q and α_r. Because of α_q, $\alpha_r \geq 0$, the set of l satisfying eq.(22) can be obtained by the following proposition.

Proposition 1: Corresponding to the region between the two span vectors h_q and h_r of k, the GFPR W_{qr} can be represented using the set of U_{qr} enclosed by boundary hyperplanes P_q and P_r, where U_{qr} is expressed as follows

$$U_{qr} = U_{qr}^1 \cup U_{qr}^2, \tag{23}$$

$$U_{qr}^1 = P_q^+ \cap P_r^-, \quad U_{qr}^2 = P_q^- \cap P_r^+. \tag{24}$$

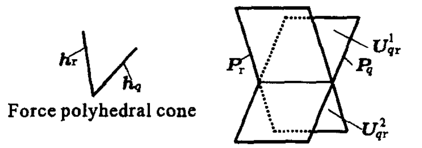

Figure 2: Force polyhedral cone and boundary hyperplanes

Proof [Necessary condition] In eq.(22), $\alpha_q,\ \alpha_r \geq 0$, thus an arbitrary $l_x \in W_{qr}$ must satisfy

$$(l_x^T A+b)h_{1q} \geq 0,\quad (l_x^T A+b)h_{1q} \leq 0, \tag{25}$$

or

$$(l_x^T A+b)h_{1q} \leq 0,\quad (l_x^T A+b)h_{1q} \geq 0. \tag{26}$$

According to eqs.(20), (21), (23) and (24), $l_x \in U_{qr}$ is obtained, so that we can conclude that

$$W_{qr} \subset U_{qr}. \tag{27}$$

[Sufficient condition] Eqs.(23) and (24) for an arbitrary $l_y \in U^1_{qr}$, give

$$(l_y^T A+b)h_{1q}-\gamma_q=0,\quad (l_y^T A+b)h_{1r}+\gamma_r=0,\quad \gamma_q,\gamma_r \geq 0. \tag{28}$$

Then, substituting eq.(28) into the equality in the right side of eq.(22), we have

$$\gamma^1_{qr}\alpha_{qr}=0,\qquad \alpha_{qr}=[\alpha_q\ \ \alpha_r]^T \geq \mathbf{0}, \tag{29}$$

$$\gamma^1_{qr} \triangleq [\gamma_q\quad -\gamma_r],\ \ \gamma_q,\ \gamma_r \geq 0. \tag{30}$$

According to eqs.(29) and (30) for an arbitrary pair of $\gamma_q,\ \gamma_r$, the $\alpha_{qr} \in \{\alpha_{qr} | \gamma^1_{qr}\alpha_{qr} = 0,\ \alpha_{qr} \geq \mathbf{0}\}$ exists to satisfy eq.(22), Thus, $l_y \in W_{qr}$ can be obtained. In the same way, $l_y \in W_{qr}$ can be obtained for an arbitrary $l_y \in U^2_{qr}$. Hence, we have

$$U_{qr} \subset W_{qr}. \tag{31}$$

According to eqs.(27) and (31), we can conclude that

$$U_{qr} = W_{qr}. \qquad \square \tag{32}$$

The length bounds of object edge are $\mathbf{0} \leq l \leq L$ from eq.(11). In the case of three fingers as shown in Fig.3, the length bounds of edges are represented as the rectangular parallelepiped. The length bounds divide the region bounded by two boundary hyperplanes P_q, P_r into two convex polyhedrons as the following:

$$V^1_{qr}=\{l|(l^TA+b)h_{1q}\geq 0,\ (l^TA+b)h_{1r}\leq 0,\ \mathbf{0}\leq l\leq L\}, \tag{33}$$

$$V^2_{qr}=\{l|(l^TA+b)h_{1q}\leq 0,\ (l^TA+b)h_{1r}\geq 0,\ \mathbf{0}\leq l\leq L\}. \tag{34}$$

Thus, the GFPR can be represented by

$$V_{qr} = V^1_{qr} \cup V^2_{qr}. \tag{35}$$

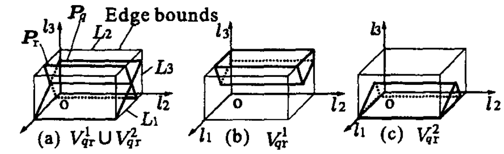

Figure 3: Two convex polyhedrons

3.3 Graspable Finger Position Regions Formed by m Hyperplanes

Now, we consider deriving a method for obtaining GFPRs formed by m hyperplanes. Substituting eq.(7) into eq.(13), for the set of k formed by m span vectors, the set of l can be represented by

$$W_l=\{l|\sum_{j=1}^{m}(l^TA+b)h_{1j}\alpha_j=0,\ \alpha_j\geq 0,\ j=1,2,\cdots,m\}. \tag{36}$$

From m span vectors of k, m hyperplanes of $P_j, j = 1,2,\cdots,m$ are obtained.

Proposition 2: For the set of k formed by m span vectors, the GFPRs W_l can be expressed by the union of all of the set U_{qr}

$$U_l = \bigcup_{q,r=1,q\neq r}^{m} (U^1_{qr} \cup U^2_{qr}), \tag{37}$$

where U_{qr} is formed by two arbitrary hyperplanes P_q and P_r, $q, r = 1, 2, \cdots, m,\ q \neq r$.

Proof [Necessary condition] From eq.(36) with $\alpha_j \geq 0,\ j = 1,2,\cdots,m$, we can see that for an arbitrary $l_x \in W_l$, the signs of two terms $(l_x^T A + b)h_{1q}$ and $(l_x^T A + b)h_{1r}$ in $(l_x^T A + b)h_{1j},\ j = 1,2,\cdots,m$, must be opposite, however the signs of the other terms $(l^T A + b)h_{1s},\ s = 1,\ 2,\cdots,\ m,\ s \neq q,\ s \neq r$ can be positive or negative. Thus, according to eqs.(20), (21), (24) and (37) the l_x exists in the sets

$$\bar{U}^1_{qr}=U^1_{qr}\cap\left(\bigcap_{s=1}^{m}(P^+_s\cup P^-_s)\right)=U^1_{qr},\quad s\neq q,\quad s\neq r, \tag{38}$$

or

$$\bar{U}^2_{qr}=U^2_{qr}\cap\left(\bigcap_{s=1}^{m}(P^+_s\cup P^-_s)\right)=U^2_{qr},\quad s\neq q,\quad s\neq r, \tag{39}$$

which denotes $l_x \in (\bar{U}^1_{qr} \cup \bar{U}^2_{qr}) = (U^1_{qr} \cup U^2_{qr})$. Thus, for all of the $q, r = 1, 2, \cdots, m,\ q \neq r$, we have

$$l_x \in \bigcup_{q,r=1,q\neq r}^{m} (U^1_{qr} \cup U^2_{qr}). \tag{40}$$

As eq.(40) stands up for an arbitrary $l_x \in W_l$, we can conclude that

$$W_l \subset U_l. \tag{41}$$

[Sufficient condition] According to eq.(37), for an arbitrary $l_y \in U_l$, in the case of $l_y \in U^1_{qr}$, we have

$$(l_y^T A+b)h_{1q}-\gamma_q=0,\ (l_y^T A+b)h_{1r}+\gamma_r=0,\ \gamma_q,\gamma_r\geq 0,$$
$$(l_y^T A+b)h_{1s}+\gamma_s=0,\ \gamma_s\geq 0 \text{ or } \gamma_s\leq 0,\ s=1,2,\cdots,m, \quad s\neq q,\ s\neq r, \tag{42}$$

and in the case of $l_y \in U^2_{qr}$, we have

$$(l_y^T A+b)h_{1q}+\gamma_q=0,\ (l_y^T A+b)h_{1r}-\gamma_r=0,\ \gamma_q,\gamma_r\geq 0,$$
$$(l_y^T A+b)h_{1s}+\gamma_s=0,\ \gamma_s\geq 0 \text{ or } \gamma_s\leq 0,\ s=1,2,\cdots,m, \quad s\neq q,\ s\neq r. \tag{43}$$

Substituting eq.(42) into the right equality of eq.(36), we have

$$\gamma_1 \alpha^T = 0, \quad \alpha \geq \mathbf{0}, \tag{44}$$
$$\gamma_1 \triangleq [\gamma_1 \ \cdots \ \gamma_q \ \cdots \ -\gamma_r \ \cdots \ \gamma_m], \tag{45}$$
$$\alpha \triangleq [\alpha_1 \ \cdots \ \alpha_q \ \cdots \alpha_r \ \cdots \ \alpha_m]^T. \tag{46}$$

Because of γ_q, $\gamma_r \geq 0$ of eq.(44), the $\alpha \in \{\alpha|\gamma_1\alpha^T = 0,\ \alpha\geq\mathbf{0}\}$ exists to satisfy eq.(36). Thus, $l_y \in U^1_{qr}$ means $l_y \in W_l$. Similarly $l_y \in U^2_{qr}$ means $l_y \in W_l$. Therefore, for all of the q, $r=1, 2, \cdots, m$, $q\neq r$, $l_y \in W_l$ can be obtained, so that

$$U_l \subset W_l. \tag{47}$$

According to eqs.(41) and (47), we can conclude that

$$U_l = W_l. \quad \square \tag{48}$$

Moreover, by considering the length bounds of object edges, from proposition 2 and eq.(35), the GFPRs are expressed as

$$V_l = \bigcup_{q,r=1,q\neq r}^{m} (V^1_{qr} \cup V^2_{qr}). \tag{49}$$

3.4 Computational Load

To determine the GFPRs on an object, at first, we use the force equilibrium condition to select the grasping candidates from combinations of the object edges. It is expected that the computational load can be reduced by this computing step. For a selected combination of edges, furthermore, we use the analytical method to determine the GFPRs exactly, where the computational load can be reduced significantly. The upper bound of computational load of the proposed approach for 2~4 fingers is evaluated by the number of multiplications and additions and shown in Table 1, where the computation load only depends on the number of fingers.

Literature [5] proposed an algorithm for computing the positions of fingertips. As shown in Fig.4, each of the finger forces acting on edges equals two forces acting on both vertices resultantly. The actual n finger positions are represented by λ_i, $0 \leq \lambda_i \leq 1$, $i = 1, 2, \cdots, n$. The maximum value and minimum value of λ_i can be

Table 1: Computational load of proposed approach

Finger number	Multiplications	Additions
$n=2$	4.736×10^3	1.740×10^3
$n=3$	4.878×10^5	1.832×10^5
$n=4$	1.184×10^7	4.441×10^6

Table 2: Computational load of method of [5] ($N \geq 2$)

Finger number	Multiplications	Additions
$n=2$	$3.780\ N \times 10^4$	$1.680\ N \times 10^4$
$n=3$	$4.010\ N^2 \times 10^5$	$1.782\ N^2 \times 10^5$
$n=4$	$1.966\ N^3 \times 10^6$	$8.736\ N^3 \times 10^5$

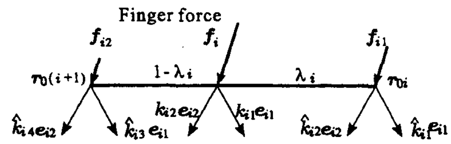

Figure 4: Composition and decomposition of f_i in [5]

solved by linear programming method. The range between the maximum value and minimum value of λ_i represents the range of ith finger position. However, the range of a finger position is dependent on the other finger positions. To obtain the whole of finger position regions, N sample points on each edge are needed to be selected. One finger is considered as the moving finger, the other fingers are touched fixedly at the selected sample points on different edges. By the algorithm in [5], the GFPR can be obtained approximately, and as the larger number of N is selected, the computational load will increase sharply with the growth of N. The upper bound of computational load of the algorithm not only depends on the number of fingers but also depends on the number of N. The upper bound of computation load of 2~4 fingers are shown in Table 2.

From Table 1 and Table 2, it can be seen that for the same combination of edges, the computational load of the proposed approach is evidently fewer than that of the algorithm from [5].

4 Determining Grasp Finger Forces

An element of GFPR V_l represents a finger position of a successful grasp. Now, we show how to obtain the finger forces for the finger position. According to eqs.(4) and (7), the finger forces of n fingers can be represented by

$$f = E_2 k = E_2 H_1 \alpha \in R^{2n}, \ \alpha \geq \mathbf{0}, \tag{50}$$
$$f \triangleq [f_1 \ f_2 \ \cdots f_n]^T, \tag{51}$$

where E_2 was given by eq.(15) and H_1 had been obtained by eq.(7). Note that the obtained set of f represents the possible forces where some f may not be permissible for grasping since the moment equilibrium condition is not considered.

The substitution of an l_f of V_l into the moment equilibrium condition of eq.(13), gives

$$(l_f^T A + b)H_1\alpha \;=\; 0, \quad \alpha \geq 0. \qquad (52)$$

Then, the α meets the moment equilibrium of eq.(52), and

$$\alpha = H_2\alpha_f = [h_{21},\ h_{22},\ \cdots,\ h_{2J}]\alpha_f \quad \alpha_f \geq 0, \qquad (53)$$

$$\alpha_f \triangleq \begin{bmatrix}\alpha_{f1} & \alpha_{f2} & \cdots & \alpha_{fJ}\end{bmatrix}^T \in R^J. \qquad (54)$$

can be obtained. Substituting the obtained α into eq.(50), corresponding finger forces

$$f_l \;=\; E_2H_1H_2\alpha_f \in R^{2n}, \quad \alpha_f \geq 0, \qquad (55)$$

can be obtained, where f_l is the successful grasp finger forces that meet the force equilibrium and the moment equilibrium. For an element of V_l, the set of f_l can be expressed as

$$F_l \;=\; \{E_2H_1H_2\alpha_f \;\mid\; \alpha_f \geq 0\}. \qquad (56)$$

5 Numerical examples

We will give numerical examples using the proposed approach to determine the GFPRs with 3 fingers.

For the object shown in Fig.5, the vertex positions of the object with respect to Σ_o are:

$$r_{o1}=\begin{bmatrix}2.000\\7.000\end{bmatrix},\ r_{o2}=\begin{bmatrix}2.000\\2.000\end{bmatrix},\ r_{o3}=\begin{bmatrix}13.000\\2.000\end{bmatrix},\ r_{o4}=\begin{bmatrix}8.000\\7.000\end{bmatrix}.$$

The direction vectors of the edges are:

$$t_1=\begin{bmatrix}0.000\\-1.000\end{bmatrix},\ t_2=\begin{bmatrix}1.000\\0.000\end{bmatrix},\ t_3=\begin{bmatrix}-0.707\\0.707\end{bmatrix},\ t_4=\begin{bmatrix}-1.000\\0.000\end{bmatrix}.$$

The lengths of the edges are:

$$L_1 = 5.000,\quad L_2 = 11.000,\quad L_3 = 7.071,\quad L_4 = 6.000.$$

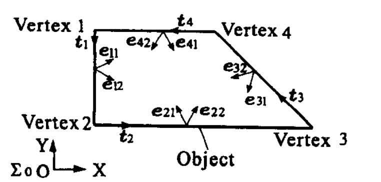

Figure 5: Polygonal object of example

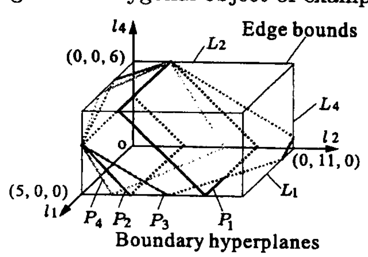

Figure 6: Boundary hyperplanes and edge bounds of three finger grasp

The coefficient of friction between the object and fingers is set as 0.5, so that we have

$$e_{11}=\begin{bmatrix}0.894\\-0.447\end{bmatrix},\ e_{12}=\begin{bmatrix}0.894\\0.447\end{bmatrix},\ e_{21}=\begin{bmatrix}0.447\\0.894\end{bmatrix},\ e_{22}=\begin{bmatrix}-0.447\\0.894\end{bmatrix},$$

$$e_{31}=\begin{bmatrix}-0.949\\-0.316\end{bmatrix},\ e_{32}=\begin{bmatrix}-0.316\\-0.949\end{bmatrix},\ e_{41}=\begin{bmatrix}-0.447\\-0.894\end{bmatrix},\ e_{42}=\begin{bmatrix}0.447\\-0.894\end{bmatrix}.$$

5.1 Selecting Edge Candidates

The combinations of edges are ${}_4C_3 = 4$ when the object is grasped by a robot hand with three fingers. According to eq.(7), all combinations of edges are the candidates. For example, for the combination of the edges 1-2-4, its k is given as

$$k \;=\; \begin{bmatrix}k_{11}\\k_{12}\\k_{21}\\k_{22}\\k_{41}\\k_{42}\end{bmatrix} = \begin{bmatrix}0.000 & 0.000 & 0.000 & 0.566\\ 0.000 & 0.000 & 0.566 & 0.000\\ 0.000 & 0.707 & 0.000 & 0.000\\ 0.707 & 0.000 & 0.424 & 0.707\\ 0.000 & 0.707 & 0.707 & 0.424\\ 0.707 & 0.000 & 0.000 & 0.000\end{bmatrix}\alpha,\ \alpha\geq 0, \qquad (57)$$

$$\alpha \;=\; \begin{bmatrix}\alpha_1 & \alpha_2 & \alpha_3 & \alpha_4\end{bmatrix}^T. \qquad (58)$$

5.2 Determining Finger Position Regions

For edges 1-2-4, corresponding to the four span vectors of k in eq.(57), four boundary hyperplanes are obtained from eq.(19) as follows:

$$P_1 \;=\; \{\ l \mid [0.000\ 0.894\ 0.894]l - 7.602 = 0\}, \qquad (59)$$

$$P_2 \;=\; \{\ l \mid [0.000\ 0.894\ 0.894]l - 3.130 = 0\}, \qquad (60)$$

$$P_3 \;=\; \{\ l \mid [0.716\ 0.537\ 0.894]l - 6.708 = 0\}, \qquad (61)$$

$$P_4 \;=\; \{\ l \mid [1.192\ 1.491\ 0.894]l - 9.091 = 0\}, \qquad (62)$$

where $l = [l_1\ \ l_2\ \ l_4]^T$ and shown in Fig.6. Taking into account $0 \leq l_i \leq L_i$, $i = 1, 2, 4$, and according to proposition 2 as well as eqs.(35) and (49), we can obtain convex polyhedrons V_{qr}^1 and V_{qr}^2, $(q, r = 1, 2, 3, 4,\ q \neq r)$, from each of both hyperplane combinations, which are

$$V_{12}^1 \;=\; \phi, \qquad (63)$$

$$V_{12}^2 \;=\; \{\ l \mid l = \begin{bmatrix}0.000 & 5.000 & 0.000 & 5.000 & 0.000\\ 3.500 & 3.500 & 0.000 & 0.000 & 8.500\\ 0.000 & 0.000 & 3.500 & 3.500 & 0.000\end{bmatrix}$$

$$\begin{bmatrix}5.000 & 0.000 & 5.000 & 0.000 & 5.000\\ 8.500 & 2.500 & 2.500 & 0.000 & 0.000\\ 0.000 & 6.000 & 6.000 & 6.000 & 6.000\end{bmatrix}\begin{bmatrix}\beta_1\\ \vdots\\ \beta_8\end{bmatrix},$$

$$\beta_1, \cdots, \beta_8 \geq 0, \quad \beta_1 + \cdots + \beta_8 = 1\}, \qquad (64)$$

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$$V_{34}^1 \;=\; \{\ l \mid l = \begin{bmatrix}0.000 & 5.000 & 3.125 & 1.875\\ 2.500 & 0.000 & 0.000 & 0.000\\ 6.000 & 3.500 & 6.000 & 6.000\end{bmatrix}\begin{bmatrix}\beta_1\\ \vdots\\ \beta_4\end{bmatrix},$$

$$\beta_1, \cdots, \beta_4 \geq 0, \quad \beta_1 + \cdots + \beta_4 = 1\}, \qquad (65)$$

$$V_{34}^2 \;=\; \{\ l \mid l = \begin{bmatrix}0.000 & 5.000 & 1.125 & 0.000\\ 2.500 & 0.000 & 11.000 & 6.098\\ 6.000 & 3.500 & 0.000 & 0.000\end{bmatrix}$$

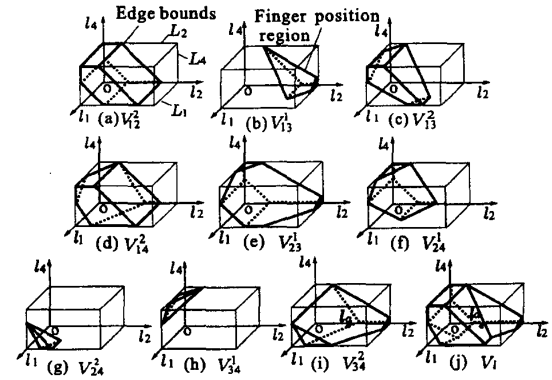

Figure 7: Graspable finger position regions for three fingers

$$\begin{bmatrix} 5.000 & 5.000 & 0.000 & 0.000 \\ 2.099 & 5.833 & 11.000 & 11.000 \\ 0.000 & 0.000 & 0.900 & 0.000 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \vdots \\ \beta_8 \end{bmatrix},$$

$$\beta_1, \cdots, \beta_8 \geq 0, \quad \beta_1 + \cdots + \beta_8 = 1\}. \tag{66}$$

The GFPR $\boldsymbol{V}_l$ therefore has the following form

$$\boldsymbol{V}_l = \bigcup_{q,r=1,q\neq r}^{4} (\boldsymbol{V}_{qr}^1 \cup \boldsymbol{V}_{qr}^2). \tag{67}$$

The regions of $\boldsymbol{V}_{qr}^1$ and $\boldsymbol{V}_{qr}^2$ $(q, r = 1,2,3,4,\ q \neq r)$ and $\boldsymbol{V}_l$ are illustrated in Fig.7, where $\boldsymbol{V}_{qr}^1$ and $\boldsymbol{V}_{qr}^2$, $(q, r = 1,2,3,4,\ q \neq r)$ are convex polyhedrons respectively, while the union set $\boldsymbol{V}_l$ is a polyhedron but not a convex one.

5.3 Determining Grasp Finger Forces

The $\boldsymbol{V}_l$ is the whole GFPR in finger position space. For an arbitrary element of the $\boldsymbol{V}_l$, the grasp finger force can be obtained correspondingly. For example, in $\boldsymbol{V}_{34}^2$ of eq.(66), when $\beta_1 = \beta_2 = \cdots = \beta_8 = 0.125$, the correspondingly finger positions $\boldsymbol{l}_a$ is

$$\boldsymbol{l}_a = [l_1 \ \ l_2 \ \ l_4]^T = [2.016 \ \ 6.191 \ \ 1.300]^T, \tag{68}$$

which is shown as Fig.7(i) and (j). According to eq.(55), the grasp finger force $\boldsymbol{f}_{l_a}$ for $\boldsymbol{l}_a$ is a polyhedral cone as

$$\boldsymbol{f}_{l_a} = \begin{bmatrix} \boldsymbol{f}_1 \\ \boldsymbol{f}_2 \\ \boldsymbol{f}_4 \end{bmatrix} = \begin{bmatrix} 0.528 & 0.157 & 0.436 & 0.000 \\ 0.127 & -0.078 & 0.218 & 0.000 \\ -0.232 & -0.340 & -0.104 & -0.202 \\ 0.464 & 0.679 & 0.446 & 0.678 \\ -0.295 & 0.182 & -0.332 & 0.202 \\ -0.591 & -0.600 & -0.665 & -0.675 \end{bmatrix} \boldsymbol{\alpha}_f, \tag{69}$$

$$\boldsymbol{\alpha}_f = \begin{bmatrix} \alpha_{f1} & \alpha_{f2} & \alpha_{f3} & \alpha_{f4} \end{bmatrix}^T, \ \boldsymbol{\alpha}_f \geq \boldsymbol{0}.$$

The finger forces for the edge span of $\boldsymbol{f}_{l_a}$ are shown in Fig.8. For Fig.8(a), (b) and (c), at least one finger force is located on the boundary of the friction cone respectively. For Fig.8(d), finger force $\boldsymbol{f}_1$ is zero, $\boldsymbol{f}_2$ and $\boldsymbol{f}_4$ meet the force equilibrium and the moment equilibrium and friction condition. Therefore, we can see that these span vectors are the boundaries of the set $\boldsymbol{f}_{l_a}$.

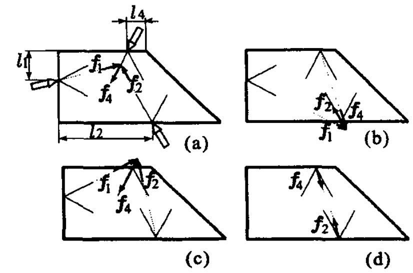

Figure 8: Fingertip forces for $\boldsymbol{l}_a$

6 Conclusion

We have presented an analytical method to determine candidates of graspable edges and graspable finger position regions on a given object.

At first, we used the force equilibrium condition to select graspable candidates from all of the combinations of the object edges. Then, for a selected candidate, the regions of graspable finger position was analyzed by using the moment equilibrium condition. It was shown that the region is bounded by plural boundary hyperplanes. Furthermore, with the combining these boundary hyperplanes, two propositions for exactly obtaining the Graspable Finger Position Region were proposed. Lastly, numerical examples were performed to show the effectiveness of the proposed approach.

The features of the analytical method presented in this paper are summarized as follows: (1) The graspable edge candidates can be selected firstly, and (2) For a selected combination of edges, we use the analytical method to determine the graspable finger position regions exactly, so that the computational load can be reduced significantly.

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