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15-451/651 Algorithm Design & Analysis, Spring 2026

Quiz 4 Solutions

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n foxes are each trying to acquire a burrow and a rabbit. Let B be the set of burrows and let R be the set of rabbits. The i-th fox only likes the subset $B_i \subseteq B$ of burrows and the subset $R_i \subseteq R$ of rabbits (these are given as input). We want to design a polynomial-time algorithm to assign burrows and rabbits to foxes so that:

1. A fox is only assigned a burrow or rabbit that they like.
2. Each burrow or rabbit is only assigned to at most one fox.
3. Maximize the number of foxes that receive both a burrow and a rabbit.

The algorithm should be based on running a maximum flow algorithm on a well-chosen graph.

Graph Construction We create a flow network with 3 "layers" (secretly 4 layers though due to the fox nodes being split):

Nodes:

- source node s
- sink node t
- burrow nodes b for all burrows in B
- rabbit nodes r for all rabbits in R
- fox nodes $f_1, \cdots, f_n$
- dummy fox nodes $f_1', \cdots, f_n'$

Edges with capacities:

- $s \to b_i$ with capacity 1 for each burrow i,
- $r_i \to t$ with capacity 1 for each rabbit i,
- $f_i \to f_i'$ with capacity 1 for all foxes i
- $b_j \to f_i$ for all admissible pairs of burrows and foxes, $b_j \in B_i$
- $f_i' \to r_j$ for all admissible pairs of foxes and rabbits, $r_j \in R_i$.

Building the Assignment Consider an integer flow on this graph. It breakd down into a list of capacity 1 paths from s to t.

The node splitting on middle fox layer guarantees that only 1 flow enters and leaves each fox.

The choices of edges from rabbits to foxes, and burrows to foxes, guarantee that the foxes are only assigned rabbits and burrows they like.

The capacity constraints on rabbits and burrows also imply that each rabbit and burrow is used at most once.

So listing the rabbit, fox, and burrows on each of the $s \to t$ paths gives the assignments.