Some Intuitions behind PCA

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1 PCA as optimization

$$\hat{\mathbf{x}}^t = \sum_{i=1}^{k} z_k^t \mathbf{u}_k$$

$$J = \frac{1}{N} \sum_{t=1}^{N} \|\mathbf{x}^t - \hat{\mathbf{x}}^t\|^2$$

$$Z = \begin{bmatrix} & | & \\ & | & \\ \dots & \mathbf{z}^t & \dots \\ & | & \\ & | & \end{bmatrix}$$

$$U = \begin{bmatrix} & \vdots & \\ - & - \; \mathbf{u}_k \; - & - \\ & \vdots & \end{bmatrix}$$

$$\begin{bmatrix} & | & \\ & | & \\ \dots & \mathbf{z}^t & \dots \\ & | & \\ & | & \end{bmatrix} \begin{bmatrix} & \vdots & \\ - & - \; \mathbf{u}_k \; - & - \\ & \vdots & \end{bmatrix} = \begin{bmatrix} & \vdots & \\ - & \hat{\mathbf{x}}^t & - \\ & \vdots & \end{bmatrix} = \hat{X}$$