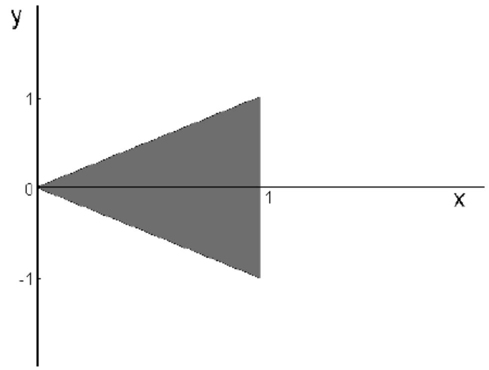
Midterm Exam Review (Part 2)

Xi Chen

Basic Probability (2003 Midterm Q 2, Spring 2005 Midterm Q1)

Suppose the joint Probability Density Function of a pair of random variables (x, y) is given by,

$p(x, y) = 1 \quad |y| < x, 0 < x < 1$
$p(x, y) = 0 \qquad$ otherwise



(a) *(4 points)* What is $p(y|x = 0.5)$?

(b) *(4 points)* Is x independent of y? (no explanation needed)

2. Which of the following statements are true ? If none of them are true, write NONE.

(a) If X and Y are independent then $E[2XY] = 2E[X]E[Y]$ and $Var[X + 2Y] = Var[X] + Var[Y]$.

(b) If X and Y are independent and $X > 1$ then $Var[X+2Y^2] = Var[X]+4Var[Y^2]$ and $E[X^2-X] \geq Var[X]$.

(c) If X are Y are not independent then $Var[X + Y] = Var[X] + Var[Y]$.

(d) If X and Y are independent then $E[XY^2] = E[X]E[Y]^2$ and $Var[X + Y] = Var[X] + Var[Y]$.

(e) If X and Y are not independent and $f(X) = X^2$ then $E[f(X)Y] = E[f(X)]E[Y]$ and $Var[X + 2Y] = Var[X] + 4Var[Y]$

Basic Probability (Spring 2005 Midterm Q1 & Q4)

3. You are playing a game with two coins. Coin 1 has a θ probability of heads. Coin 2 has a 2θ probability of heads. You flip these coins several times and record your results:

Coin	Result
1	Head
2	Tail
2	Tail
2	Tail
2	Head

(a) What is the log-likelihood of the data given θ ?

(b) What is the maximum likelihood estimate for θ ?

1. Consider the following joint distribution over the random variables A, B, and C.

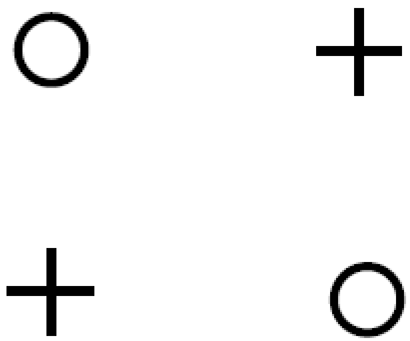
A	B	C	P(A,B,C)
0	0	0	1/8
0	1	0	1/8
0	0	1	1/8
0	1	1	1/8
1	0	0	1/8
1	1	0	1/8
1	0	1	1/8
1	1	1	1/8

(a) True or False: A is conditionally independent of B given C.

(b) If you answered part (a) with TRUE, make a change to the top two rows of this table to create a joint distribution in which the answer to (a) is FALSE.

If you answered part (a) with FALSE, make a change to the top two rows of this table to create a joint distribution in which the answer to (a) is TRUE.

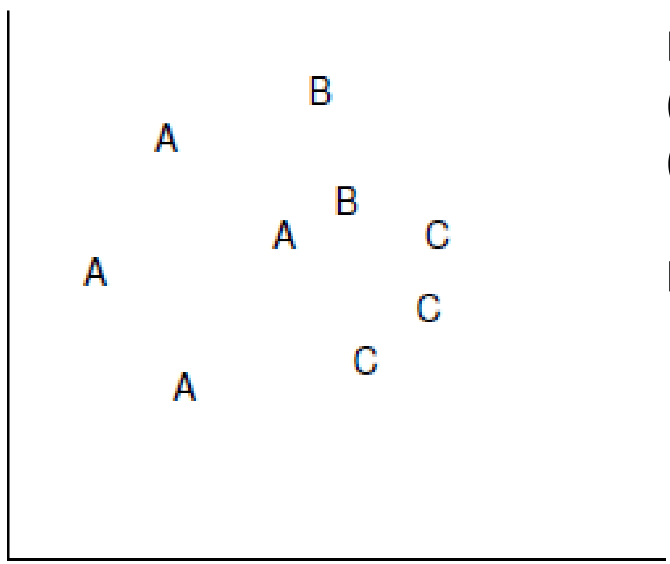
Decision Boundary (Fall 2010 Midterm Problem 1, Spring 2005 Problem 3)



Circle which of the classier that will achieve zero training error on this data set.

(a) Logistic regression

(b) Depth-2 ID3 decision trees



Decision Boundary for

(a) Logistic regression

(b) Decision trees

If we use leave one out training, which instance might be classified wrongly?

Bayes Networks (Fall 2008 Final Exam Problem 9)

Consider the Bayesian network shown in Figure 5. All the variables are boolean.

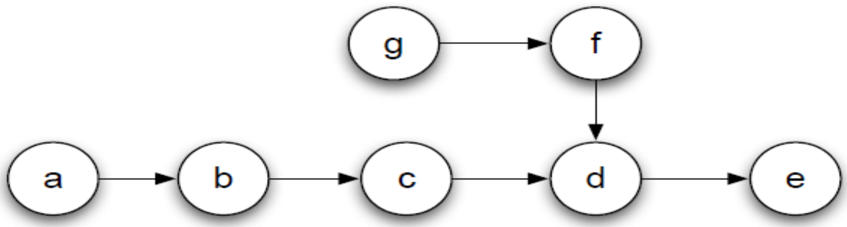


Figure 5: Bayesian network for Question 9.2 and 9.3

9.1 Likelihood

Write the expression for the joint likelihood of the network in its factored form. (2 points)

9.2 D-separation

1. Let $X = \{c\}, Y = \{b, d\}, Z = \{a, e, f, g\}$. Is $X \perp Z | Y$? If yes, explain why. If no, show a path from X to Z that is not blocked. (2 points)

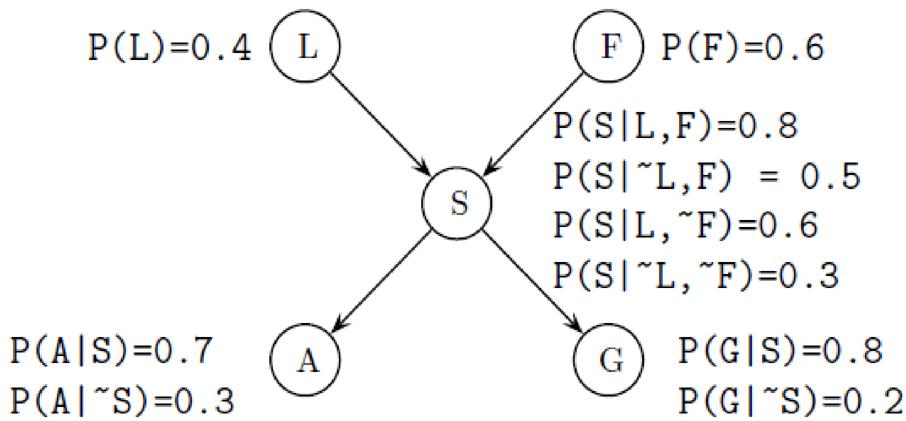
2. Suppose you are allowed to choose a set W such that $W \subset Z$. Then define $Z^* = Z/W$ and $Y^* = Y \cup W$. What is the smallest set W such that $X \perp Z^* | Y^*$ is true? (2 points)

Suppose that we do not know the directionality of the edges $a - b$ and $b - c$, and we are trying to learn that by observing the conditional probability $p(a|b, c)$. Some of the entries in the table are observed and noted. Fill in the rest of the conditional probability table so that we obtain the directionality that we see in the graph, i.e, $a \rightarrow b$ and $b \rightarrow c$. (2 points)

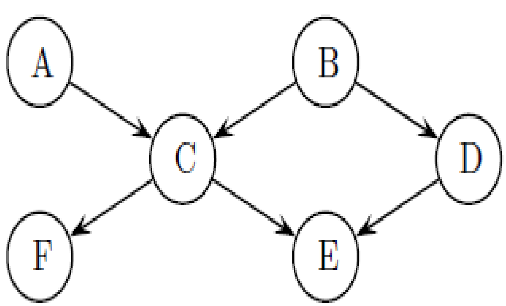
$P(a = 1 \mid b = 0, c = 0)$	0.8
$P(a = 1 \mid b = 0, c = 1)$	
$P(a = 1 \mid b = 1, c = 0)$	0.4
$P(a = 1 \mid b = 1, c = 1)$	

Bayes Networks (Fall 2003 Final Exam Problem 4 Part 2)

Inference. A student of the Machine Learning class notices that people driving SUVs (S) consume large amounts of gas (G) and are involved in more accidents than the national average (A). He also noticed that there are two types of people that drive SUVs: people from Pennsylvania (L) and people with large families (F). After collecting some statistics, he arrives at the following Bayesian network.



(c) What is $P(S)$? (d) What is $P(S|A)$?

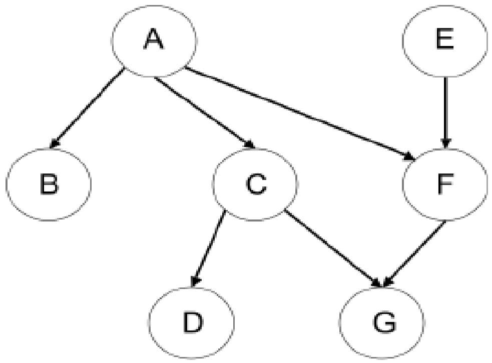


I<A,{},B>

I<A,{E},D>

I<A,{F},D>

Bayes Networks (Fall 2004 Final Exam Problem 7 Part 2)



(d) [**T/F** - (*1 pt*)] I$<A, \{\}, E>$

(e) [**T/F** - (*1 pt*)] I$<A, G, E>$

(f) [**T/F** - (*1 pt*)] I$<C, \{A, G\}, F>$

(g) [**T/F** - (*1 pt*)] I$<B, \{C, E\}, F>$

Gaussian Naive Bayes Classifier (Spring 2007 Midterm Problem 4)

In each of these datasets there are two classes, '+' and 'o'.

- Each class has the same number of points.
- Each data point has two real valued features, the X and Y coordinates.

Draw the decision boundary that a Gaussian Naive Bayes classifier will learn.