

PRACTICE EXERCISES - LAB 8 (Recursion)

1. We can define the sum from 1 to x (i.e. $1 + 2 + \dots + x$) recursively as follows for integer $x \geq 1$:

$$\begin{array}{ll} 1, & \text{if } x = 1 \\ x + \text{sum from 1 to } x-1 & \text{if } x > 1 \end{array}$$

Complete the following Python program to compute the sum $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10$ **recursively**:

```
def main():
    # compute and print 1 + 2 + ... + 10
    print sum(10)

def sum(x):
    # you complete this function recursively

main()
```

2. Revise the Fibonacci program so that it asks the user for which Fibonacci number he or she wants. Then use this value to instead of 6 in the program. Use your program to compute the 10th, 20th, 30th and 40th Fibonacci numbers. Why does it take so much longer to compute the higher Fibonacci numbers?

3. We can determine how many digits a positive integer has by repeatedly dividing by 10 (without keeping the remainder) until the number is less than 10, consisting of only 1 digit. We add 1 to this value for each time we divided by 10. Here is the recursive algorithm:

1. If $n < 10$ return 1.
2. Otherwise, return $1 + \text{the number of digits in } n/10$ (ignoring the fractional part).

(OVER)

Implement this recursive algorithm in Python and test it using a main function that calls this with the values 15, 105, and 15105. (HINT: Remember that if n is an integer, $n/10$ will be an integer without the fractional part.)

4. (HARDER) Write a **recursive** Python function that has a parameter representing a list of integers and returns the maximum stored in the list. Thinking recursively, the maximum is either the first value in the list or the maximum of the rest of the list, whichever is larger. If the list only has 1 integer, then its maximum is this single value, naturally.

Helpful Python syntax:

If A is a list of integers, and you want to set the list B to all of the integers in A except the first one, you can write

```
B = A[1:len(A)]
```

(This sets B to the integers in A starting at index 1 and ending at index $\text{len}(A)-1$, the last index. The integer in the first position of A at index 0 is not included.)