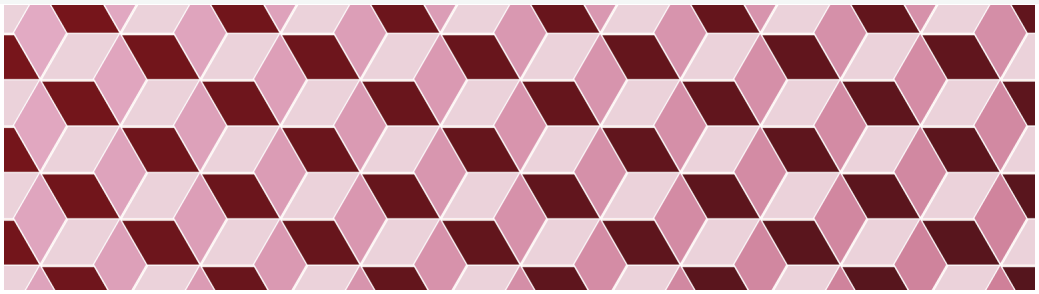


15-104 Introduction to Computing for Creative Practice

Fall 2021

32 Recursive Searching

Instructor: Tom Cortina, tcortina@cs.cmu.edu, GHC 4117, 412-268-3514

1

Linear Search Revisited

- Recall that we could use iteration (loops) to implement a linear search on an array of data elements:

```
// search for element in array arr starting at index
function lin_search(arr, element, index) {
    if (index < 0 || index >= arr.length) return -1;
    var i = index;
    while (i < arr.length) {
        if (arr[i] == element) return i;
        i++;
    }
    return -1;
}
```

2

Recursive Linear Search

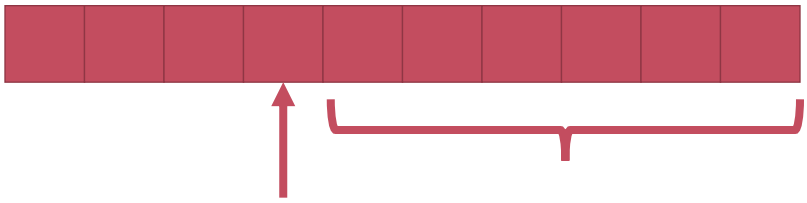
Searching for element in array arr starting at index:

- Is index out of range (i.e. less than 0 or at least arr.length)?
  - If so, return what?
    - -1 (can't possibly find it) — BASE CASE
- Is arr[index] the element we're looking for?
  - If so, return what?
    - index (that's where it is) — BASE CASE
- Otherwise, what do we do?
  - Return the result of a search for element in array arr starting at index+1. — RECURSIVE CASE

3

Another way to look at it

Searching for element in array arr starting at index:



index

If arr[index] is not the element,
then the result of a search starting at index+1
is also the answer to a search starting at index.

4

Recursive Linear Search

```
// search for element in array arr starting at index,
// recursively
function lin_search(arr, element, index) {
    if (index < 0 || index >= arr.length) {
        return -1;
    }
    if (arr[index] == element) {
        return index;
    }
    return lin_search(arr, element, index+1);
}
```

5

Trace

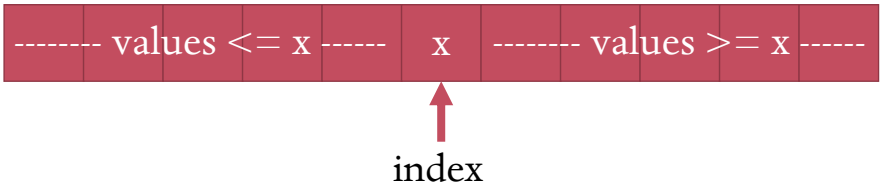
```
          0    1    2    3    4    5    6
arr = [13, 83, 23, 90, 54, 47, 61]

lin_search(arr, 90, 1)
     lin_search(arr, 90, 2)
          lin_search(arr, 90, 3)
          3
     3
3
```

6

A Faster Search: Binary Search

- What if we knew that the data in the array was sorted?
- We can take advantage of this fact to search much faster.
- General idea, look at the middle data value in the array:



index

If arr[index] = element, then we found it.
If element < arr[index], then it could only be in the left half if it is there at all. So search that half the same way.
If element > arr[index], then it could only be in the right half if it is there at all. So search that half the same way.

7

A Faster Search: Binary Search

- To search for an element in a SORTED array:
  - Find the index of the middle position of the array.
  - If the array has no elements, then return -1.
  - If the value at that index is the element we're looking for, then we're done. Return that index.
  - Otherwise:
    - If the element we're looking for is less than the value at the middle position, search the left half of the array recursively.
    - If the element we're looking for is greater than the value at the middle position, search the right half of the array recursively.

8

Recursive Binary Search

```
function bin_search(arr, element, lo, hi) {
    // search for element in arr between indices lo and hi
    if (lo > hi) {  return -1; }        // not found
    var mid = int((lo + hi) / 2);       // int in case even # of values
    if (arr[mid] == element) {
        return mid;         // found
    } else if (arr[mid] > element) {
        // return the result of a search of the left "half"
        return bin_search(arr, element, lo, mid - 1);
    } else if (arr[mid] < element) {
        // return the result of a search of the right "half"
        return bin_search(arr, element, mid + 1, hi);
    }
}
```

To start a search: bin_search(arr, element, 0, arr.length-1)

9

Trace

```
          0    1    2    3    4    5    6
arr = [13, 23, 47, 54, 61, 83, 90]            // sorted!

bin_search(arr, 61, 0, 6)
     lo = 0, hi = 6, mid = 3, arr[3] = 54 🙁
     bin_search(arr, 61, 4, 6)
          lo = 4, hi = 6, mid = 5, arr[5] = 83 🙁
          bin_search(arr, 61, 4, 4)
               lo = 4, hi = 4, mid = 4, arr[4] = 61 🙂
          4
     4
4
```

10

Trace

```
          0    1    2    3    4    5    6
arr = [13, 23, 47, 54, 61, 83, 90]                    // sorted!
bin_search(arr, 38, 0, 6)
     lo = 0, hi = 6, mid = 3, arr[3] = 54
     bin_search(arr, 38, 0, 2)
          lo = 0, hi = 2, mid = 1, arr[1] = 23
          bin_search(arr, 38, 2, 2)
               lo = 2, hi = 2, mid = 2, arr[2] = 47
                bin_search(arr, 38, 2, 1)
                -1
          -1
     -1
-1
```

11

Efficiency

If you have n elements, how many values do you have to examine in the worst case?

n	number of elements examined
1	1
3	2
7	3
15	4
31	5

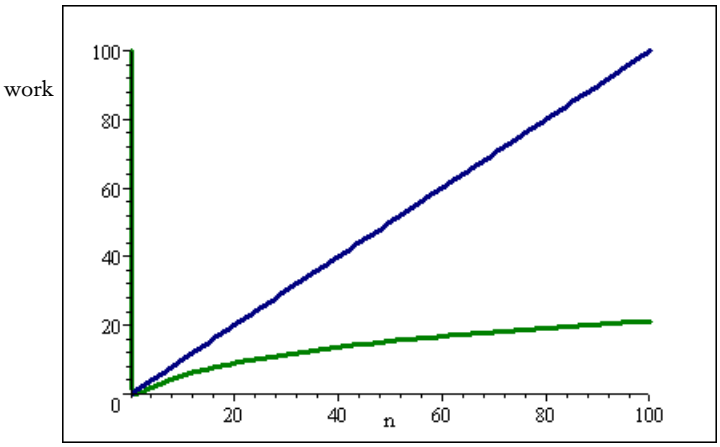
In general:

n	$\log_2 n$

If you double the number of elements, binary search requires only 1 more element examined!

12

Linear vs. Logarithmic



(source: Stack Overflow)

13

Big O

- Binary Search is an **O(log n)** algorithm.
- But it does require the data to be sorted before its use.
  (if the data needs to be sorted, this adds to the computational complexity of the search)
- If you have a sorted array with 1 million elements, you only need to examine about 20 elements!
  - Since 1 million $\cong 2^{20}$, $\log_2(2^{20}) = 20 \log_2(2) = 20$.
- If you have a sorted array with 1 billion elements, you only need to examine about 30 elements! WOW!
  - Since 1 billion $\cong 2^{30}$, $\log_2(2^{30}) = 30 \log_2(2) = 30$.

14