Revisiting Digitization, Robustness, and Decidability for Timed Automata

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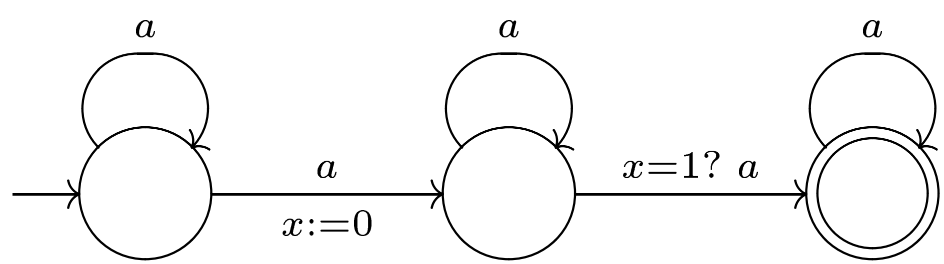
Timed Automata

- Untimed automata with clocks.
- Timed trace semantics: sequences of events with non-decreasing real-valued timestamps. E.g., $u = \langle (0.3, a), (2, b), (2, c), (3.1, a) \rangle$.

$$[\![A]\!] \mathrel{\widehat{=}} \text{ set of timed traces accepted by } A.$$

- Standard real-time modelling formalism.

A :



Shortcomings

- PSPACE-complete emptiness problem ($[\![A]\!] = \emptyset$?) (Alur-Dill 94).
- Undecidable universality problem ($[\![A]\!] = \mathbf{TT}$?) (idem).
- Excessive ‘precision’.

Various restrictions on timed automata proposed to remedy these points...

Digitization Techniques

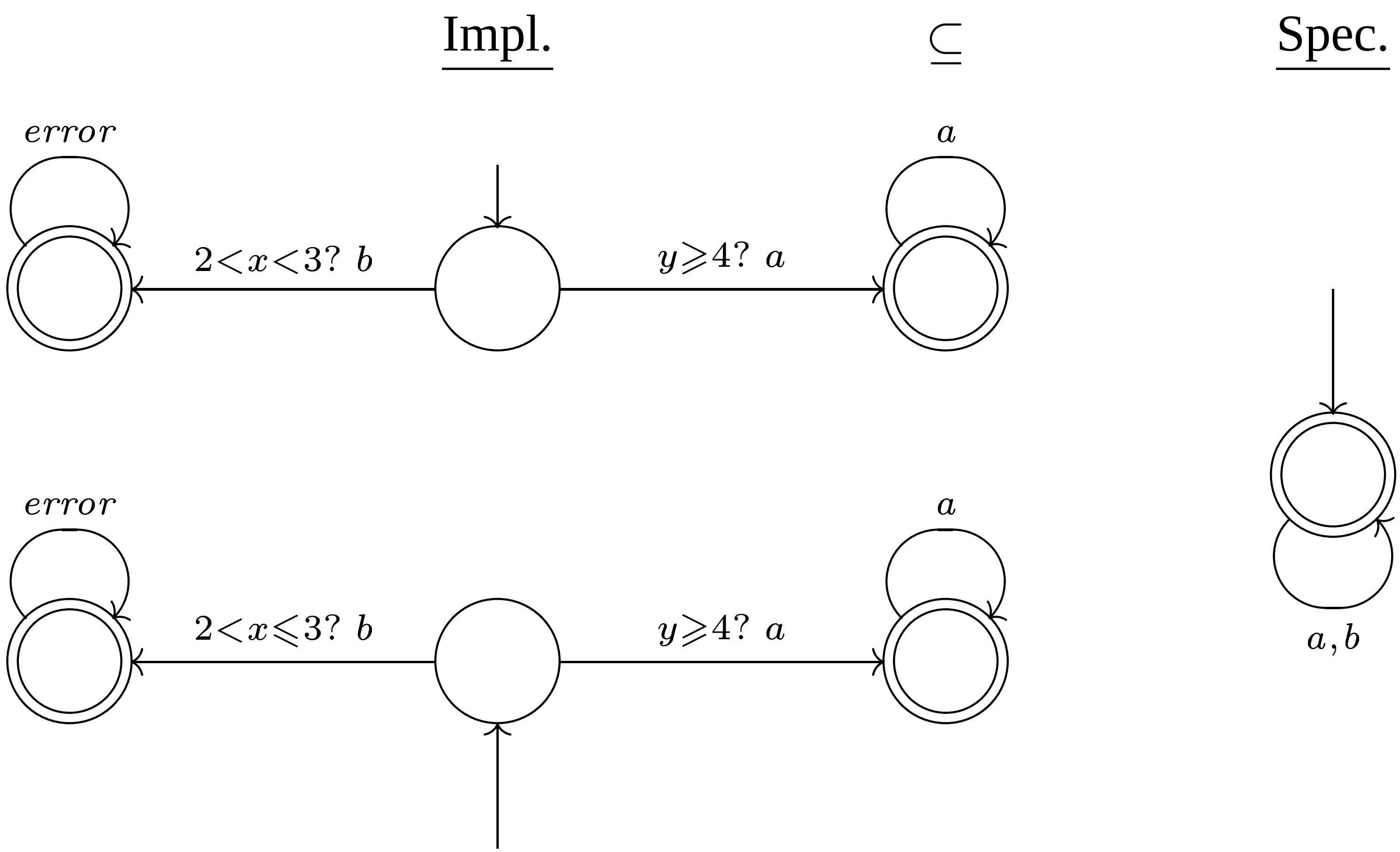
Introduced by Henzinger-Manna-Pnueli 92.

- **Under appropriate conditions**, reduce dense-time language inclusion problems to discrete time:

$$[\![A]\!] \subseteq [\![B]\!] \iff \mathbb{Z}[\![A]\!] \subseteq \mathbb{Z}[\![B]\!].$$

- Very successful and widespread. Useful in practice.

Digitization: An Example

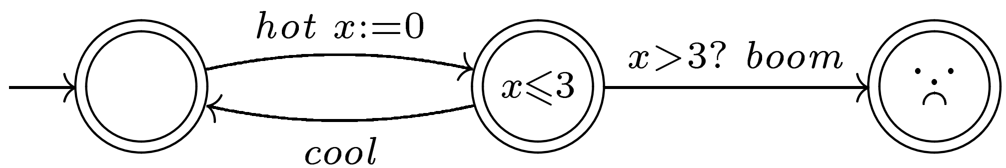


Digitization: Prerequisites

- **Prerequisites**: Implementation must be *closed under digitization*,
  Specification must be *closed under inverse digitization.*
- Closure under digitization is **decidable**.
- Closure under inverse digitization is **undecidable**.

Are Timed Automata Too Expressive?

Example: Nuclear meltdown if in ‘hot’ state for strictly longer than $3s$.
Is the following system safe?



- ‘Infinite precision’ of timed automata also originally blamed for undecidability of universality problem.
- Require ‘safety margins’: make timed automata **robust**.
- Robustness also vital for ensuring the soundness and convergence of numerical approximation tools.

Robust Timed Automata

- What is **robustness**? If $u \in [\![A]\!]$, then all timed traces 'sufficiently close' to u should also be in $[\![A]\!]$.
  (If a behaviour is 'safe', small perturbations of it should also be safe.)

- Robustness corresponds to the removal of equality testing:
  - 'Syntactic robustness' $\leadsto$ open timed automata.
  - 'Semantic robustness' $\leadsto$ robust timed automata (Gupta-Henzinger-Jagadeesan 97).

The d-Topology

$u = \langle (t_1, a_1), \ldots, (t_m, a_m) \rangle$, $u' = \langle (t'_1, a'_1), \ldots, (t'_n, a'_n) \rangle$.

$$d(u, u') = \infty, \text{ if } \langle a_1, \ldots, a_m \rangle \neq \langle a'_1, \ldots, a'_n \rangle,$$
$$d(u, u') = \max\{|t_i - t'_i| : 1 \leqslant i \leqslant m\}, \text{ if } \mathsf{untime}(u) = \mathsf{untime}(u').$$

Two traces are 'close' if they have the same sequence of events, occurring at neighbouring times.

(GHJ 97: All 'reasonable' metrics actually yield the same topology.)

The Robust Semantics for Timed Automata

A **tube** is a d-open set of timed traces.

The robust semantics assigns sets of tubes to timed automata, rather than sets of timed traces.

A tube u is accepted if $[\![A]\!]$ is dense in u.

- Tube-emptiness problem is decidable (Gupta-Henzinger-Jagadeesan 97).
- It was believed that tube-universality might be decidable. Eventually disproved (Henzinger-Raskin 00).
- Current understanding is that robust semantics yields roughly same theory as standard semantics (idem for hybrid automata). **Not so!**

Convert Robust Semantics to Timed Traces

Can equivalently capture the robust semantics by considering only the largest accepted tube:

$$\widetilde{[\![A]\!]} \mathrel{\widehat{=}} \left(\overline{[\![A]\!]}\right)^{\mathrm{int}} .$$

In this way, both $[\![A]\!]$ and $\widetilde{[\![A]\!]}$ are sets of timed traces, and can directly be compared.

Robust vs. Open Timed Automata

Open timed automata have only strict inequalities (e.g., $x < 3$ rather than $x \leqslant 3$) as clock constraints.

- Open timed automata: **Syntactic** removal of equality.
- Robust timed automata: **Semantic** removal of equality.

Both types of automata are 'acceptance-robust': whenever they accept a trace, they also accept all sufficiently close neighbouring traces.

– Are their respective expressive powers comparable?

Robust vs. Standard: Incomparable Expressive Powers

- There exists a timed automaton A such that, for every timed automaton B, $\widetilde{[\![A]\!]} \neq [\![B]\!]$.

- (Also: There exists an open timed automaton B such that, for every timed automaton A, $\widetilde{[\![A]\!]} \neq [\![B]\!]$.)

Universality

- Undecidablity of **robust universality problem** established by Henzinger-Raskin 00 (over **strongly** monotonic time).
  Universality of open timed automata left there as open question.
- Universality of open timed automata recently settled (OW 03):
  - **Undecidable** over **strongly** monotonic time.
  - **Decidable** over **weakly** monotonic time.

  Strongly monotonic: time strictly increasing — no two events have same timestamp.

  Weakly monotonic: time merely non-decreasing. Events can occur simultaneously.

Universality over Weakly Monotonic Time

Fact: open timed automata are closed under inverse digitization.

Universality: $\mathbf{TT} = [\![A]\!]? \iff \mathbf{TT} \subseteq [\![A]\!]? \iff \mathbb{Z}\mathbf{TT} \subseteq \mathbb{Z}[\![A]\!]?$

But $\mathbb{Z}[\![A]\!]$ is regular! Thus decidable.

Robust timed automata are also closed under inverse digitization. Thus

$$\mathbf{TT} = \widetilde{[\![A]\!]}? \iff \mathbf{TT} \subseteq \widetilde{[\![A]\!]}? \iff \mathbb{Z}\mathbf{TT} \subseteq \mathbb{Z}\widetilde{[\![A]\!]}?$$

Yet robust universality (over weakly monotonic time) turns out to be . . . **undecidable**! What is going on?

Discrete Robust Languages Are Non-Regular!

It turns out that $\mathbb{Z}\widetilde{[\![A]\!]}$ is (in general) **not regular**.

In particular, robust integral universality ($\mathbb{Z}\widetilde{[\![A]\!]} = \mathbb{Z}\mathbf{TT}$?) **undecidable**.

– Open question: is robust integral emptiness ($\mathbb{Z}\widetilde{[\![A]\!]} = \emptyset$?) decidable?

(Recall: robust emptiness ($\widetilde{[\![A]\!]} = \emptyset$?) is decidable.)

Summary

Digitization and **robustness** are important and well-studied topics.

- Closure under digitization **decidable**.
- Closure under inverse digitization **undecidable**.
- These two results **reversed** under the **robust semantics**.
- Expressive powers of robust and standard semantics **incomparable**.
- **Robust semantics much less tractable**: Undecidable (non-regular) discrete-time theory, contrary to standard semantics.
- Consequence: impossible to combine **digitization techniques** with **robust semantics**.
- Better introduce robustness explicitly — **syntactically**.
- Positive side: robust semantics is still **recursive**.

Future Work

- What about **hybrid** automata?
- Is robust integral emptiness decidable?