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Efficient BDD Representation for Reachability Analysis of Timed Automata

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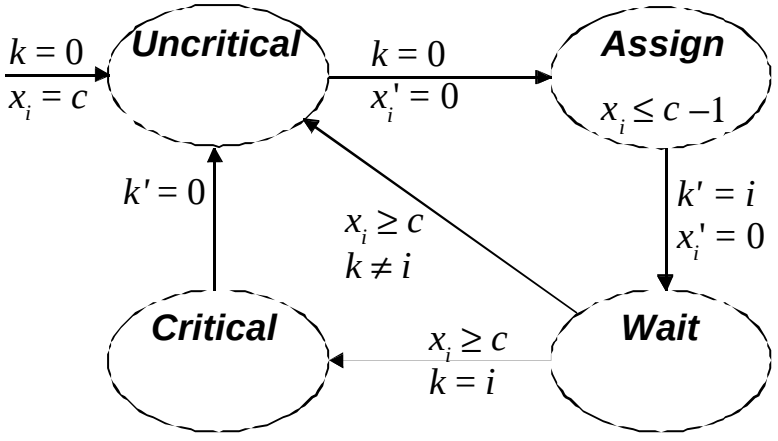
Problems of existing approaches

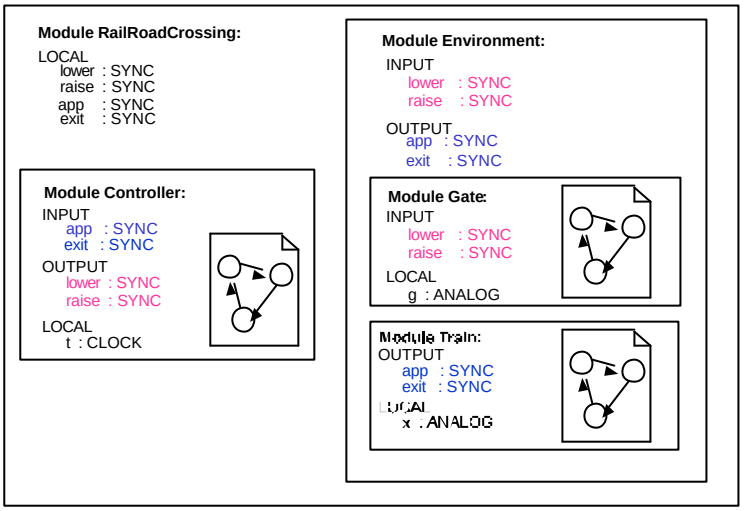
- **Modeling**
  - flat set of communicating automata
  - structure of system is lost in model
  - ➔ confusing, hard to understand, not modular
- **Verification**
  - explicit enumeration of discrete states, exponential
  - clock valuations are represented by matrices
  - non-convex sets require more than one matrices
  - CDDs are used, but without regarding variable ordering
  - ➔ inefficient, exponential effort
- **Case studies**
  - scalable benchmark examples with regular structure
  - few models with realistic, unregular structure
  - ➔ existing case studies to small

Example: Fischer's protocol

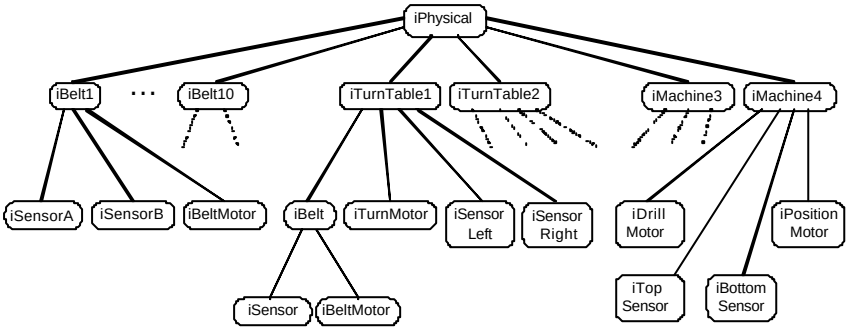


Timed Automata for process i

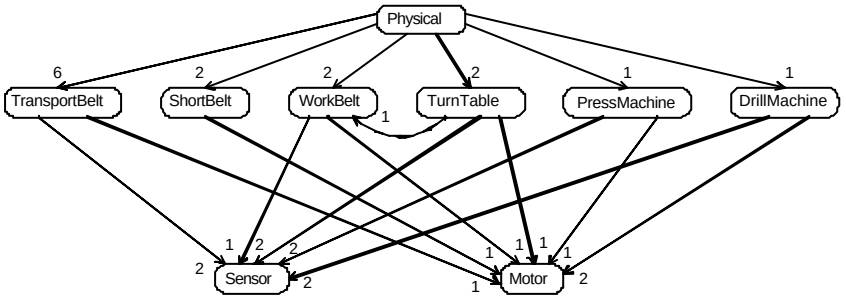
Modularity in Cottbus Timed Automata



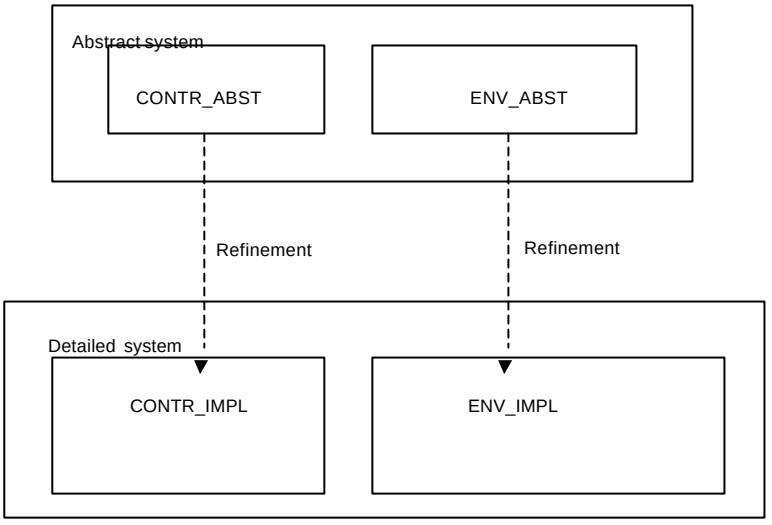
Hierarchy



Reuse of components



Refinement steps for large systems



Problems of existing approaches

- **Modeling** ✓
  - hierarchical model, abstraction layers
  - reuse of components, substitution of modules
  - ➔ structured, understandable, maintainable, because modular, useful for verification
- **Verification**
  - explicit enumeration of discrete states, exponential
  - clock valuations are represented by matrices
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Verification of real-time systems

- **Formalism for modular modeling**
  - Theoretical basis: timed and hybrid automata
  - Module concept
  - ➔ Good for modeling large systems
- **Reachability analysis using BDD representation**
  - Integer semantics
  - Estimate-based variable ordering
  - ➔ Very efficient
- **Refinement checking**
  - Simulation relation
  - ➔ Modular Proofs

Reachability analysis

- Verification of safety properties
  - ➔ Performance problems with existing tools
- 1st problem: explicit discrete states
  - ➔ BDD representation (own package)
- 2nd problem: separated clock representation
  - ➔ discrete TA-semantics, using also BDDs
- 3rd problem: variable orderings
  - ➔ heuristic using communication structure

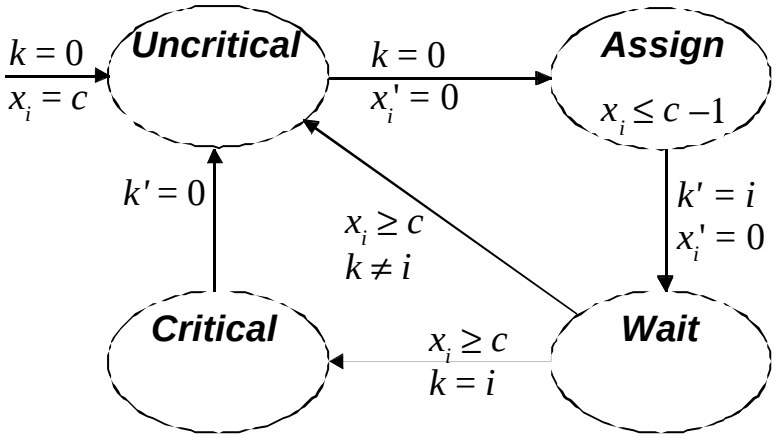
Contribution to efficiency

- Evaluation of our method for three examples (Fischer, FDDI, CSMA/CD)
- 1. there ***exists*** a variable ordering for polynomial size complexity
- 2. our tool is ***able to find*** such variable ordering automatically
- 3. empirical evidence
- industrial case study: the approach works
- the approach is applicable for DDs in general

Complexity results

Protocol	BDD size	CDD size (location-first)	CDD size (smallest)
Fischer	$\Theta(c^3 n^2 \lg c)$	$\Omega(2^n)$	$\Theta(n^3)$
CSMA/CD	$\Theta(n\ s^3 \lg\ l)$	$\Omega(3^n)$	$\Theta(n^2)$
Token ring FDDI	$\Theta(n^2\ ttrt^2 \lg\ ttrt) = \Theta(n^4 \lg n)$	$\Theta(n^2)$	$\Theta(n^2)$

Complexity analysis: Fischer's protocol

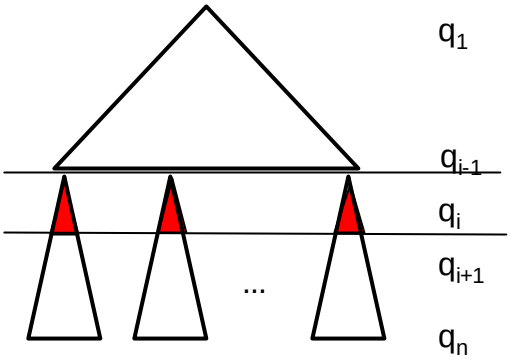


Timed Automata for process i

Communication graph

- General characteristics of „good“ variable orderings:
  - Communicating components have neighboring positions
  - Components which communicate with many other components at first
- Automatic ordering:
  - Estimation for the size of the BDD evaluates different variable orderings

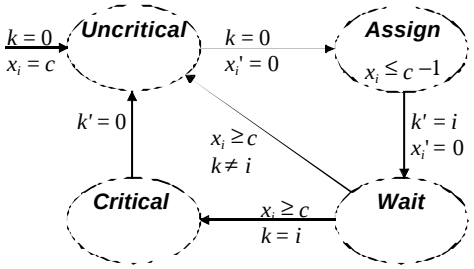
BDD structure and size estimation



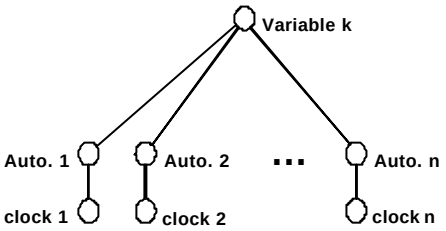
$$\sum_{i=1}^{n} \left(2^{|q_i|} - 1\right) \cdot \left(\prod_{k \in Comm_A(i)} |Q_k|\right)$$

Example: Fischer's mutex protocol

TA for process i:

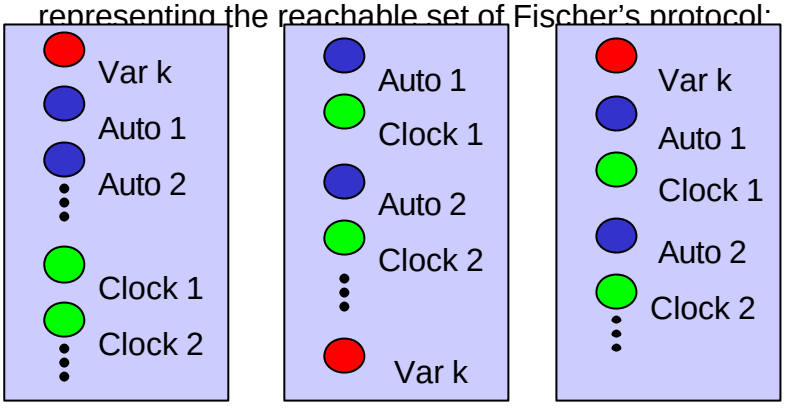


Communication graph:

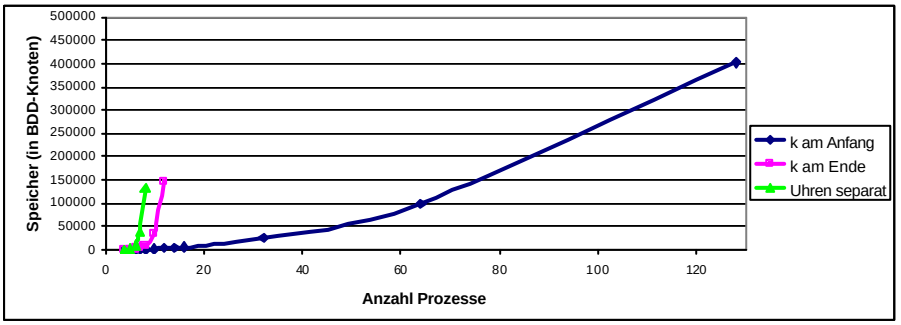


Different variable orderings

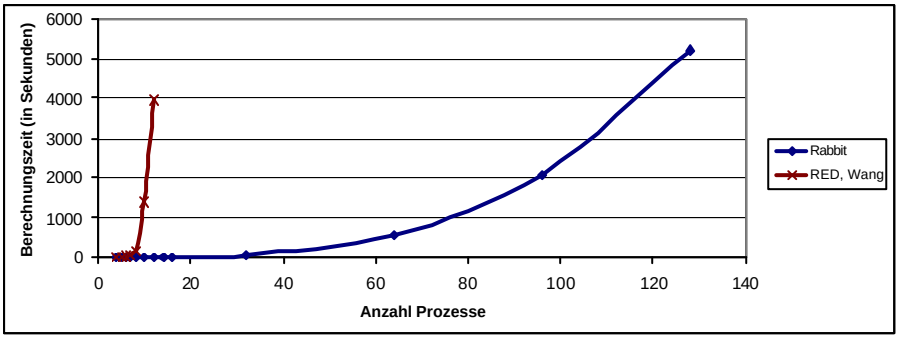
Consider 3 different variable orderings for the BDD representing the reachable set of Fischer's protocol:



Measurement: Fischer‘s protocol



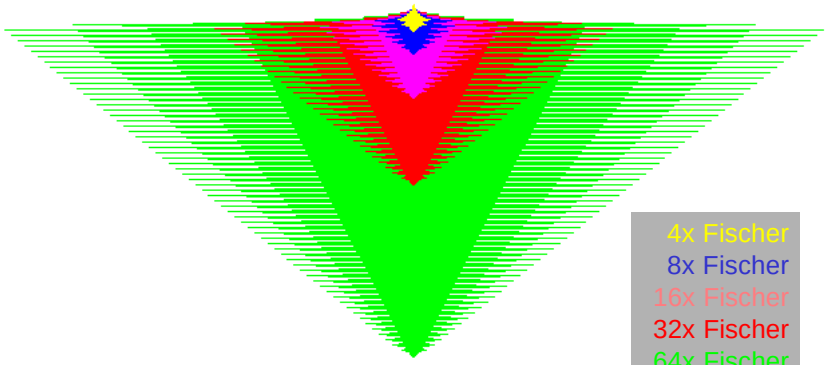
Comparison with other approaches



BDD for the reachable set

32x Fischer

BDDs for the reachable sets



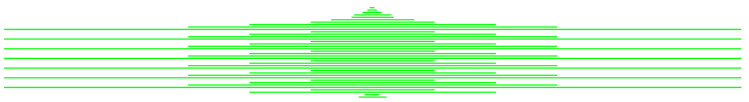
4x Fischer
8x Fischer
16x Fischer
32x Fischer
64x Fischer

BDDs for the reachable sets

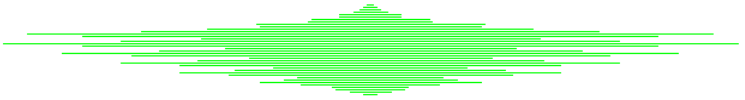
k in front versus k at end

AND 4

Estimate vs. real size



estimated BDD



real BDD

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  - ➔ structured, understandable, maintainable, because modular
- **Verification** ✓
  - unique symbolic representation of states AND clock valuations
  - proved integer semantics, proved upper bound for trans rel
  - method for complexity analysis, polynomial is possible
  - method for variable ordering, good orderings can be computed
  - ➔ efficient, polynomial effort for some examples
- **Case studies**
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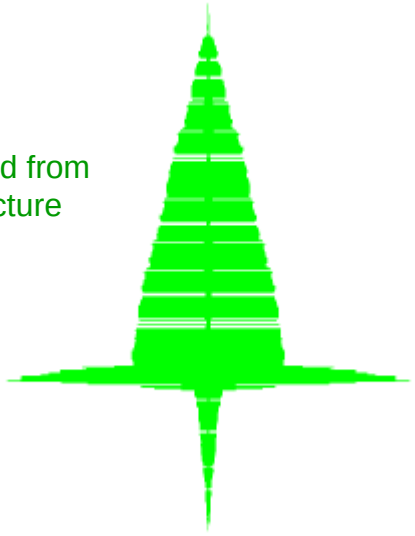
Video production cell

Complete production line example (1)

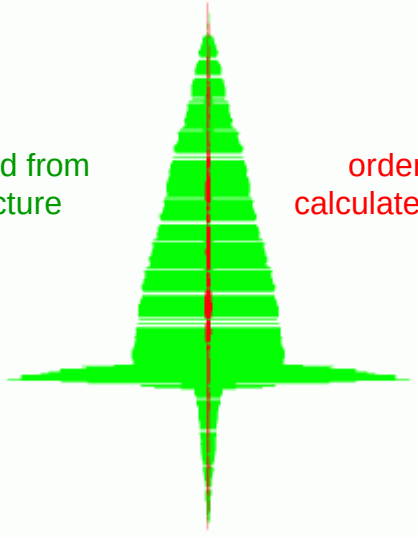
ordering derived from modular structure



Complete production line example (2)

ordering derived from modular structure

ordering based on calculated size estimates



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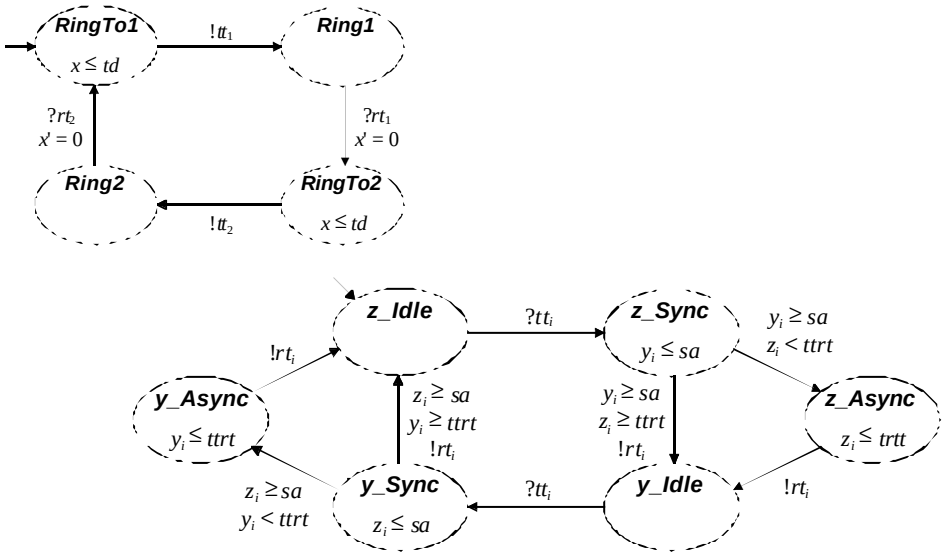
- **Case studies** ✓
  - scalable benchmark examples => polynomial effort
  - large production cell model => verifiable using modular structure
  - ➔ approach works, is relevant for practice, executable controller can be synthesized from proven model

Results

- **Modular formalism for modelling**
- **Efficient verification using BDD representation**
  - Complexity analysis of reach sets: good ordering ***exists***
  - Good variable orderings ***can be found*** by tool using modular structure of model
- **Tool framework for timed and hybrid automata**
  - Double Description Method for the hybrid case
  - Binary Decision Diagrams for the timed case
- **Different verification strategies for modular proofs**
  - Reachability analysis for safety properties
  - Refinement check via simulation relation
- **Several case studies:**
  - AND circuit, Fischer's protocol, CSMA/CD, FDDI,
  - Production cell, controller synthesis

FDDI protocol



CSMA/CD protocol