15-744: Computer Networking

L-14 Network Topology

Today's Lecture

- Structural generators
- Power laws
- HOT graphs
- Graph generators
- Assigned reading
  - On Power-Law Relationships of the Internet Topology
  - A First Principles Approach to Understanding the Internet's Router-level Topology

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Outline

- Motivation/Background

- Power Laws

- Optimization Models

- Graph Generation


Why study topology?

- Correctness of network protocols typically independent of topology
- Performance of networks critically dependent on topology
  - e.g., convergence of route information
- Internet impossible to replicate
- Modeling of topology needed to generate test topologies

Internet topologies

AT&T

MCI

SPRINT

AT&T

MCI   SPRINT

Router level

Autonomous System (AS) level
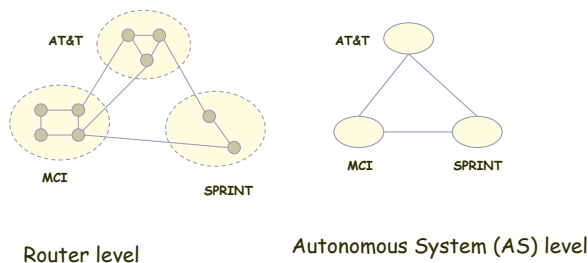
More on topologies..

- Router level topologies reflect physical connectivity between nodes
  - Inferred from tools like *traceroute* or well known public measurement projects like Mercator and Skitter
- AS graph reflects a peering relationship between two providers/clients
  - Inferred from inter-domain routers that run BGP and publlic projects like Oregon Route Views
- Inferring both is difficult, and often inaccurate

Hub-and-Spoke Topology

- Single hub node
  - Common in enterprise networks
  - Main location and satellite sites
  - Simple design and trivial routing
- Problems
  - Single point of failure
  - Bandwidth limitations
  - High delay between sites
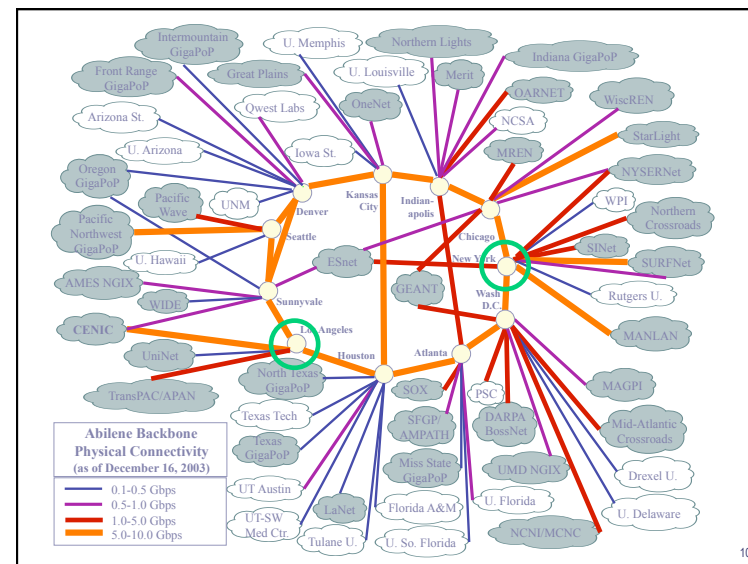 - Costs to backhaul to hub
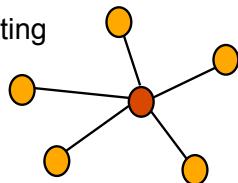
Simple Alternatives to Hub-and-Spoke

- Dual hub-and-spoke
  - Higher reliability
  - Higher cost
  - Good building block

- Levels of hierarchy
  - Reduce backhaul cost
  - Aggregate the bandwidth
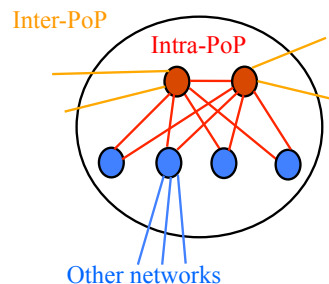 - Shorter site-to-site delay
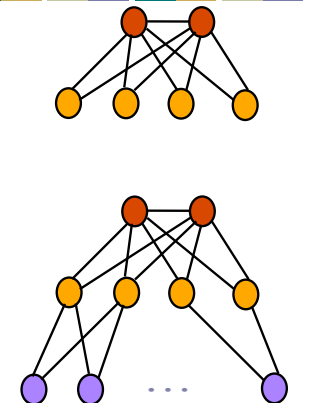
Abilene Internet2 Backbone

Points-of-Presence (PoPs)

- Inter-PoP links
  - Long distances
  - High bandwidth
- Intra-PoP links
  - Short cables between racks or floors
  - Aggregated bandwidth
- Links to other networks
  - Wide range of media and bandwidth

Inter-PoP

Intra-PoP

Other networks

Deciding Where to Locate Nodes and Links

- Placing Points-of-Presence (PoPs)
  - Large population of potential customers
  - Other providers or exchange points
  - Cost and availability of real-estate
  - Mostly in major metropolitan areas
- Placing links between PoPs
  - Already fiber in the ground
  - Needed to limit propagation delay
  - Needed to handle the traffic load

Trends in Topology Modeling

Observation	Modeling Approach
• Long-range links are expensive	• Random graph (Waxman88)
• Real networks are not random, but have obvious hierarchy	• Structural models (GT-ITM Calvert/Zegura, 1996)
• Internet topologies exhibit power law degree distributions (Faloutsos et al., 1999)	• Degree-based models replicate power-law degree sequences
• Physical networks have hard technological (and economic) constraints.	• Optimization-driven models topologies consistent with design tradeoffs of network engineers

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Waxman model (Waxman 1988)

- Router level model
- Nodes placed at random in 2-d space with dimension L
- Probability of edge (u,v):
  - $ae^{-d/(bL)}$, where d is Euclidean distance (u,v), a and b are constants
- Models locality

$u \quad d(u,v) \quad v$
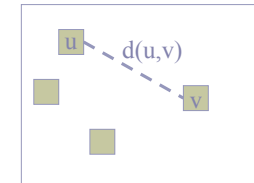
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Real world topologies

- Real networks exhibit
  - Hierarchical structure
  - Specialized nodes (transit, stub..)
  - Connectivity requirements
  - Redundancy
- Characteristics incorporated into the Georgia Tech Internetwork Topology Models (GT-ITM) simulator (E. Zegura, K.Calvert and M.J. Donahoo, 1995)

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Transit-stub model (Zegura 1997)

- Router level model
- Transit domains
  - placed in 2-d space
  - populated with routers
  - connected to each other
- Stub domains
  - placed in 2-d space
  - populated with routers
  - connected to transit domains
- Models hierarchy
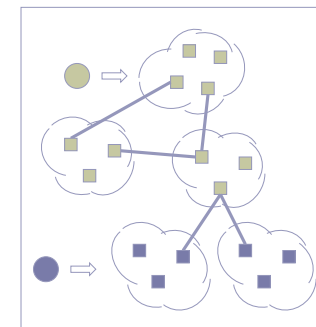
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So…are we done?

- No!
- In 1999, Faloutsos, Faloutsos and Faloutsos published a paper, demonstrating power law relationships in Internet graphs
- Specifically, the node degree distribution exhibited power laws
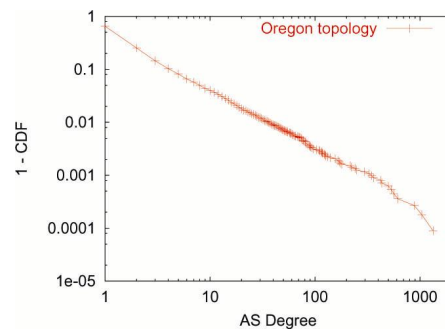
That Changed Everything…..

Outline

- Motivation/Background

- Power Laws

- Optimization Models

- Graph Generation

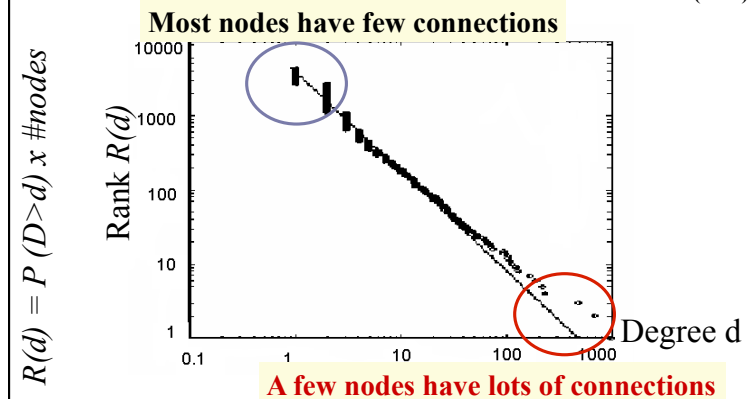## Power laws in AS level topology

Power Laws and Internet Topology

Source: Faloutsos et al. (1999)

Most nodes have few connections

$R(d) = P(D>d) \times \#nodes$

Rank $R(d)$

Degree d

A few nodes have lots of connections

- Router-level graph & Autonomous System (AS) graph
- Led to active research in *degree-based* network models

GT-ITM abandoned..

- GT-ITM did not give power law degree graphs
- New topology generators and explanation for power law degrees were sought
- Focus of generators to match degree distribution of observed graph

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Inet (Jin 2000)

- Generate degree sequence
- Build spanning tree over nodes with degree larger than 1, using preferential connectivity
  - randomly select node u not in tree
  - join u to existing node v with probability d(v)/Σd(w)
- Connect degree 1 nodes using preferential connectivity
- Add remaining edges using preferential connectivity
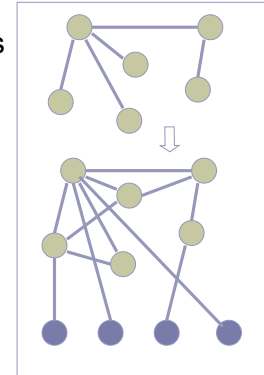
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Power law random graph (PLRG)

- Operations
  - assign degrees to nodes drawn from power law distribution
  - create kv copies of node v; kv degree of v.
  - randomly match nodes in pool
  - aggregate edges

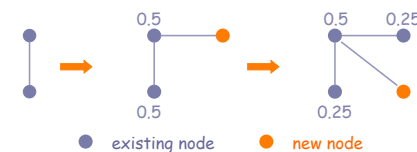may be disconnected, contain multiple edges, self-loops

- contains unique giant component for right choice of parameters

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Barabasi model: fixed exponent

- incremental growth
  - initially, m0 nodes
  - step: add new node i with m edges
- linear preferential attachment
  - connect to node i with probability $k_i / \sum k_j$

existing node   new node
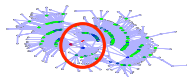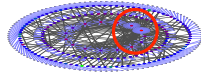
may contain multi-edges, self-loops

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Features of Degree-Based Models

Preferential Attachment   Expected Degree Sequence

- Degree sequence follows a power law (by construction)
- High-degree nodes correspond to highly connected central "hubs", which are crucial to the system
- Achilles' heel: robust to random failure, fragile to specific attack

Does Internet graph have these properties?

- No…(There is no Memphis!)
- Emphasis on degree distribution - structure ignored
- Real Internet very structured
- Evolution of graph is highly constrained

Problem With Power Law

- ... but they're descriptive models!

- No correct physical explanation, need an understanding of:
  - the driving force behind deployment
  - the driving force behind growth

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Li et al.

- Consider the explicit design of the Internet
  - Annotated network graphs (capacity, bandwidth)
  - Technological and economic limitations
  - Network performance
- Seek a theory for Internet topology that is explanatory and not merely descriptive.
  - Explain high variability in network connectivity
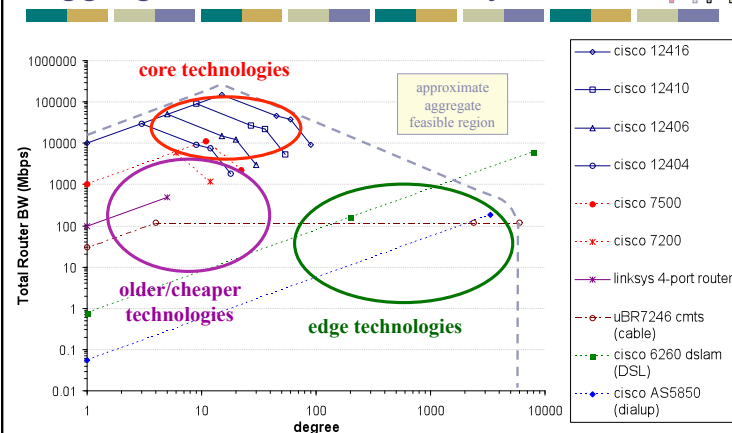 - Ability to match large scale statistics (e.g. power laws) is only secondary evidence
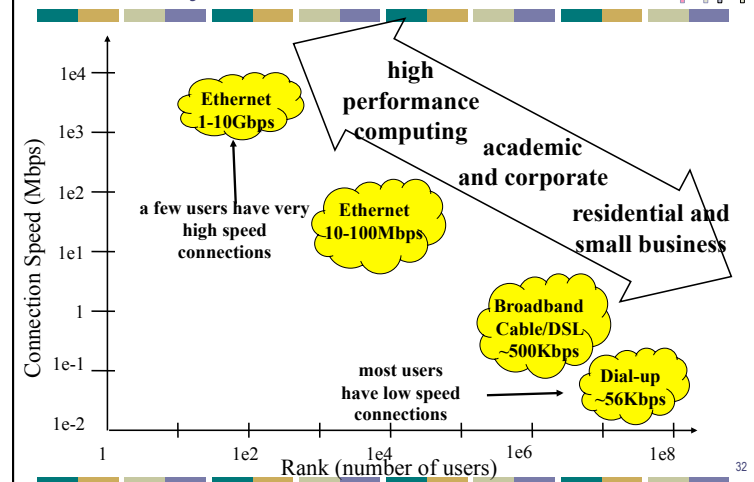
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Router Technology Constraint
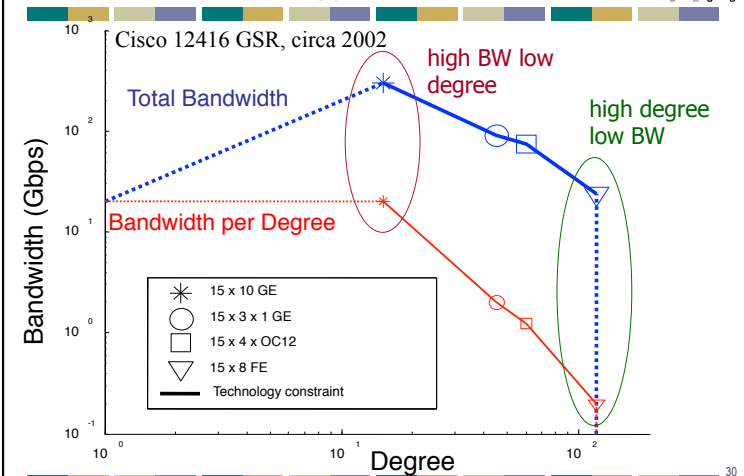
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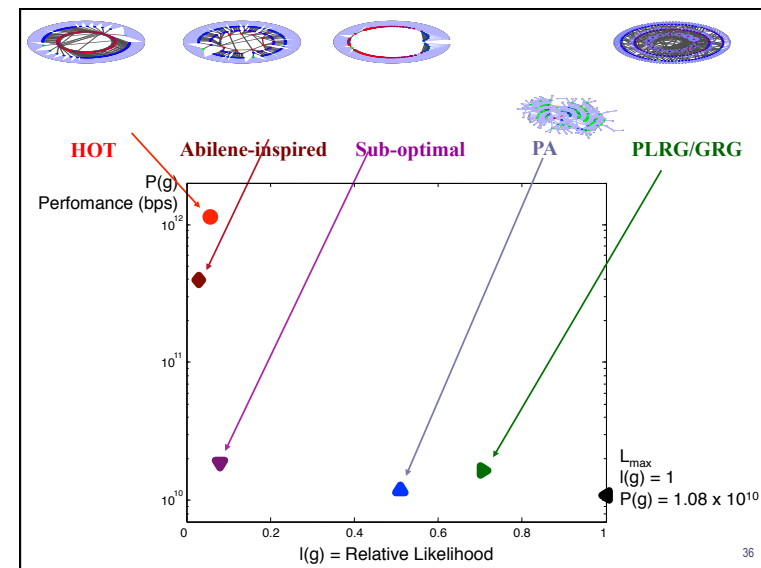## Aggregate Router Feasibility

Source: Cisco Product Catalog, June 2002

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Variability in End-User Bandwidths
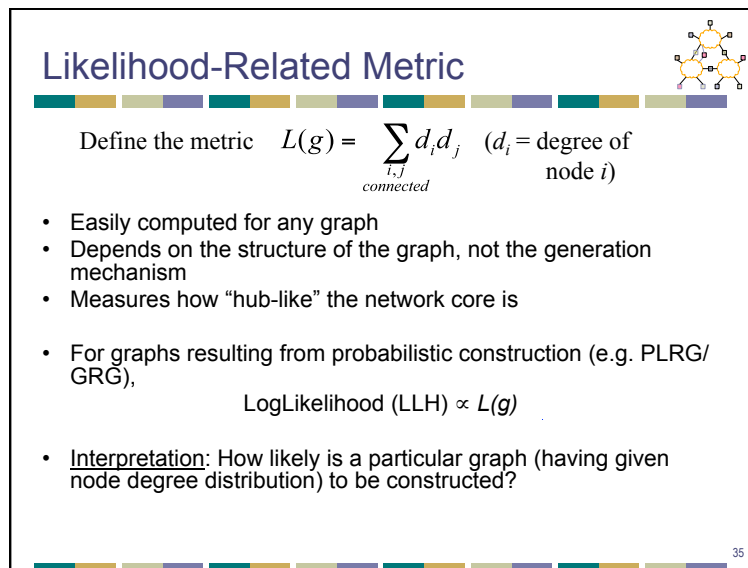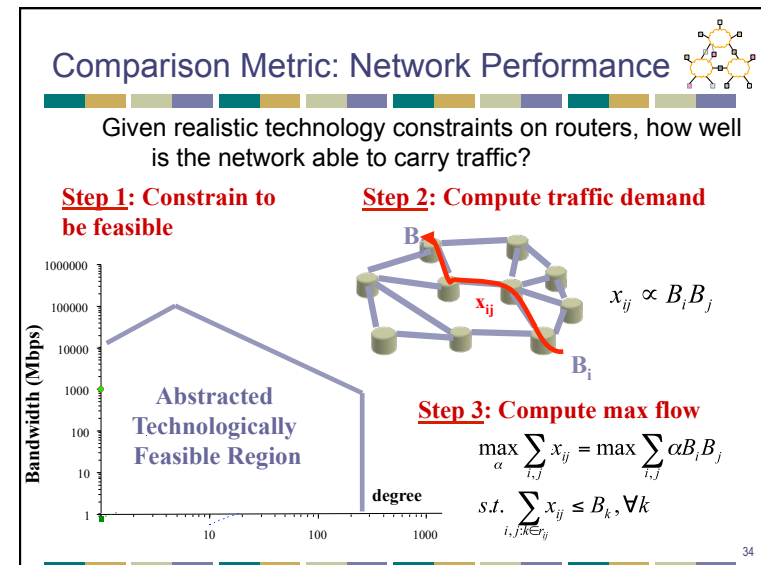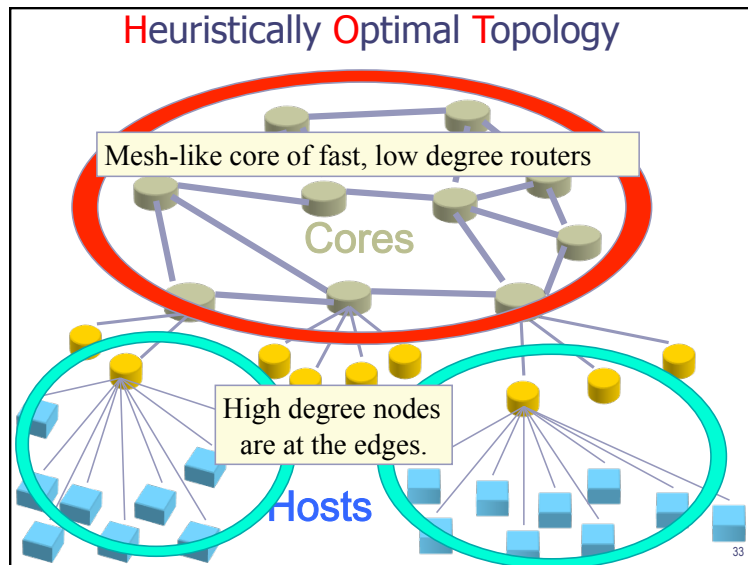

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Slide 33

Heuristically Optimal Topology

Mesh-like core of fast, low degree routers

Cores

High degree nodes are at the edges.

Hosts

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Slide 34

Comparison Metric: Network Performance

Given realistic technology constraints on routers, how well is the network able to carry traffic?

Step 1: Constrain to be feasible

Bandwidth (Mbps)

Abstracted Technologically Feasible Region

degree

Step 2: Compute traffic demand

$$x_{ij} \propto B_i B_j$$

$\mathbf{B}$
$\mathbf{x_{ij}}$
$\mathbf{B_i}$

Step 3: Compute max flow

$$\max_{\alpha} \sum_{i,j} x_{ij} = \max \sum_{i,j} \alpha B_i B_j$$

$$s.t. \sum_{i,j:k \in r_{ij}} x_{ij} \le B_k, \forall k$$

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Slide 35

Likelihood-Related Metric

Define the metric
$$L(g) = \sum_{\substack{i,j \\ connected}} d_i d_j \quad (d_i = \text{degree of node } i)$$

- Easily computed for any graph
- Depends on the structure of the graph, not the generation mechanism
- Measures how "hub-like" the network core is

- For graphs resulting from probabilistic construction (e.g. PLRG/GRG),

  LogLikelihood (LLH) $\propto L(g)$

- <u>Interpretation</u>: How likely is a particular graph (having given node degree distribution) to be constructed?

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Slide 36

HOT   **Abilene-inspired**   **Sub-optimal**   **PA**   **PLRG/GRG**

P(g)
Perfomance (bps)

10^{12}

10^{11}

10^{10}

L_{max}
$l(g) = 1$
$P(g) = 1.08 \times 10^{10}$

0   0.2   0.4   0.6   0.8   1

$l(g)$ = Relative Likelihood

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Structure Determines Performance

HOT   **PA**   **PLRG/GRG**

$P(g) = 1.13 \times 10^{12}$ $P(g) = 1.19 \times 10^{10}$ $P(g) = 1.64 \times 10^{10}$
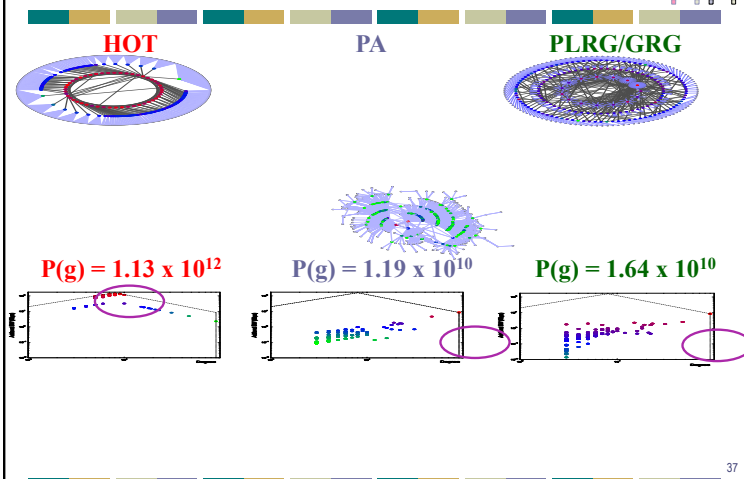
Summary Network Topology

- Faloutsos[3] [SIGCOMM99] on Internet topology
  - Observed many "power laws" in the Internet structure
    - Router level connections, AS-level connections, neighborhood sizes
  - Power law observation refuted later, Lakhina [INFOCOM00]

- Inspired many degree-based topology generators
  - Compared properties of generated graphs with those of measured graphs to validate generator
  - What is wrong with these topologies? Li et al [SIGCOMM04]
    - Many graphs with similar distribution have different properties
    - Random graph generation models don't have network-intrinsic meaning
    - Should look at fundamental trade-offs to understand topology
      - Technology constraints and economic trade-offs
    - Graphs arising out of such generation better explain topology and its properties, but are unlikely to be generated by random processes!

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Graph Generation

- Many important topology metrics
  - Spectrum
  - Distance distribution
  - Degree distribution
  - Clustering…
- No way to reproduce most of the important metrics
- No guarantee there will not be any other/ new metric found important

dK-series approach

- Look at inter-dependencies among topology characteristics
- See if by reproducing most basic, simple, but not necessarily practically relevant characteristics, we can also reproduce (capture) all other characteristics, including practically important
- Try to find the one(s) defining *all others*

0K

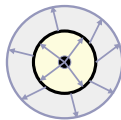Average degree *<k>*

1K

Degree distribution *P(k)*

2K

Joint degree distribution *P(k_1,k_2)*

3K

"Joint edge degree" distribution $P(k_1,k_2,k_3)$

3K, more exactly

Wedges:
$P_\wedge(k_1,k_2,k_3)$

Triangles:
$P_\Delta(k_1,k_2,k_3)$

4K

$P_1(k_1,k_2,k_3,k_4)$ $P_2(k_1,k_2,k_3,k_4)$ $P_3(k_1,k_2,k_3,k_4)$

$P_4(k_1,k_2,k_3,k_4)$ $P_5(k_1,k_2,k_3,k_4)$ $P_6(k_1,k_2,k_3,k_4)$
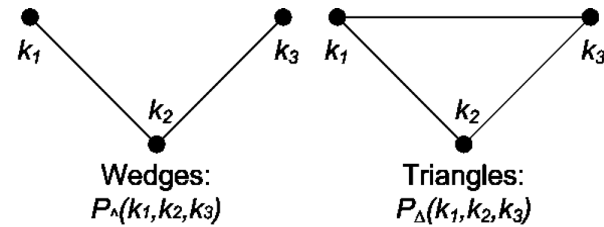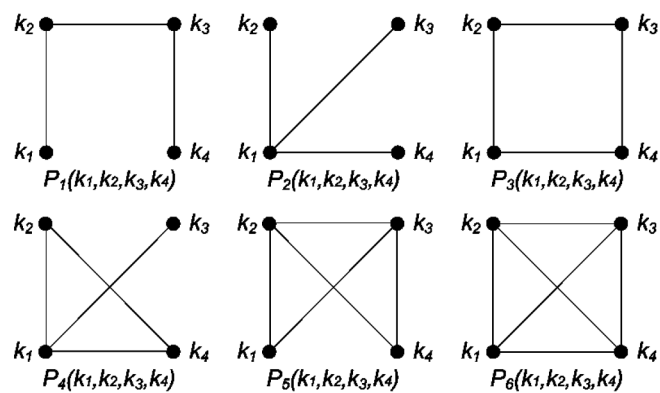
Definition of *dK*-distributions

dK-distributions are degree correlations within simple connected graphs of size *d*

Nice properties of properties P_d

- **Constructability**: we can construct graphs having properties P_d (*dK*-graphs)
- **Inclusion**: if a graph has property P_d, then it also has all properties P_i, with $i < d$ (*dK*-graphs are also *iK*-graphs)
- **Convergence**: the set of graphs having property P_n consists only of one element, *G* itself (*dK*-graphs converge to *G*)

Rewiring

Graph Reproduction

(a) 0K-graph

(b) 1K-graph

(c) 2K-graph

(d) 3K-graph

(e) original HOT graph
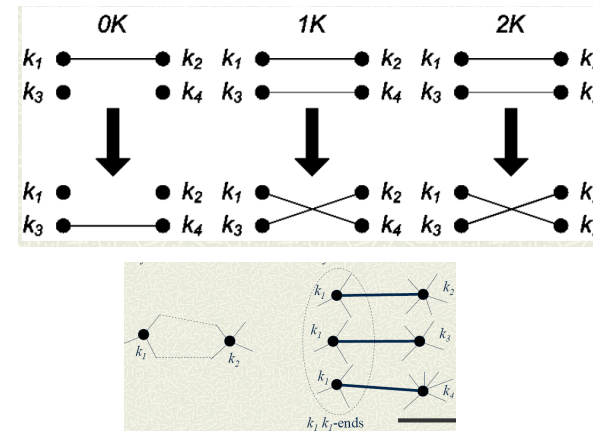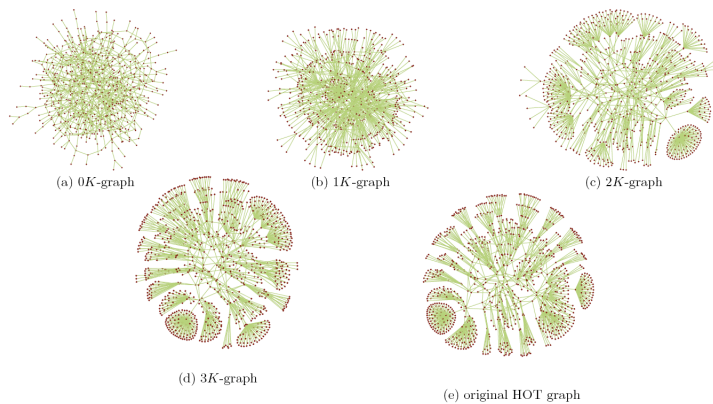
The elephant in the room…

- How good is the underlying data on which these studies are based?
- Impact
  - Sampling bias → traceroute of shortest paths on random graph can produce power-law distribution [Lakhina03]
  - Similar issues with AS-level views

Better Measurements?

- Rocketfuel [sigcomm02]
  - Better router alias resolution
  - Detailed maps based on multiple viewpoints
- Reverse traceroute [nsdi10]
  - Improves ability to view peer-to-peer links
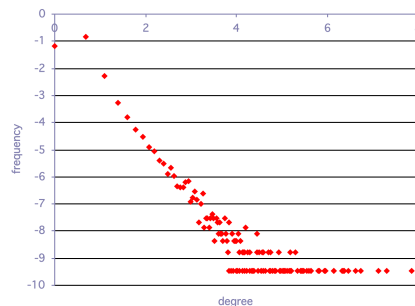- RouteViews and BGP collection efforts

Next Lecture

- Overlay networks
- Challenges in deploying new protocols
- Required readings:
  - Active network vision and reality: lessons from a capsule-based system
- Optional readings:
  - Resilient Overlay Networks
  - Future Internet Architecture: Clean-Slate Versus Evolutionary Research

Power Laws
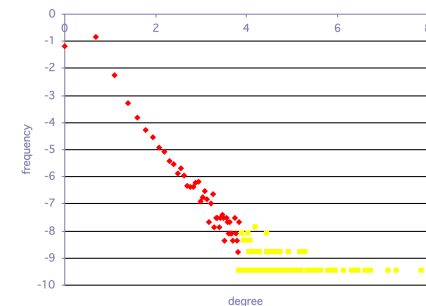
- Faloutsos[3] (Sigcomm'99)
  - frequency vs. degree

topology from BGP tables of 18 routers

Power Laws

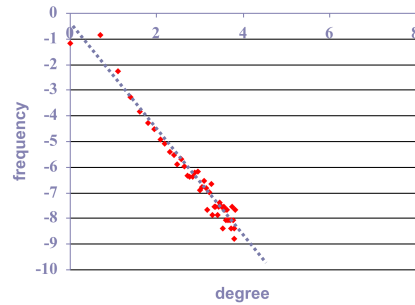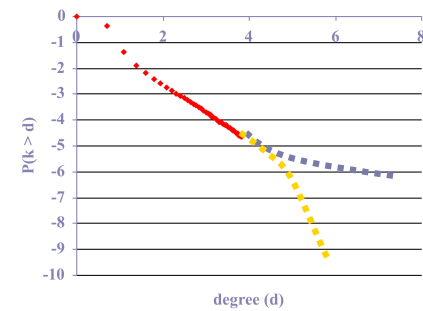- Faloutsos[3] (Sigcomm'99)
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Power Laws

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topology from BGP tables of 18 routers

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Power Laws

- Faloutsos
  - frequency vs. degree
  - empirical ccdf $P(d>x) \sim x^{-a}$
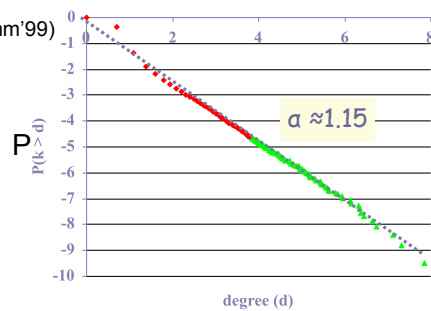

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Power Laws

- Faloutsos[3] (Sigcomm'99)
  - frequency vs. degree
  - empirical ccdf $(d>x) \sim x^{-a}$

$a \approx 1.15$

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