# 15-744: Computer Networking

L-14 Network Topology

## Sensor Networks

- Structural generators
- Power laws
- HOT graphs
- Graph generators
- Assigned reading
  - On Power-Law Relationships of the Internet Topology
  - A First Principles Approach to Understanding the Internet's Router-level Topology



## Outline

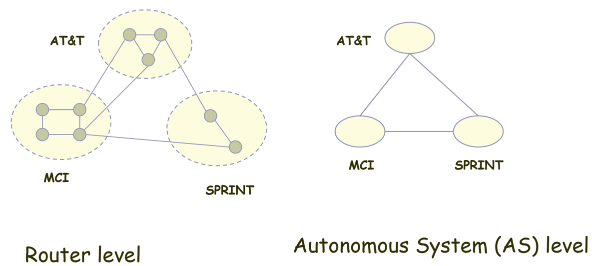
- Motivation/Background
- Power Laws
- Optimization Models
- Graph Generation



## Why study topology?

- Correctness of network protocols typically independent of topology
- Performance of networks critically dependent on topology
  - e.g., convergence of route information
- Internet impossible to replicate
- Modeling of topology needed to generate test topologies

## Internet topologies

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## More on topologies..

- Router level topologies reflect physical connectivity between nodes
  - Inferred from tools like *traceroute* or well known public measurement projects like Mercator and Skitter
- AS graph reflects a peering relationship between two providers/clients
  - Inferred from inter-domain routers that run BGP and publlic projects like Oregon Route Views
- Inferring both is difficult, and often inaccurate

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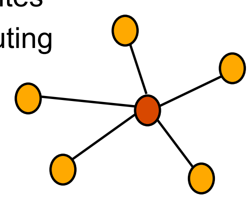
## Hub-and-Spoke Topology

- Single hub node
  - Common in enterprise networks
  - Main location and satellite sites
  - Simple design and trivial routing
- Problems
  - Single point of failure
  - Bandwidth limitations
  - High delay between sites
  - Costs to backhaul to hub



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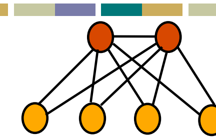
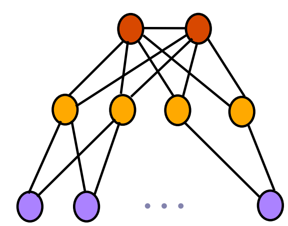
## Simple Alternatives to Hub-and-Spoke

- Dual hub-and-spoke
  - Higher reliability
  - Higher cost
  - Good building block



- Levels of hierarchy
  - Reduce backhaul cost
  - Aggregate the bandwidth
  - Shorter site-to-site delay



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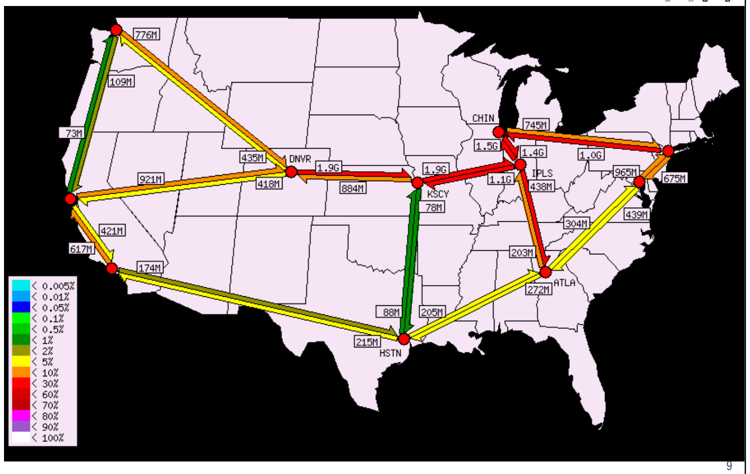
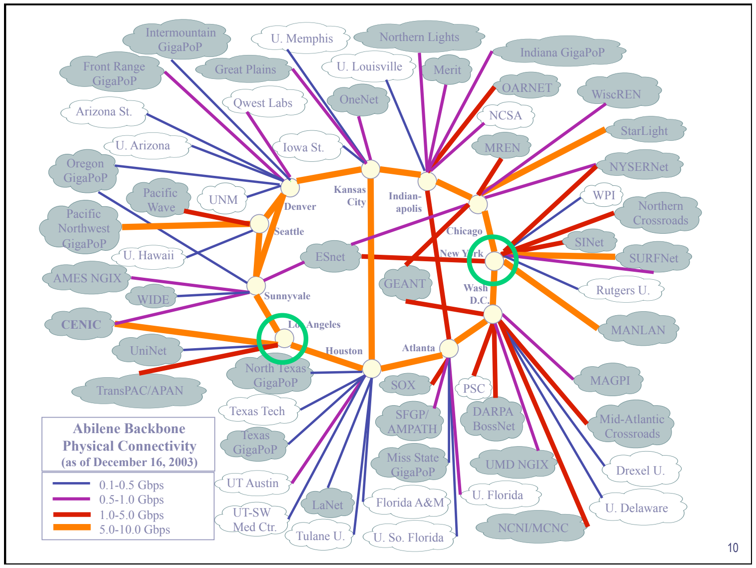
## Abilene Internet2 Backbone



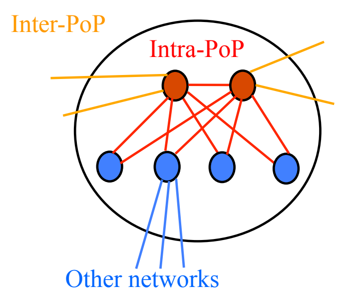





## Points-of-Presence (PoPs)

- Inter-PoP links
  - Long distances
  - High bandwidth
- Intra-PoP links
  - Short cables between racks or floors
  - Aggregated bandwidth
- Links to other networks
  - Wide range of media and bandwidth






## Deciding Where to Locate Nodes and Links

- Placing Points-of-Presence (PoPs)
  - Large population of potential customers
  - Other providers or exchange points
  - Cost and availability of real-estate
  - Mostly in major metropolitan areas
- Placing links between PoPs
  - Already fiber in the ground
  - Needed to limit propagation delay
  - Needed to handle the traffic load

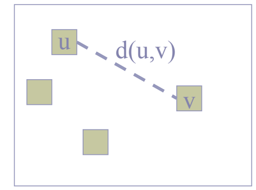
# Trends in Topology Modeling

| Observation | Modeling Approach |
|---|---|
| • Long-range links are expensive | • Random graph (Waxman88) |
| • Real networks are not random, but have obvious hierarchy | • Structural models (GT-ITM Calvert/Zegura, 1996) |
| • Internet topologies exhibit power law degree distributions (Faloutsos et al., 1999) | • Degree-based models replicate power-law degree sequences |
| • Physical networks have hard technological (and economic) constraints. | • Optimization-driven models topologies consistent with design tradeoffs of network engineers |



# Waxman model (Waxman 1988)

- Router level model
- Nodes placed at random in 2-d space with dimension L
- Probability of edge (u,v):
  - ae^{-d/(bL)}, where d is Euclidean distance (u,v), a and b are constants
- Models locality





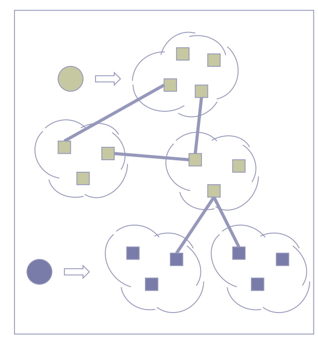

# Real world topologies

- Real networks exhibit
  - Hierarchical structure
  - Specialized nodes (transit, stub..)
  - Connectivity requirements
  - Redundancy
- Characteristics incorporated into the Georgia Tech Internetwork Topology Models (GT-ITM) simulator (E. Zegura, K.Calvert and M.J. Donahoo, 1995)



# Transit-stub model (Zegura 1997)

- Router level model
- Transit domains
  - placed in 2-d space
  - populated with routers
  - connected to each other
- Stub domains
  - placed in 2-d space
  - populated with routers
  - connected to transit domains
- Models hierarchy



## So…are we done?

- No!
- In 1999, Faloutsos, Faloutsos and Faloutsos published a paper, demonstrating power law relationships in Internet graphs
- Specifically, the node degree distribution exhibited power laws

That Changed Everything…..

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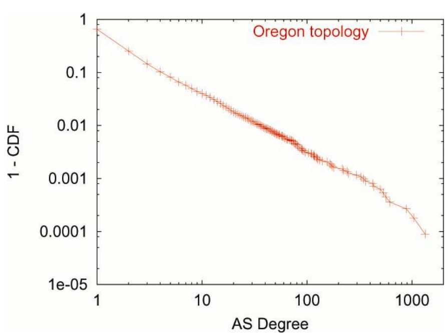
## Power laws in AS level topology

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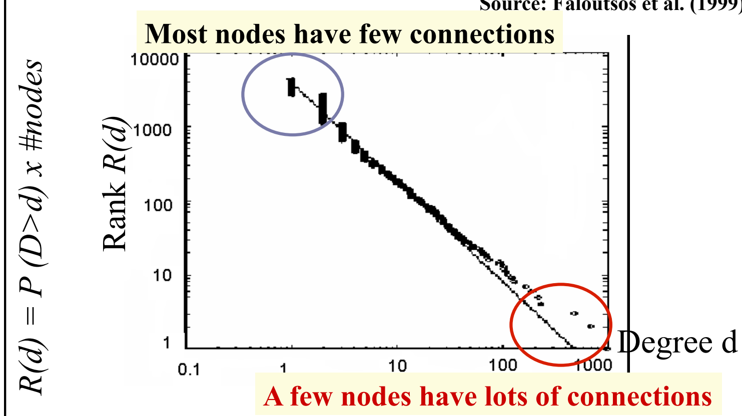
Power Laws and Internet Topology

Source: Faloutsos et al. (1999)




- Router-level graph & Autonomous System (AS) graph
- Led to active research in *degree-based* network models

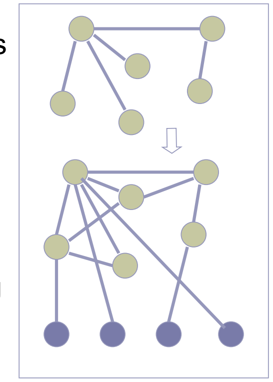
## GT-ITM abandoned..

- GT-ITM did not give power law degree graphs
- New topology generators and explanation for power law degrees were sought
- Focus of generators to match degree distribution of observed graph



## Inet (Jin 2000)

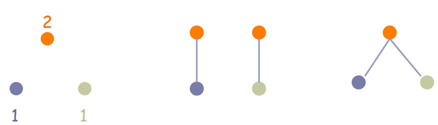
- Generate degree sequence
- Build spanning tree over nodes with degree larger than 1, using preferential connectivity
  - randomly select node u not in tree
  - join u to existing node v with probability d(v)/Σd(w)
- Connect degree 1 nodes using preferential connectivity
- Add remaining edges using preferential connectivity





## Power law random graph (PLRG)

- Operations
  - assign degrees to nodes drawn from power law distribution
  - create kv copies of node v; kv degree of v.
  - randomly match nodes in pool
  - aggregate edges



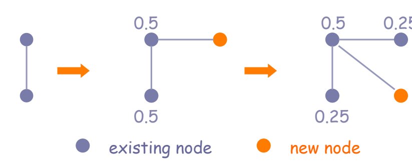

may be disconnected, contain multiple edges, self-loops

- contains unique giant component for right choice of parameters



## Barabasi model: fixed exponent

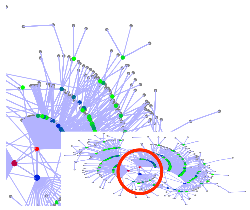
- incremental growth
  - initially, m0 nodes
  - step: add new node i with m edges
- linear preferential attachment
  - connect to node i with probability ki / ∑ kj



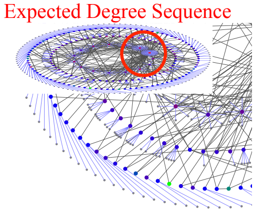

may contain multi-edges, self-loops

## Features of Degree-Based Models

Preferential Attachment



Expected Degree Sequence



- Degree sequence follows a power law (by construction)
- High-degree nodes correspond to highly connected central "hubs", which are crucial to the system
- Achilles' heel: robust to random failure, fragile to specific attack

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## Does Internet graph have these properties?

- No…(There is no Memphis!)
- Emphasis on degree distribution - structure ignored
- Real Internet very structured
- Evolution of graph is highly constrained

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## Problem With Power Law

- ... but they're descriptive models!
- No correct physical explanation, need an understanding of:
  - the driving force behind deployment
  - the driving force behind growth

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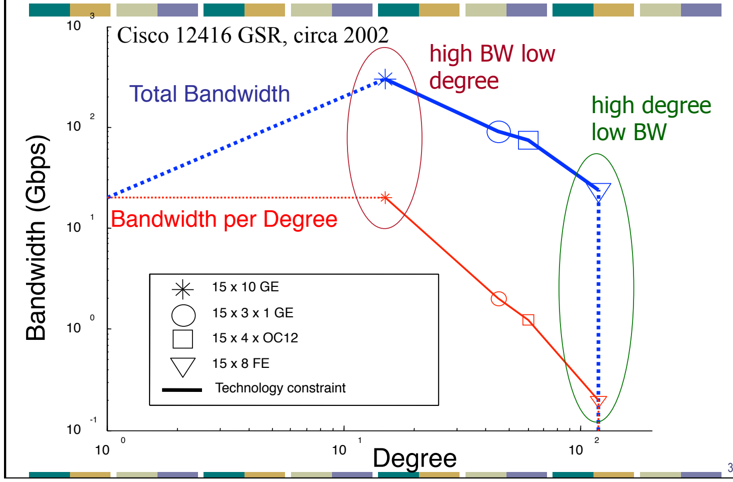
# Li et al.

- Consider the explicit design of the Internet
  - Annotated network graphs (capacity, bandwidth)
  - Technological and economic limitations
  - Network performance
- Seek a theory for Internet topology that is explanatory and not merely descriptive.
  - Explain high variability in network connectivity
  - Ability to match large scale statistics (e.g. power laws) is only secondary evidence

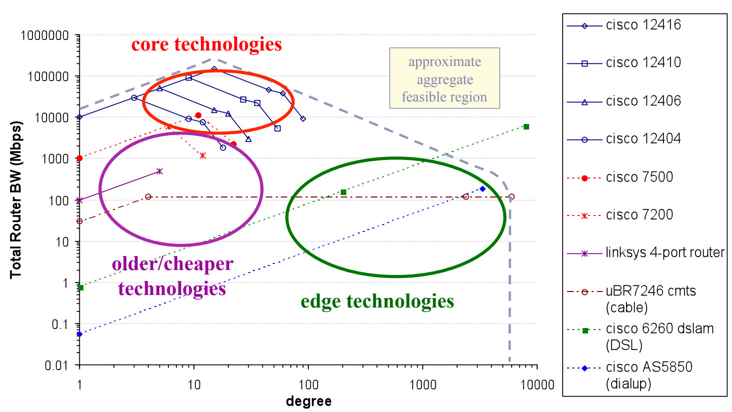


# Router Technology Constraint





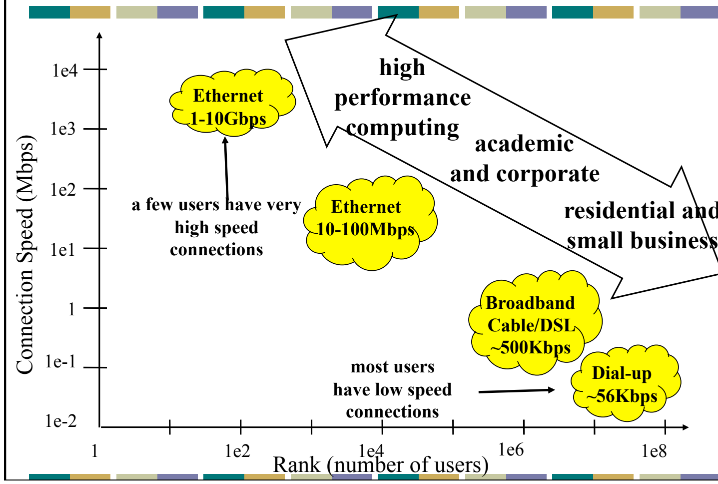

# Aggregate Router Feasibility



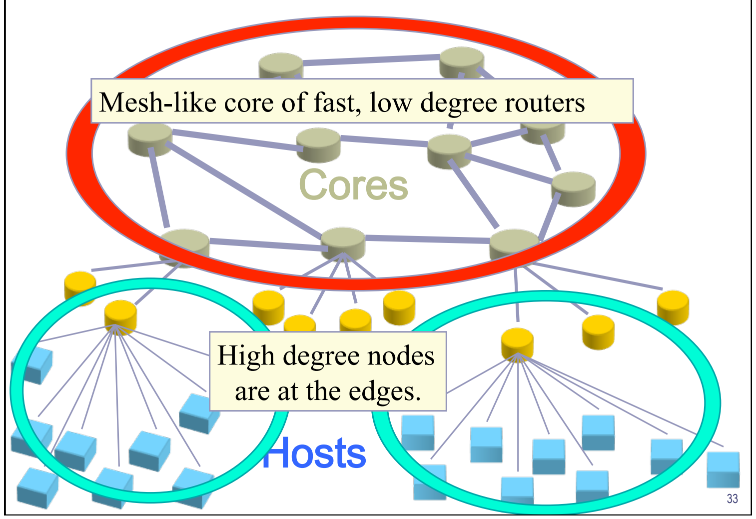

Source: Cisco Product Catalog, June 2002



# Variability in End-User Bandwidths



# Heuristically Optimal Topology

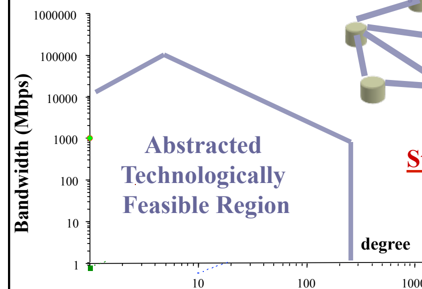




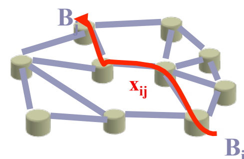

## Comparison Metric: Network Performance

Given realistic technology constraints on routers, how well is the network able to carry traffic?

**Step 1: Constrain to be feasible**




**Step 2: Compute traffic demand**




$$x_{ij} \propto B_i B_j$$

**Step 3: Compute max flow**

$$\max_{\alpha} \sum_{i,j} x_{ij} = \max \sum_{i,j} \alpha B_i B_j$$

$$s.t. \sum_{i,j:k \in r_{ij}} x_{ij} \le B_k, \forall k$$



## Likelihood-Related Metric

Define the metric $L(g) = \sum_{i,j \text{ connected}} d_i d_j$ ($d_i$ = degree of node $i$)

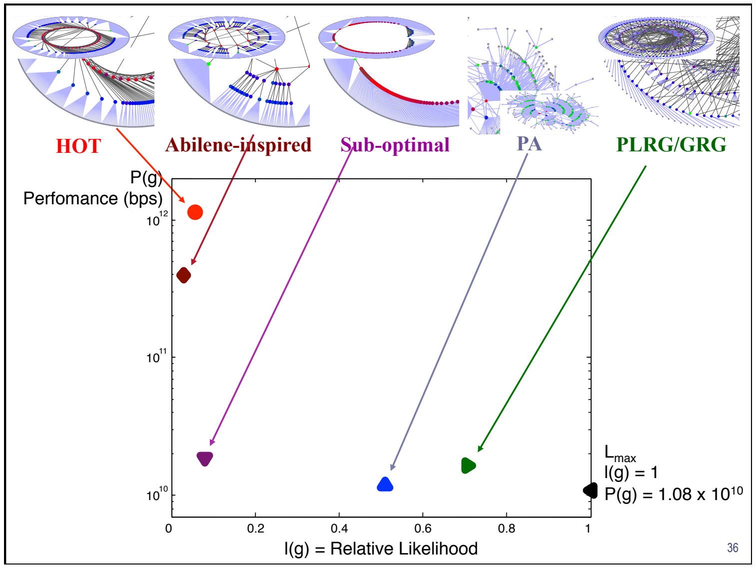
- Easily computed for any graph
- Depends on the structure of the graph, not the generation mechanism
- Measures how "hub-like" the network core is

- For graphs resulting from probabilistic construction (e.g. PLRG/GRG),

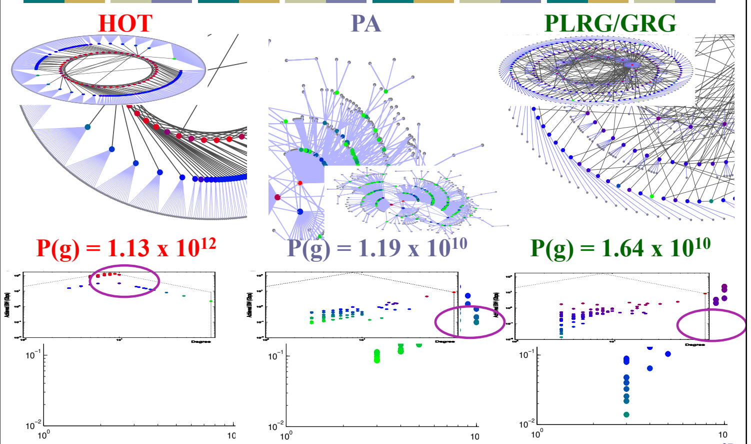
  LogLikelihood (LLH) $\propto L(g)$

- Interpretation: How likely is a particular graph (having given node degree distribution) to be constructed?





## Structure Determines Performance






## Summary Network Topology

- Faloutsos[3] [SIGCOMM99] on Internet topology
  - Observed many “power laws” in the Internet structure
    - Router level connections, AS-level connections, neighborhood sizes
  - Power law observation refuted later, Lakhina [INFOCOM00]
- Inspired many degree-based topology generators
  - Compared properties of generated graphs with those of measured graphs to validate generator
  - What is wrong with these topologies? Li et al [SIGCOMM04]
    - Many graphs with similar distribution have different properties
    - Random graph generation models don’t have network-intrinsic meaning
    - Should look at fundamental trade-offs to understand topology
      - Technology constraints and economic trade-offs
    - Graphs arising out of such generation better explain topology and its properties, but are unlikely to be generated by random processes!



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## Graph Generation

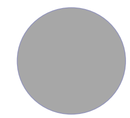
- Many important topology metrics
  - Spectrum
  - Distance distribution
  - Degree distribution
  - Clustering…
- No way to reproduce most of the important metrics
- No guarantee there will not be any other/ new metric found important

## dK-series approach

- Look at inter-dependencies among topology characteristics
- See if by reproducing most basic, simple, but not necessarily practically relevant characteristics, we can also reproduce (capture) all other characteristics, including practically important
- Try to find the one(s) defining *all others*

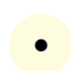
## *0K*

Average degree <*k*>



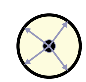
## *1K*

Degree distribution *P(k)*



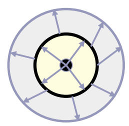
## *2K*

Joint degree distribution $P(k_1,k_2)$

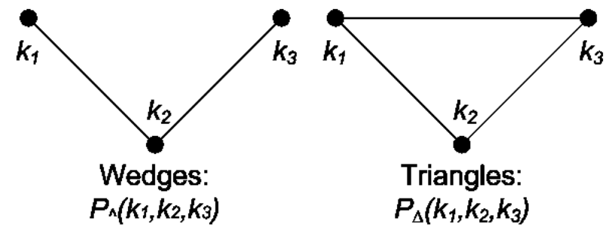


## *3K*

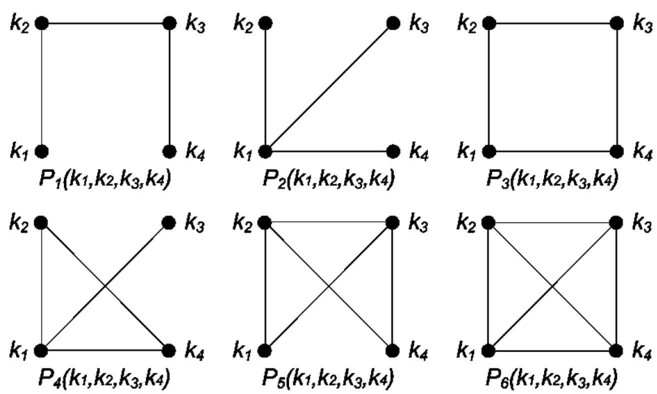
"Joint edge degree" distribution $P(k_1,k_2,k_3)$



## *3K*, more exactly




## *4K*



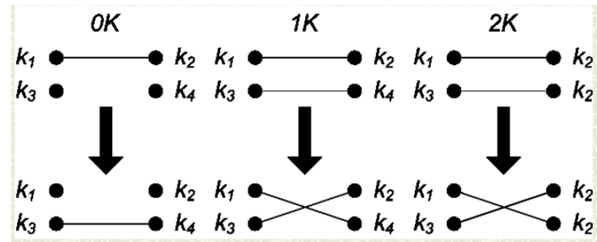

## Definition of *dK*-distributions

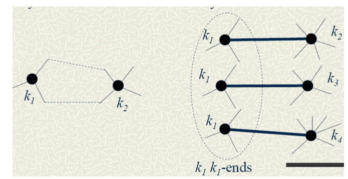
*dK*-distributions are degree correlations within simple connected graphs of size *d*

## Nice properties of properties $P_d$

- **Constructability**: we can construct graphs having properties $P_d$ ($dK$-graphs)
- **Inclusion**: if a graph has property $P_d$, then it also has all properties $P_i$, with $i < d$ ($dK$-graphs are also $iK$-graphs)
- **Convergence**: the set of graphs having property $P_n$ consists only of one element, $G$ itself ($dK$-graphs converge to $G$)

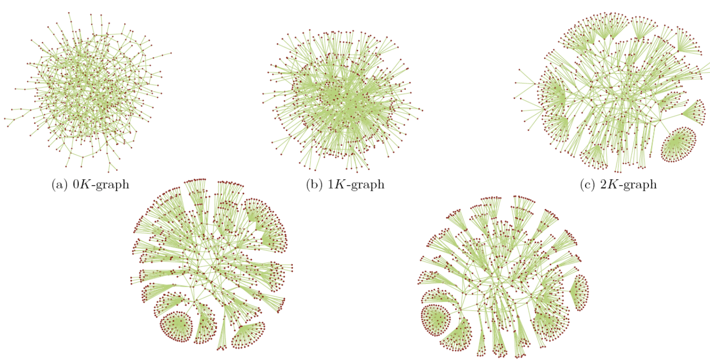
## Rewiring






## Graph Reproduction

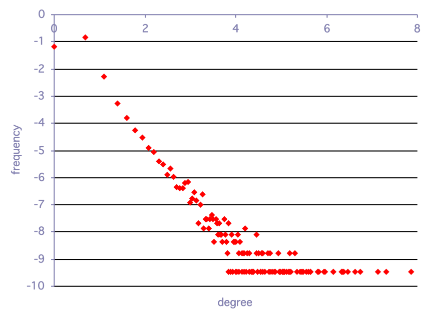


(a) $0K$-graph (b) $1K$-graph (c) $2K$-graph

(d) $3K$-graph (e) original HOT graph

## Power Laws

- Faloutsos[3] (Sigcomm'99)
  - frequency vs. degree




topology from BGP tables of 18 routers



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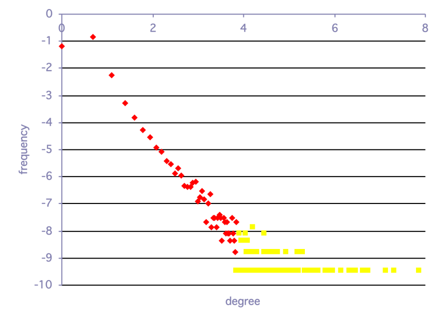


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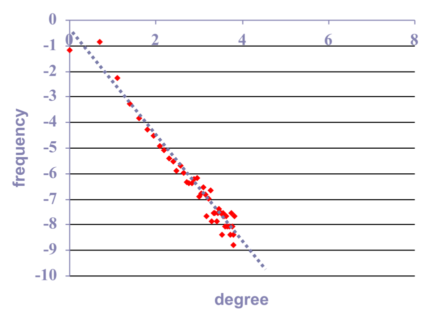


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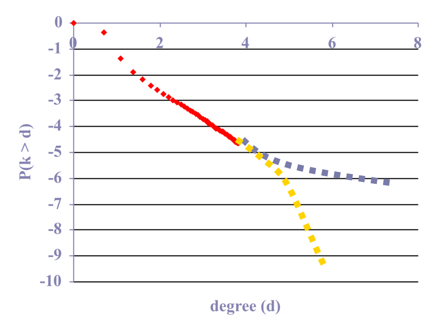
## Power Laws

- Faloutsos
  - frequency vs. degree
  - empirical ccdf $P(d>x) \sim x^{-a}$



# Power Laws

- Faloutsos[3] (Sigcomm'99)
  - frequency vs. degree
  - empirical ccdf P(d>x) ~ $x^{-a}$