CVPR 99 Tutorial on
3D Photography

Passive 3D Photography

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Talk Outline

1. *Visual Cues*
2. *Classical Vision Algorithms*
3. *State of the Art (video)*

Visual Cues

Motion

Visual Cues

Motion

Shading

Merle Norman Cosmetics, Los Angeles

Visual Cues

Motion

Shading

Texture

***The Visual Cliff*, by William Vandivert, 1960**

Visual Cues

Motion

Shading

Texture

Focus

From *The Art of Photography*, Canon

Visual Cues

Motion

Shading

Texture

Focus

Others:

- Highlights
- Shadows
- Silhouettes
- Inter-reflections
- Symmetry
- Light Polarization
- ...

Reconstruction Algorithms

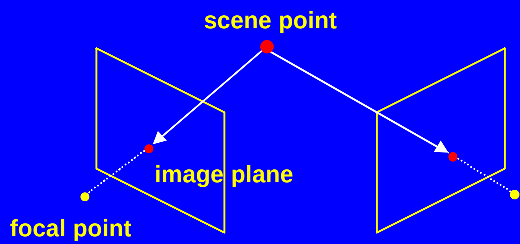
Shape From X

- ✓ Stereo (shape from parallax)
- ✓ Structure from motion
- ✓ Shape from shading
- ✓ Photometric stereo
- Shape from texture
- Shape from focus/defocus
- Shape from silhouettes, ...

Stereo

The Stereo Problem

- Reconstruct scene geometry from two or more *calibrated* images



Stereo

The Stereo Problem

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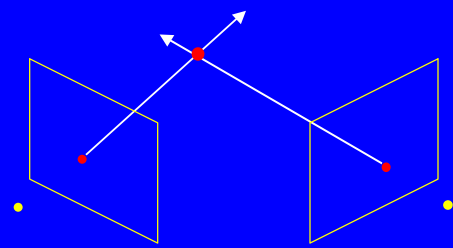


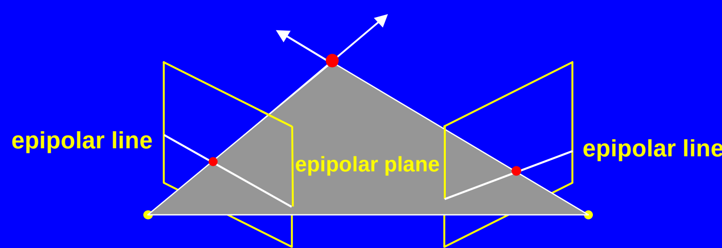
Basic Principle: Triangulation

- Gives reconstruction as intersection of two rays
- Requires *point correspondence*

Stereo Correspondence

Determine Pixel Correspondence

- **Pairs of points that correspond to same scene point**



Epipolar Constraint

- **Reduces correspondence problem to 1D search along *conjugate epipolar lines***

Stereo Matching Algorithms

Match Pixels in Conjugate Epipolar Lines

- **This is the tough part**
  - > specularities (non-Lambertian surfaces)
  - > ambiguity (low-contrast regions)
  - > missing data (occlusions)
  - > intensity error (quantization, sensor error)
  - > position error (camera calibration)
- **Numerous approaches**
  - > winner-take all
  - > dynamic programming [Ohta 85]
  - > smoothness functionals
  - > more images (trinocular, N-ocular) [Okutomi 93]

Structure from Motion

The SFM Problem

- **Reconstruct scene geometry and camera motion from two or more images**

Assume

- **Pixel correspondence**
  - > via tracking
- **Projection model**
  - > classic methods are orthographic

Orthographic Projection

$$\underset{2\times1}{\mathbf{u}} = \underset{2\times3}{\Pi}\,\underset{3\times1}{\mathbf{X}} + \underset{2\times1}{\mathbf{t}}$$

image point, projection matrix, scene point, image offset

Trick

- **Choose scene origin to be centroid of 3D points**
- **Choose image origins to be centroid of 2D points**
- **Allows us to drop the camera translation:**

$$\underset{2\times1}{\mathbf{u}} = \underset{2\times3}{\Pi}\,\underset{3\times1}{\mathbf{X}}$$

Shape by Factorization [Tomasi & Kanade, 92]

projection of *n* features in one image:

$$\underset{2\times n}{\begin{bmatrix} \mathbf{u}_1 & \mathbf{u}_2 & \cdots & \mathbf{u}_n \end{bmatrix}} = \underset{2\times n}{\prod} \underset{3\times n}{\begin{bmatrix} \mathbf{X}_1 & \mathbf{X}_2 & \cdots & \mathbf{X}_n \end{bmatrix}}$$

projection of *n* features in *f* images

$$\underset{2f\times n}{\begin{bmatrix} \mathbf{u}_1^1 & \mathbf{u}_2^1 & \cdots & \mathbf{u}_n^1 \\ \mathbf{u}_1^2 & \mathbf{u}_2^2 & \cdots & \mathbf{u}_n^2 \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{u}_1^f & \mathbf{u}_2^f & \cdots & \mathbf{u}_n^f \end{bmatrix}} = \underset{2f\times 3}{\begin{bmatrix} \text{Đ}^1 \\ \text{Đ}^2 \\ \vdots \\ \text{Đ}^f \end{bmatrix}} \underset{3\times n}{\begin{bmatrix} \mathbf{X}_1 & \mathbf{X}_2 & \cdots & \mathbf{X}_n \end{bmatrix}}$$

W measurement **M** motion **S** shape

Shape by Factorization [Tomasi & Kanade, 92]

known — $\underset{2f\times n}{\mathbf{W}} = \underset{2f\times 3}{\mathbf{M}} \underset{3\times n}{\mathbf{S}}$

Factorization Technique

- *W* is at most rank 3 (assuming no noise)
- We can use *singular value decomposition* to factor W:

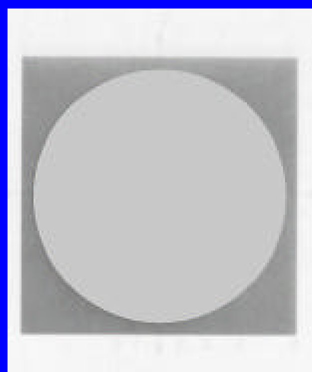
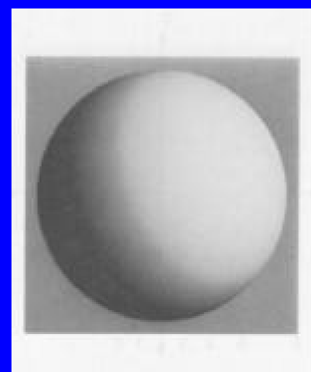
$$\underset{2f\times n}{\mathbf{W}} = \underset{2f\times 3}{\mathbf{M}'} \underset{3\times n}{\mathbf{S}'}$$

- *S'* differs from *S* by a linear transformation *A*:

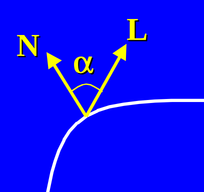
$$\mathbf{W} = \mathbf{M}'\mathbf{S}' = (\mathbf{M}\mathbf{A}^{-1})(\mathbf{A}\mathbf{S})$$

- Solve for *A* by enforcing constraints on *M*

Shape from Shading

Shape from Shading [Horn, 1970]



Classical Approach

- Reflected light depends only on α

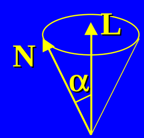
$$radiance = k\cos a$$

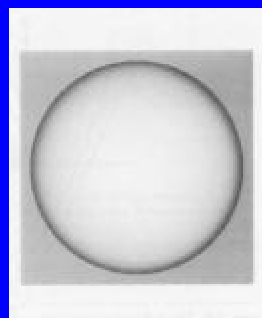
- Surface: Z = f(X,Y)
- Normal: $\mathbf{N} = [\mathbf{p} \quad \mathbf{q} \quad -1] = \left[\frac{\partial \mathbf{f}}{\partial \mathbf{x}} \quad \frac{\partial \mathbf{f}}{\partial \mathbf{y}} \quad -1\right]$

This Means We Assume

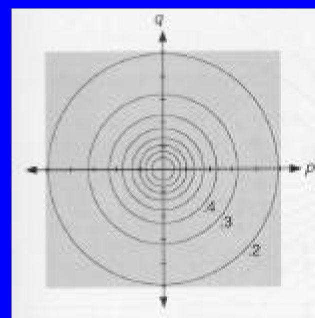
- Orthographic projection
- Lambertian surface
- Known light source at infinity
- No shadows
- Uniform surface material

The Reflectance Map





Image



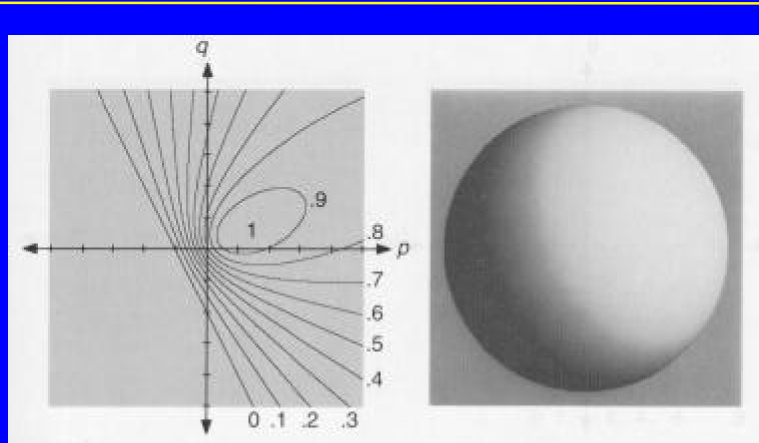
Reflectance Map: R
(gradient space)

$$\mathbf{N} = [p \quad q \quad -1]$$

$$\mathbf{L} = [p_l \quad q_l \quad -1]$$

$$I = k\cos\alpha = k\frac{(pp_l + qq_l + 1)}{\sqrt{p^2+q^2+1}\sqrt{p_l^2+q_l^2+1}}$$

The Reflectance Map



Reflectance Map

Image

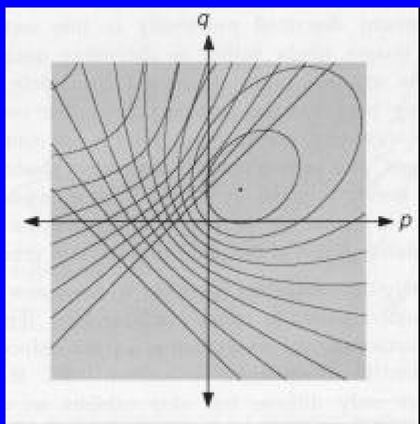
Finding a Unique Solution

Three Approaches

- **Characteristic Strip Method [Horn, 77]**
  - > select a few points where normal is known
  - > grow solution by moving direction of $\nabla \mathbf{R}$
- **Variational Method [Ikeuchi & Horn, 81]**
  - > start with an initial guess of surface shape
  - > define energy function
  - > refine to minimize energy function
- **Photometric Stereo [Woodham 80]**
  - > use more images

Photometric Stereo

Two Images Under Different Lighting



Need Three Images for Unique Solution

Photometric Stereo: Matrix Formulation

Write Equations in Matrix Form

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{L}}_1^T \\ \hat{\mathbf{L}}_2^T \\ \hat{\mathbf{L}}_3^T \end{bmatrix} \bullet \begin{bmatrix} k\hat{\mathbf{N}} \\ k\hat{\mathbf{N}} \\ k\hat{\mathbf{N}} \end{bmatrix}$$

$$\mathbf{I}_{3\times1} \quad \mathbf{L}_{3\times3} \quad \tilde{\mathbf{N}}_{3\times1}$$

$$\Rightarrow \quad \tilde{\mathbf{N}} = \mathbf{L}_1^{-1}\,\mathbf{I} \qquad k = \left\|\tilde{\mathbf{N}}\right\|$$

Advantage:

- **Can solve for variable reflectance *k***

Resources

Computer Vision Home Page

- **http://www.cs.cmu.edu/afs/cs/project/cil/ftp/html/vision.html**

Computer Vision Textbooks

- D. H. Ballard and C. M. Brown, *Computer Vision*, Prentice-Hall, 1982.
- O. Faugeras, *Three-Dimensional Computer Vision*, MIT Press, 1993.
- B. K. P. Horn, *Robot Vision*, McGraw-Hill, 1986.
- R. Jain, R. Kasturi and B. G. Schunck, *Machine Vision*, McGraw-Hill, 1995.
- R. Klette, K. Schluns and A. Koschan, *Computer Vision: Three-Dimensional Data from Images*, Springer-Verlag, 1998.
- V. S. Nalwa, *A Guided Tour of Computer Vision*, Addison-Wesley, 1993.
- M. Sonka, V. Hlavac and R. Boyle, *Image Processing, Analysis, and Machine Vision*, Brooks/Cole Publishing, 1999.
- E. Trucco and A. Verri, *Introductory Techniques for 3-D Computer Vision*, Prentice-Hall, 1998.
- D. Marr, *Vision*, Freeman, 1982.
- J. Koenderink, *Solid Shape*, MIT Press, 1990.

Bibliography

Stereo

- Yuichi Ohta & Takeo Kanade, "Stereo by Intra- and Inter-Scanline Search Using Dynamic Programming", IEEE Trans. on Pattern Analysis and Machine Intelligence, 7(2), 1985, pp. 129-154.
- Masatoshi Okutomi & Takeo Kanade, ”A Multiple-Baseline Stereo", IEEE Trans. on Pattern Analysis and Machine Intelligence", 15(4), 353-363, 1985.

Structure-from-Motion

- Carlo Tomasi & Takeo Kanade, ”Shape and Motion from Image Streams Under Orthography: A Factorization Method", Int. Journal of Computer Vision, 9(2), 1992, pp. 137-154.

Shape from Shading

- B. Horn and M. Brooks, “Shape from Shading”, 1989, MIT Press.
- L. Wolff, S. Shafer, and G. E. Healey, “Physics-Based Vision: Shape Recovery”, 1992, Jones and Bartlett.
- R. J. Woodham, “Photometric Method for Determining Surface Orientation from Multiple Images”, Optical Engineering, 1980, pp. 139-144.

Video