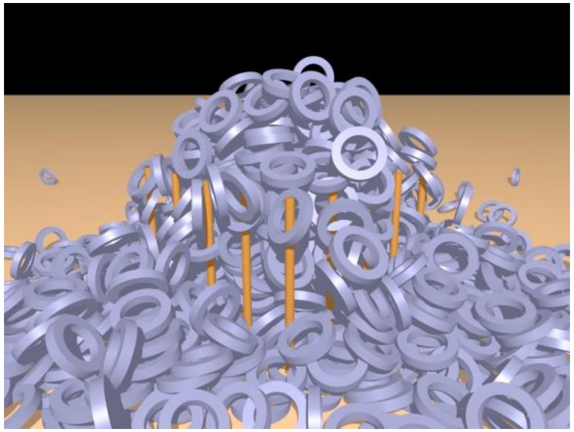
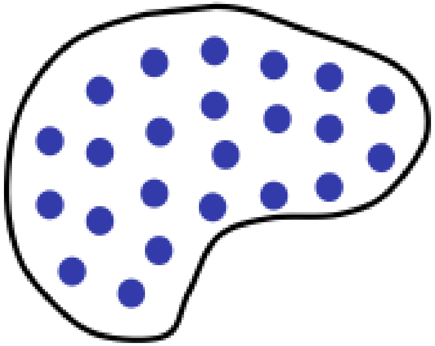
Rigid Body Dynamics
Part II



A rigid body

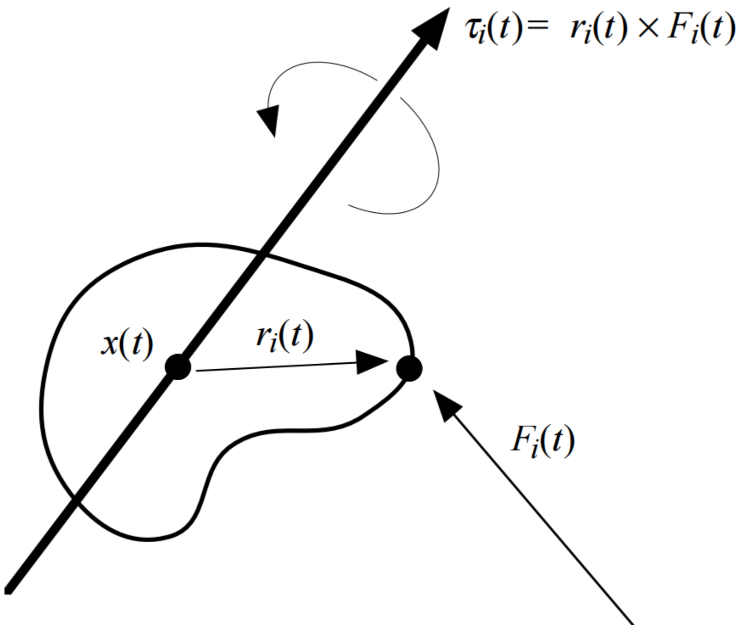


- Collection of particles
- Distance between any two particles is always constant

Forces and Torques

- Forces on individual particles generate torques
  - (consequence of constant inter-particle distance)

Net torque on body: $\boldsymbol{\tau} = \sum \boldsymbol{\tau}_i = \sum \boldsymbol{r_i} \times \boldsymbol{f}_i$



Linear and Angular Accelerations

- From conservation of linear momentum:

$$\boldsymbol{p} = M\boldsymbol{v}; \dot{\boldsymbol{p}} = \boldsymbol{F};\ \dot{\boldsymbol{v}} = \frac{1}{\boldsymbol{M}}\boldsymbol{F}$$

- From Conservation of angular momentum:

$$\boldsymbol{L} = \boldsymbol{I\omega}; \dot{\boldsymbol{L}} = \boldsymbol{\tau};\ \dot{\boldsymbol{\omega}} = \boldsymbol{I}^{-1}(\boldsymbol{\tau} - \boldsymbol{\omega} \times \boldsymbol{I\omega})$$

Numerical Integration

- COM Acceleration → Velocity → Position
  - Easy: $\boldsymbol{v}_{t+1} = \boldsymbol{v}_t + \Delta t \dot{\boldsymbol{v}}$; $\boldsymbol{x}_{t+1} = \boldsymbol{x}_t + \Delta t \boldsymbol{v}_{t+1}$

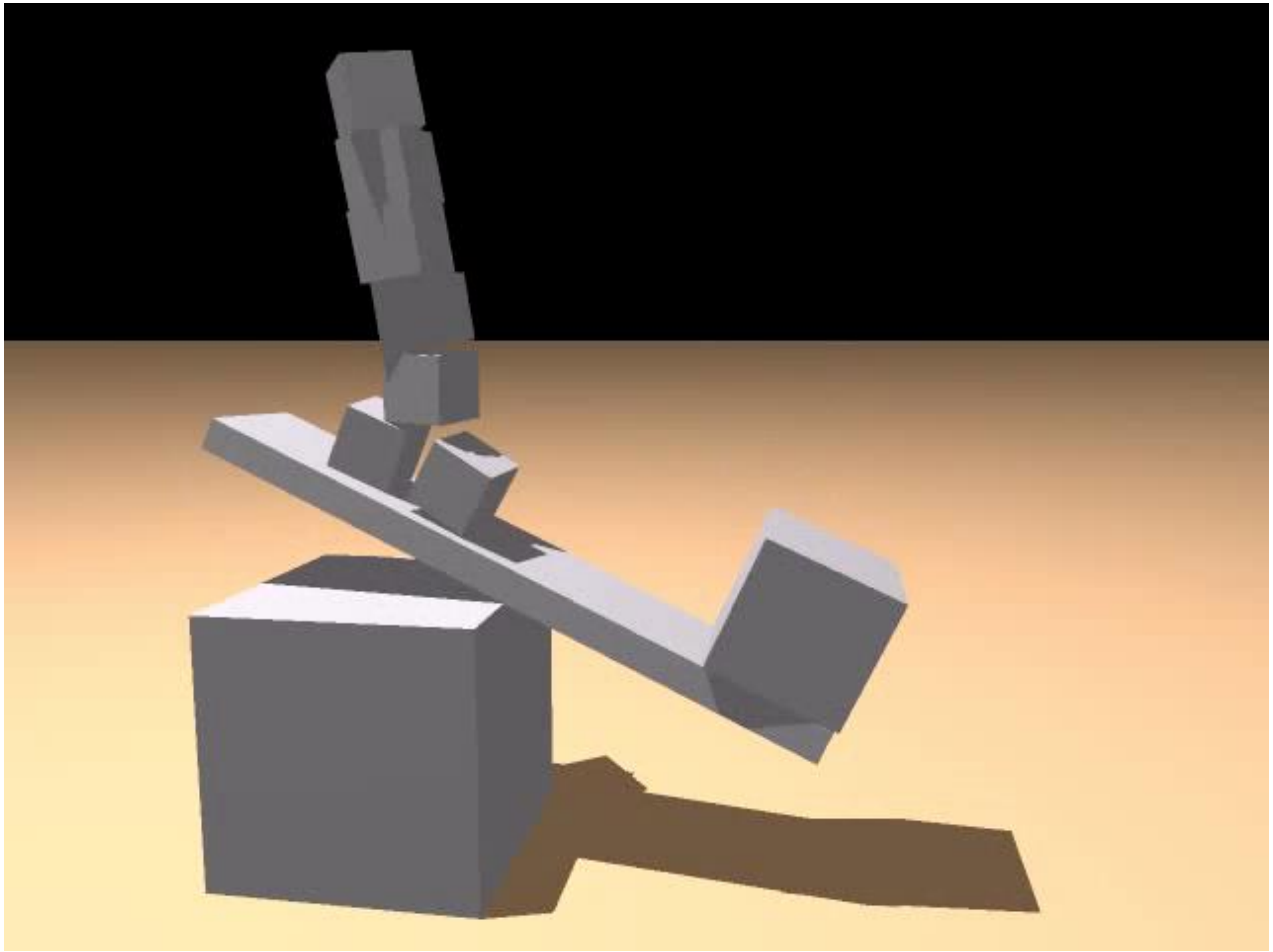
- Angular Acceleration → Angular Velocity
  - Easy: $\boldsymbol{\omega}_{t+1} = \boldsymbol{\omega}_t + \Delta t \dot{\boldsymbol{\omega}}$

- Angular Velocity to Rotations?
  - A bit trickier: $\boldsymbol{R}_{t+1} = \boldsymbol{R}_t + \Delta t \dot{\boldsymbol{R}}_{t+1}$?

Computing forces

- Given a set of forces, you know how to compute the motion of a rigid body
- Where do forces come from?
  - User interaction
  - Gravity

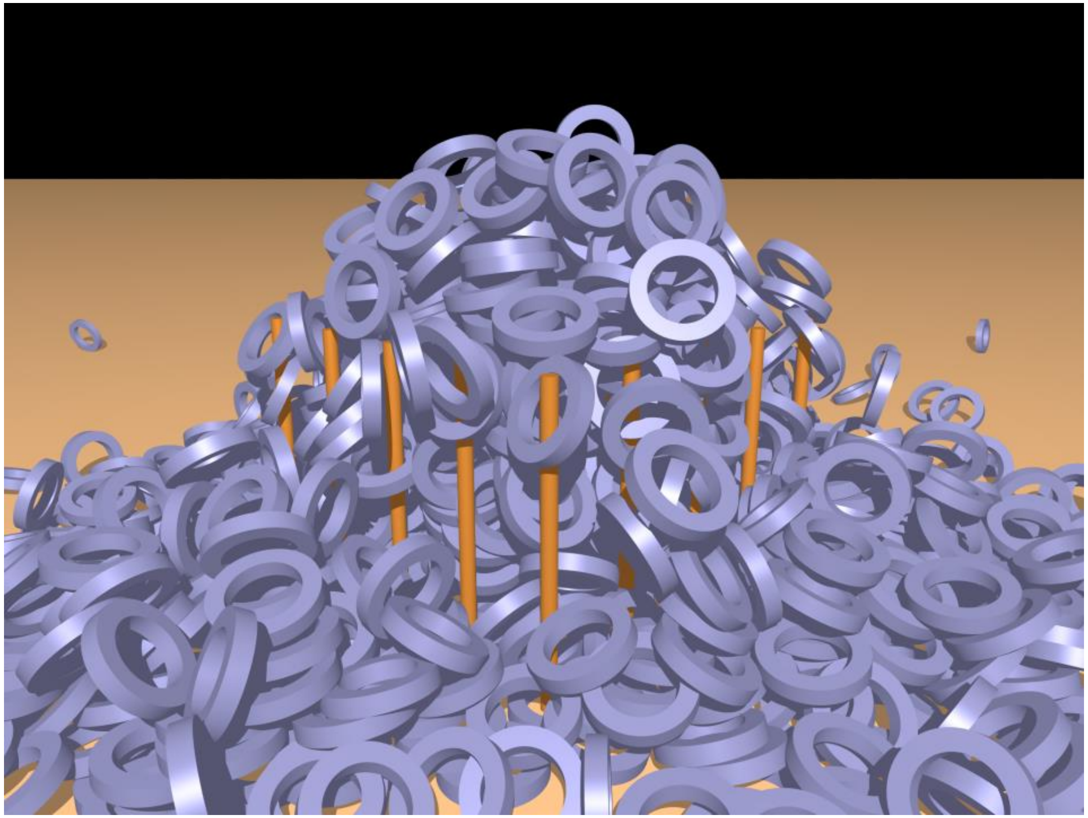
Rigid Bodies



Computing forces

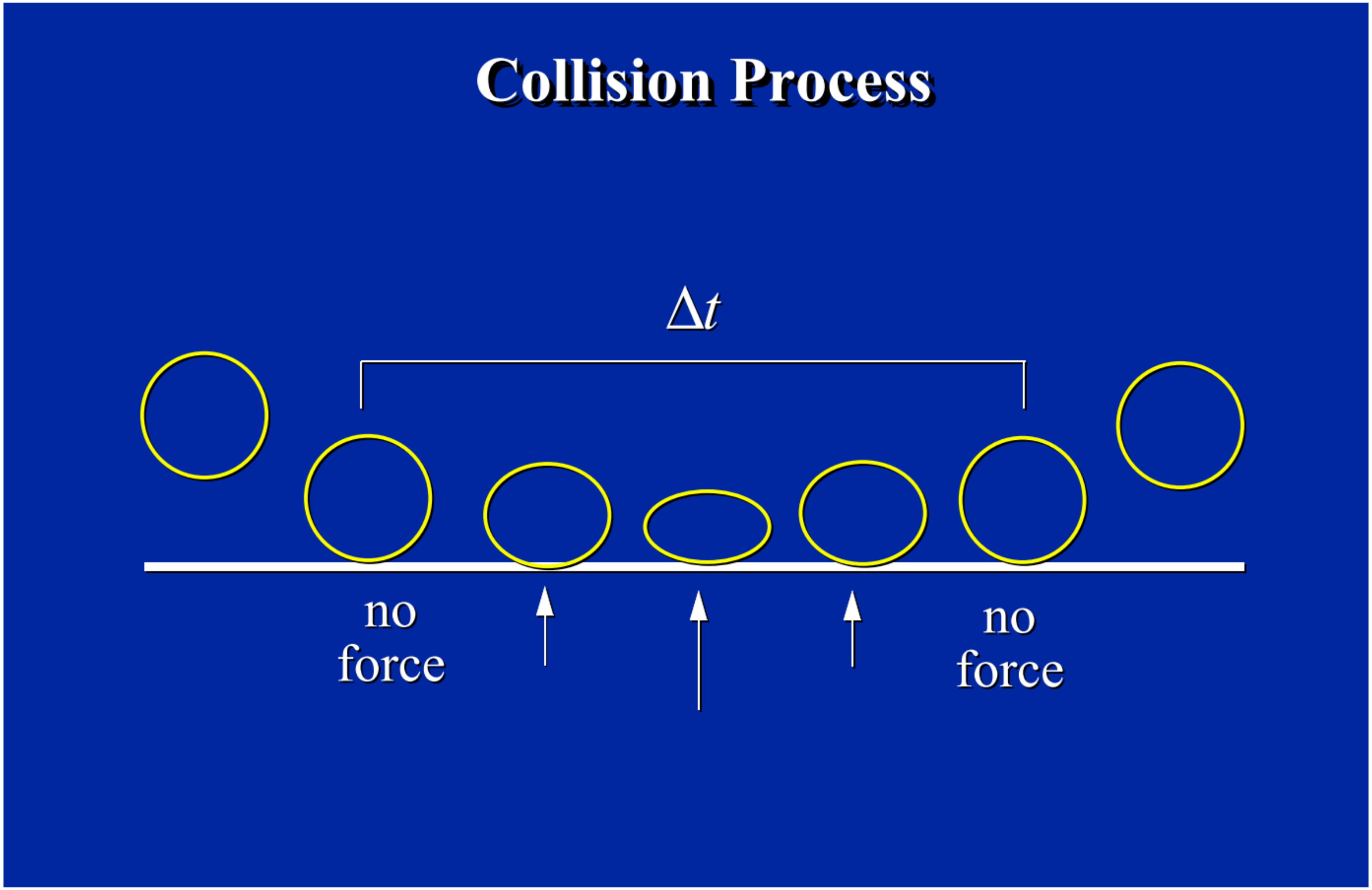
- Given a set of forces, you know how to compute the motion of a rigid body
- Where do forces come from?
  - User interaction
  - Gravity
  - Collisions and contacts

Collision Response

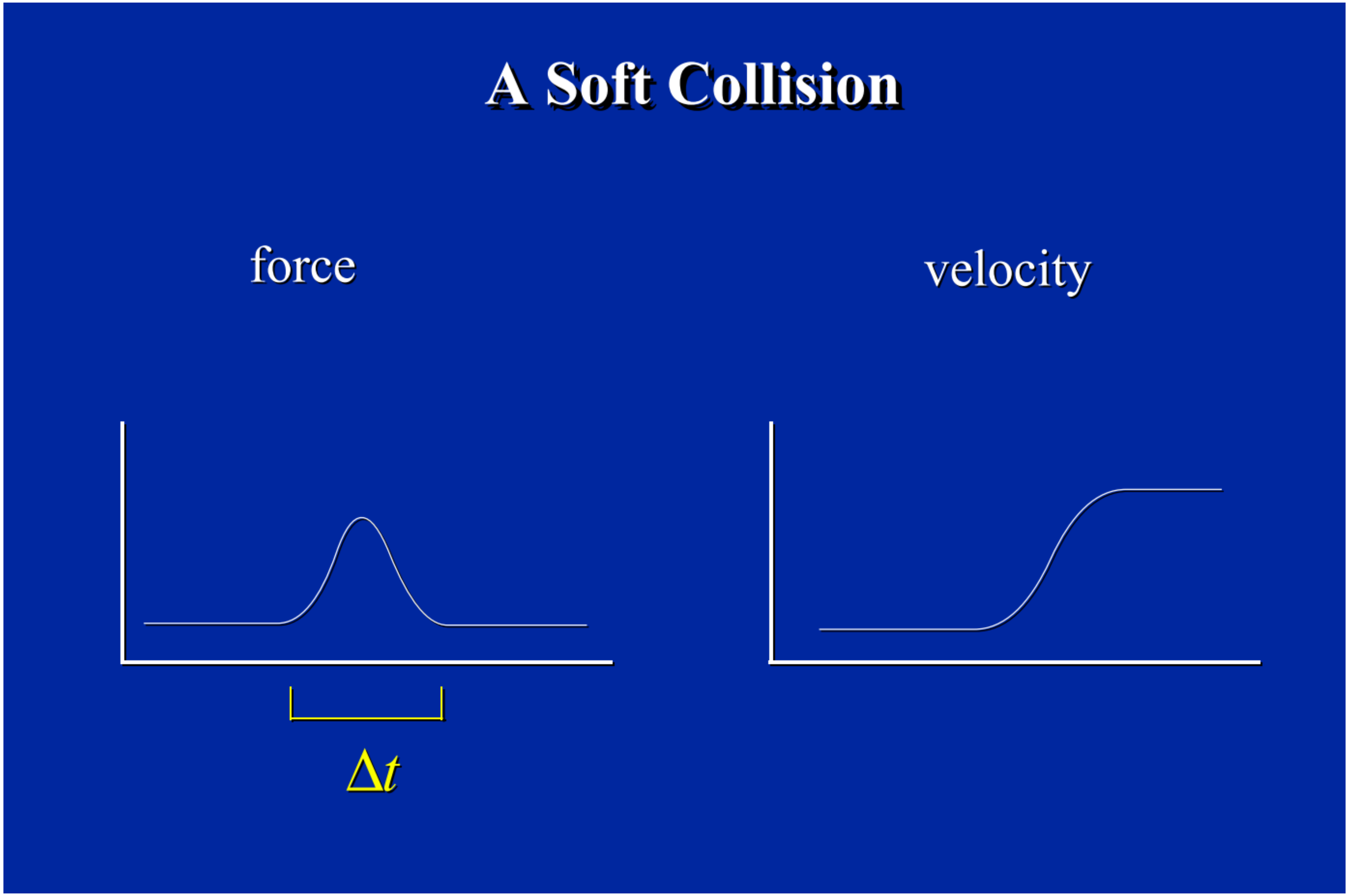


"Nonconvex Rigid Bodies with Stacking", Guendelman et al., SIGGRAPH 2003

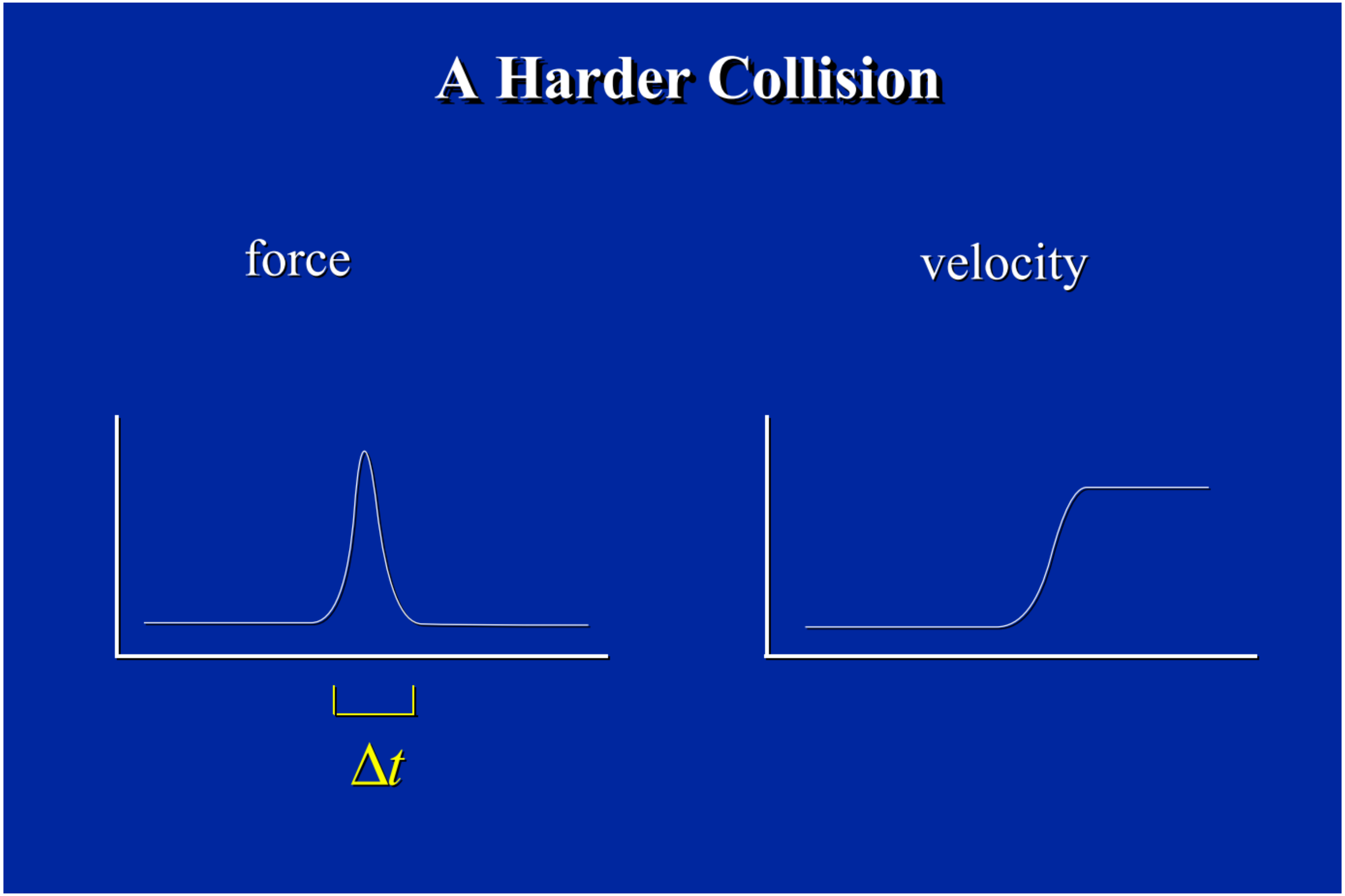
Collision Response



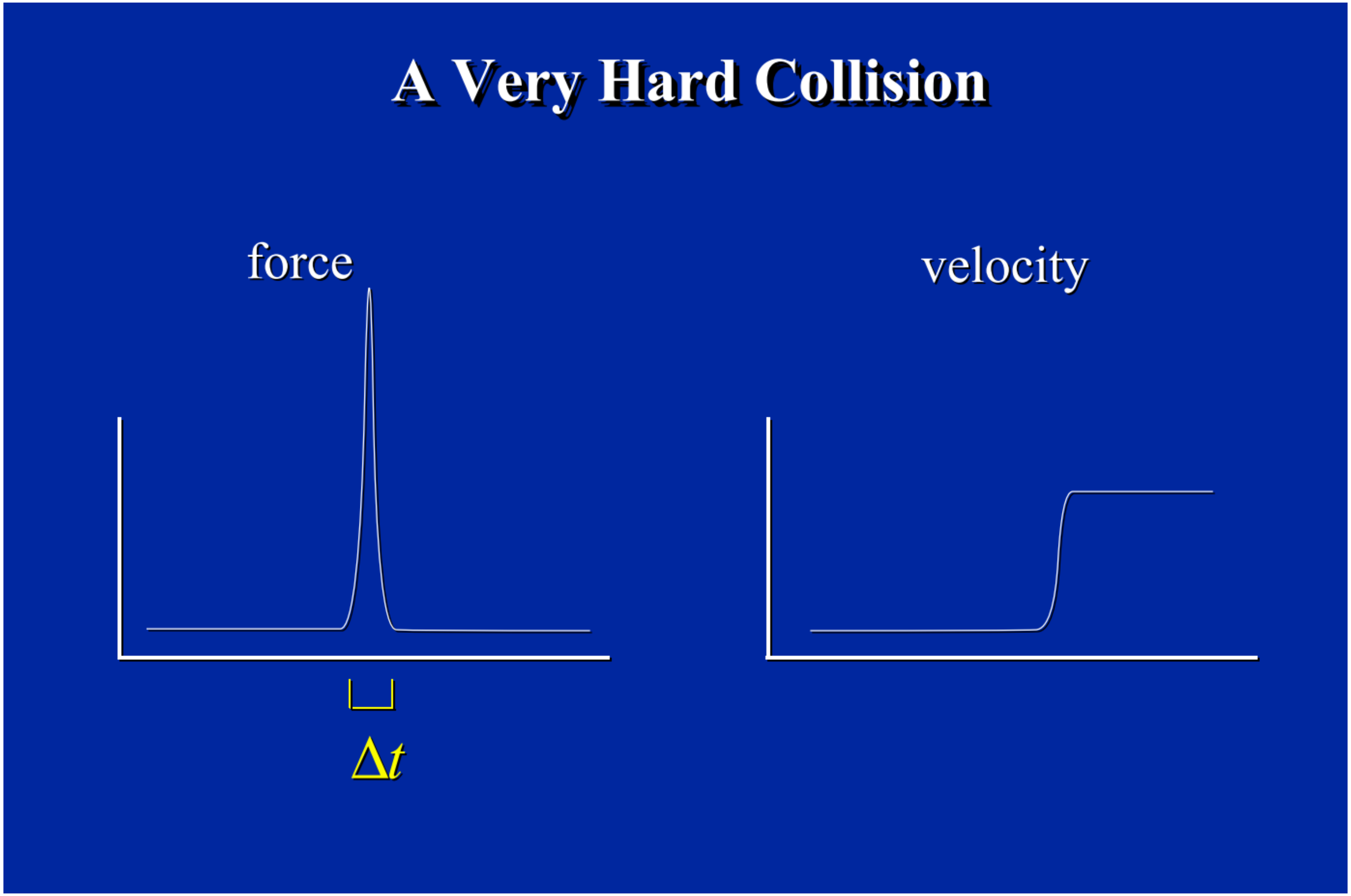
Collision Response



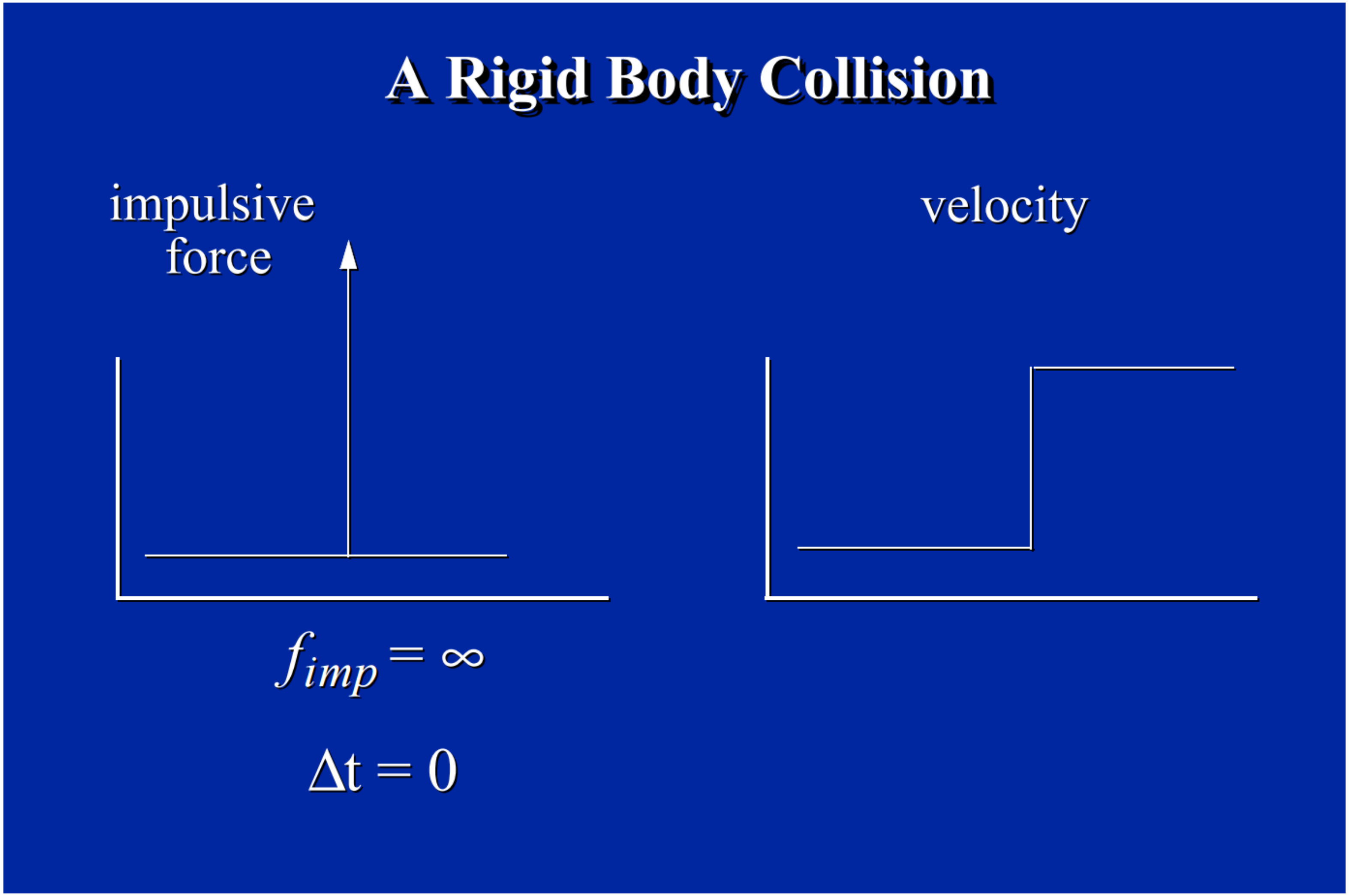
Collision Response



Collision Response



Collision Response



Collision Response

- An impulse changes velocities instantaneously

$$\boldsymbol{J} = \delta t \boldsymbol{F}$$

- They can therefore be used to model rigid body collisions

- Derivation on the whiteboard…

Computing forces

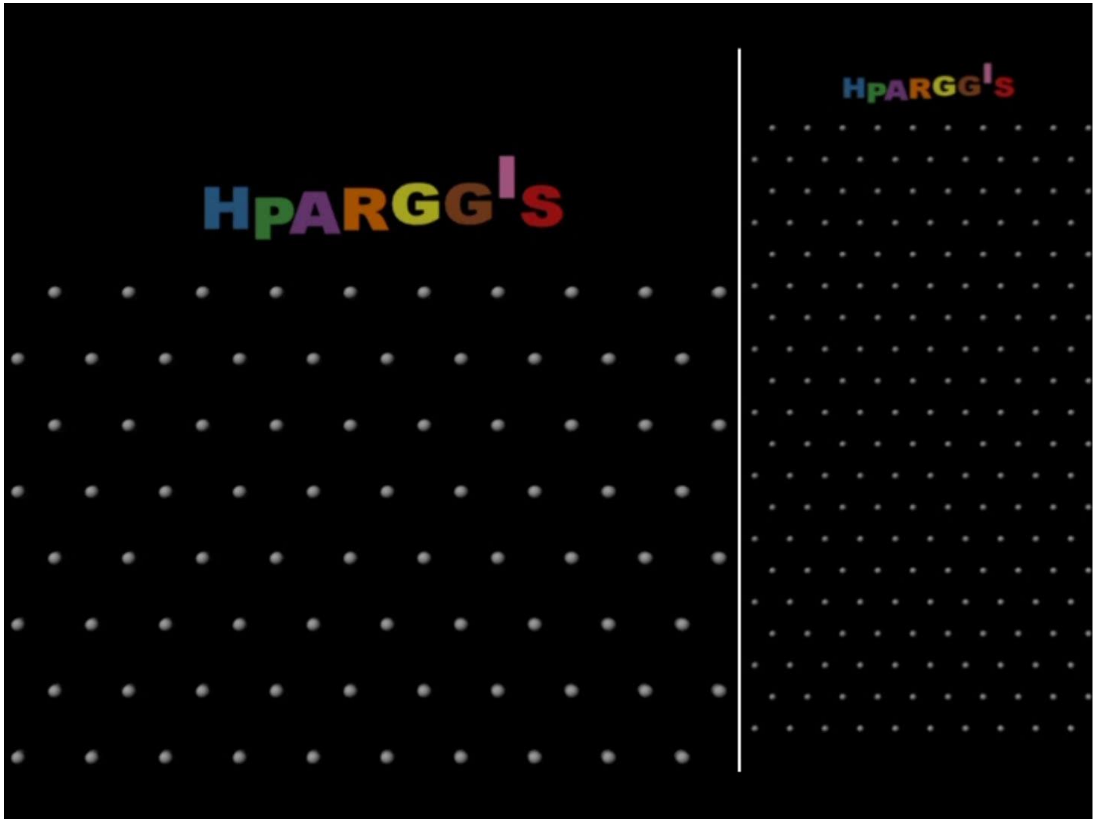
- Given a set of forces, you know how to compute the motion of a rigid body
- Where do forces come from?
  - User interaction
  - Gravity
  - Collisions and contacts
  - Articulation

Articulated Rigid Body Dynamics

Derivation on the whiteboard…

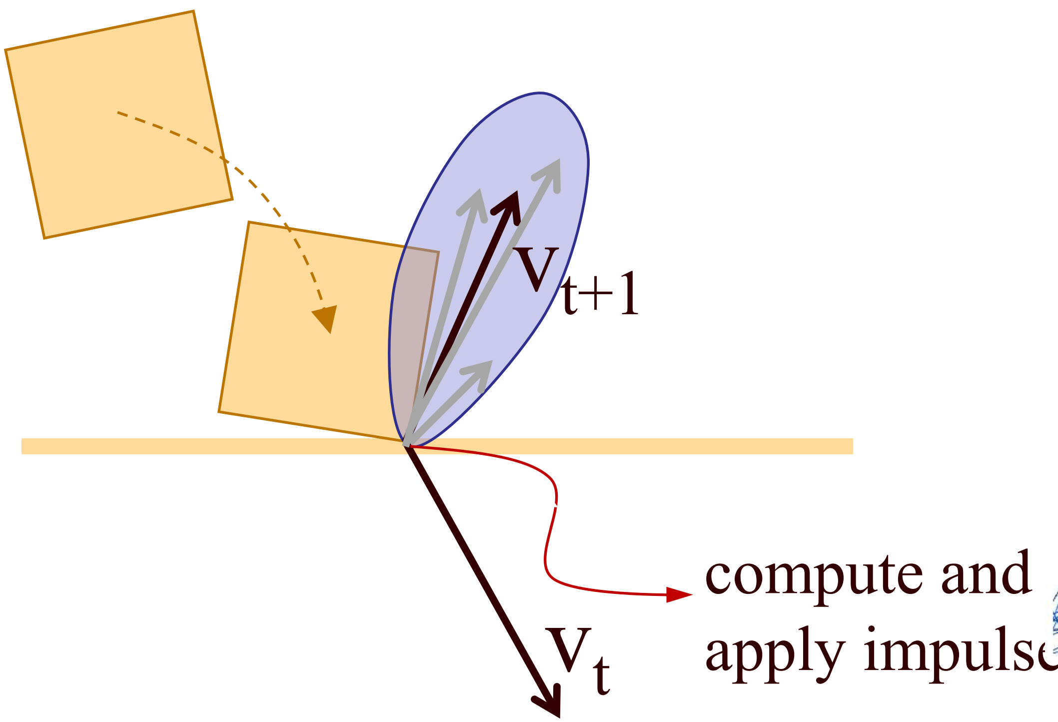
Artistic control over rigid body simulations

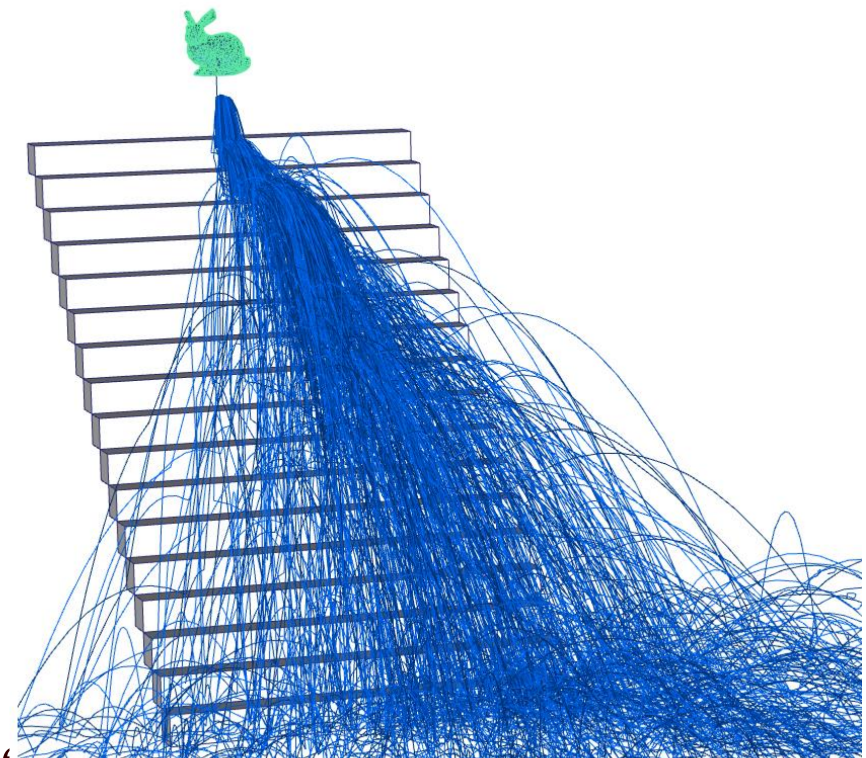
Many-Worlds Browsing for Control of Multibody Dynamics Twigg and James, 2007



Many Worlds Browsing…

Sampling Plausible Worlds

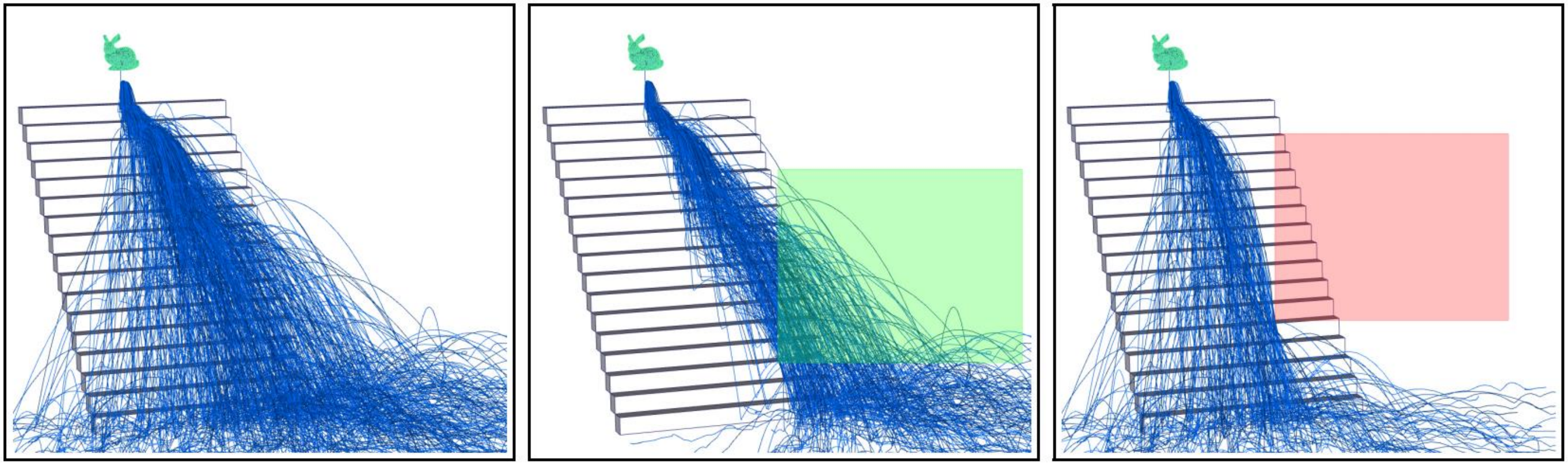




[O'Sullivan et al., 2003]

Many Worlds Browsing…

Interactive Browsing – various criteria



Input Positive Negative

Many Worlds Browsing