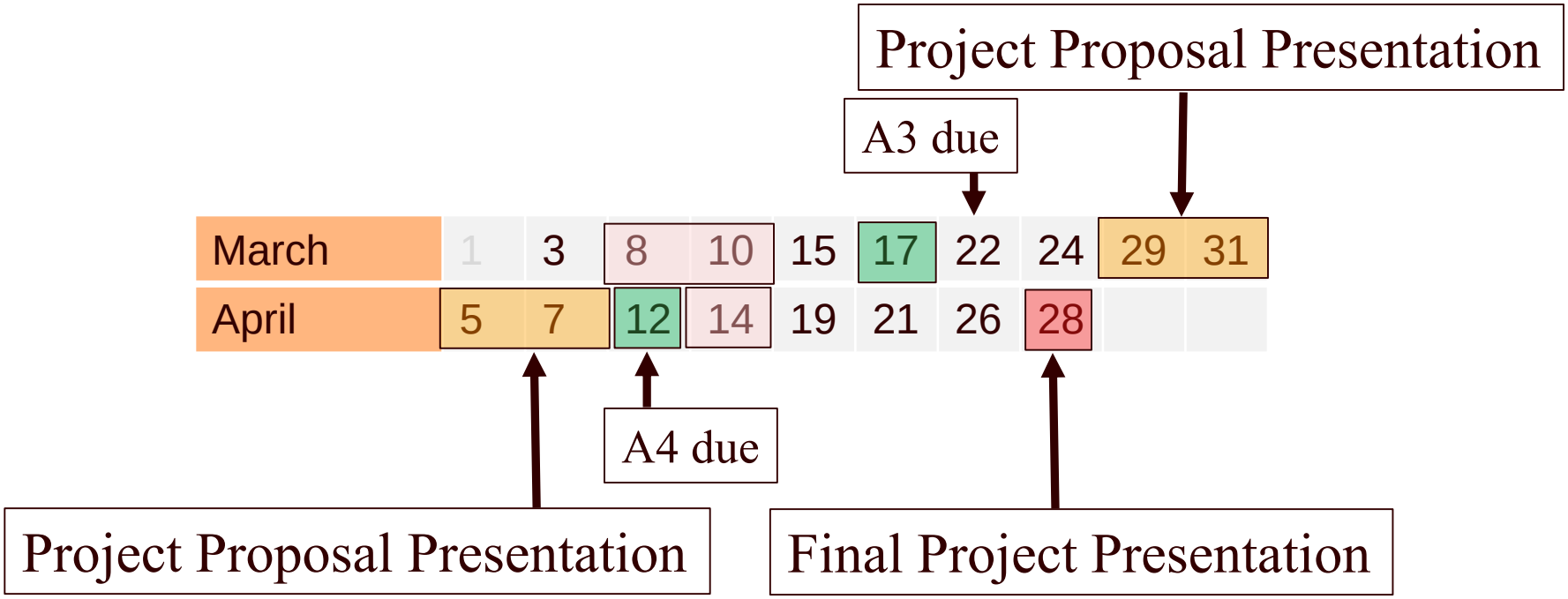
Fun With Elastica
Part II

Schedule


Project Proposal Presentation
A3 due
March
1
3
8
10
15
17
22
24
29
31
April
5
7
12
14
19
21
26
28
A4 due
Project Proposal Presentation
Final Project Presentation

Project Proposal Presentation

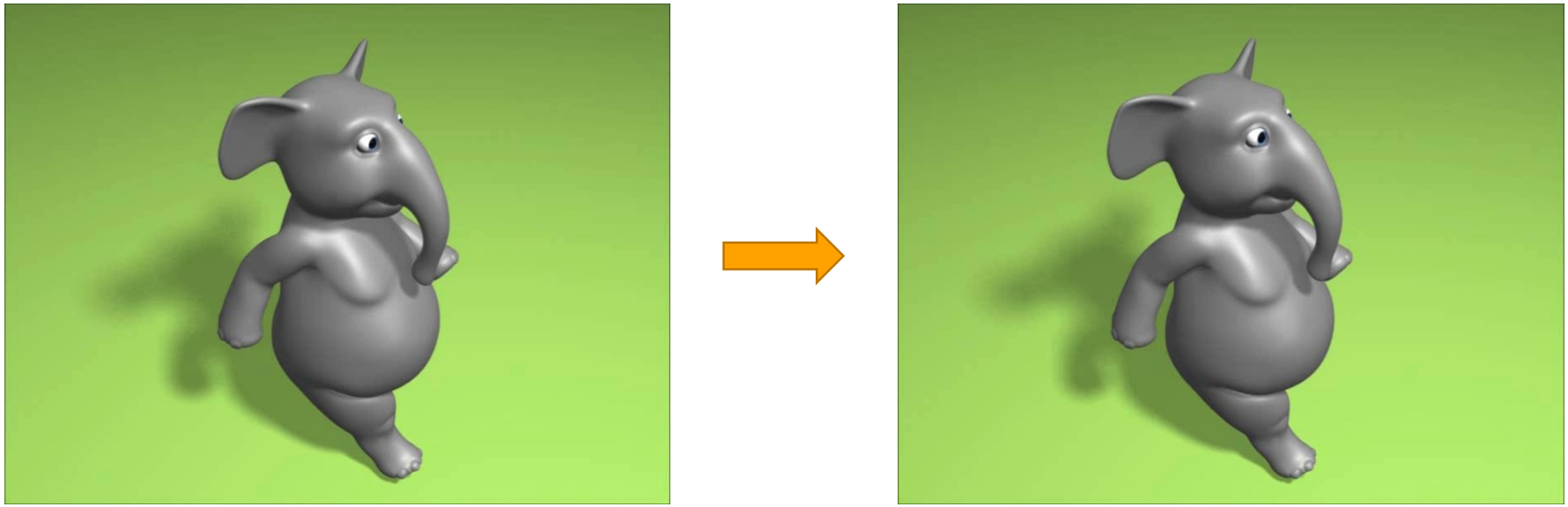
- Plan for 20 min presentation if solo, 30 min if you are working in a team
  - Problem and motivation
  - Background
    - Choose 1-2 related research papers, and describe in detail – talk to us if unsure which papers to look at
  - Proposed technical method
  - Expected results (low and high bar)
- Dates: March 29, 31, April 5,7
  - Let me know if you have a preference ASAP

From Last Class

Find ~~$\mathbf{x}$~~ $\mathbf{p}$ $\quad s.t.$

$$\mathbf{M}\left(\frac{\overset{\mathbf{x(p)}}{\cancel{\mathbf{x}}}}{h^2} - \frac{\mathbf{x}_n}{h^2} - \frac{\mathbf{v}_n}{h}\right) = \mathbf{f}_{\text{int}}\left(\mathbf{X}, \overset{\mathbf{x(p)}}{\cancel{\mathbf{x}}}\right) + \mathbf{f}_{\text{ext}}\left(\overset{\mathbf{x(p)}}{\cancel{\mathbf{x}}}\right)$$

Simulation in Rig Subspace



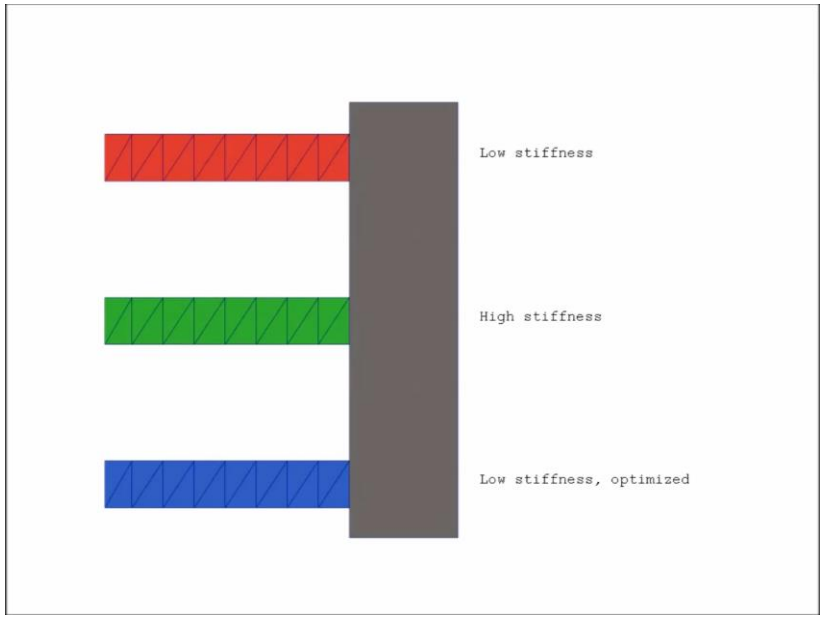
From Last Class

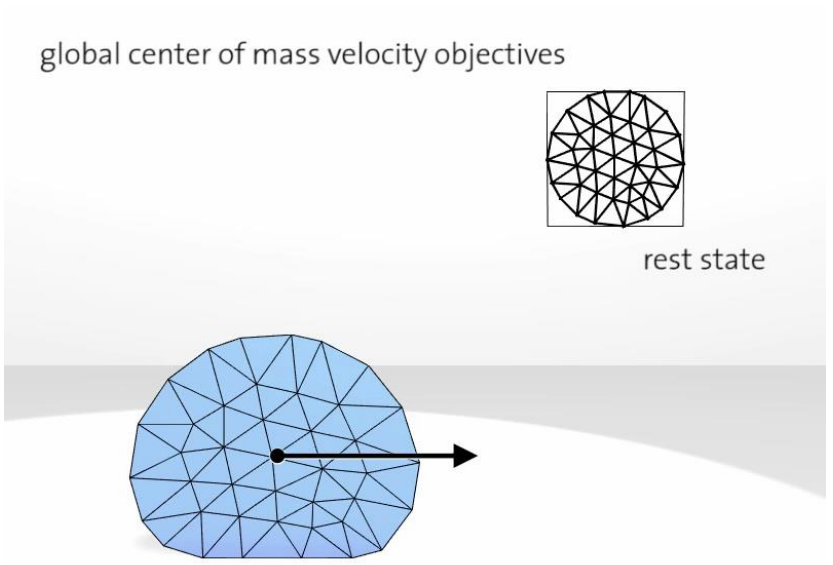
Find $\mathbf{x}, \mathbf{X}$ to minimize $\mathbf{g}(\mathbf{x}, t)$

$$s.t.$$

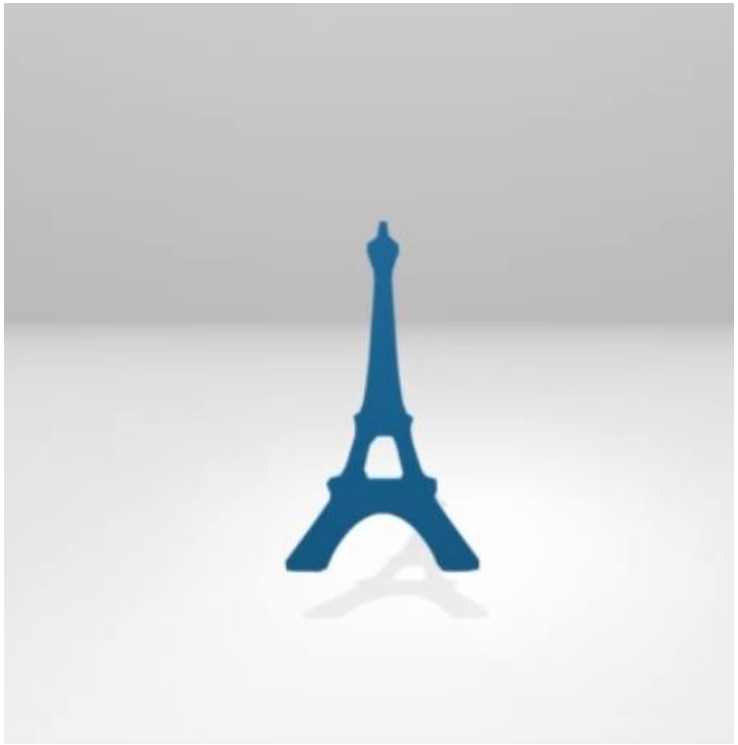
$$\mathbf{M}\left(\frac{\mathbf{x}}{h^2} - \frac{\mathbf{x}_n}{h^2} - \frac{\mathbf{v}_n}{h}\right) = \mathbf{f}_{\text{int}}(\mathbf{X}, \mathbf{x}) + \mathbf{f}_{\text{ext}}(\mathbf{x})$$

Optimizing Rest Configuration





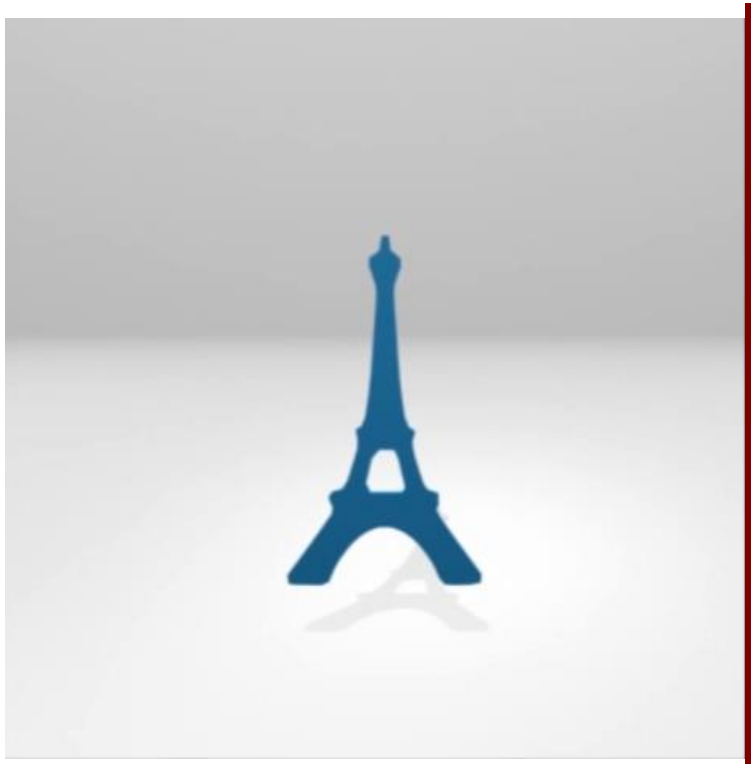
On to the real world…

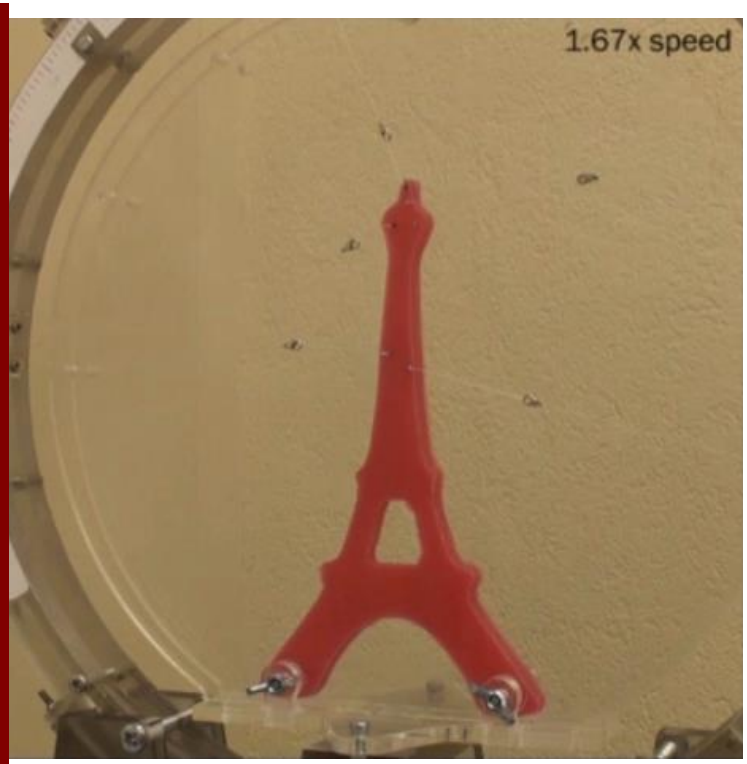


Why?

Animating Physical Objects

Computational Design of Actuated Deformable Characters, Skouras et al., 2013

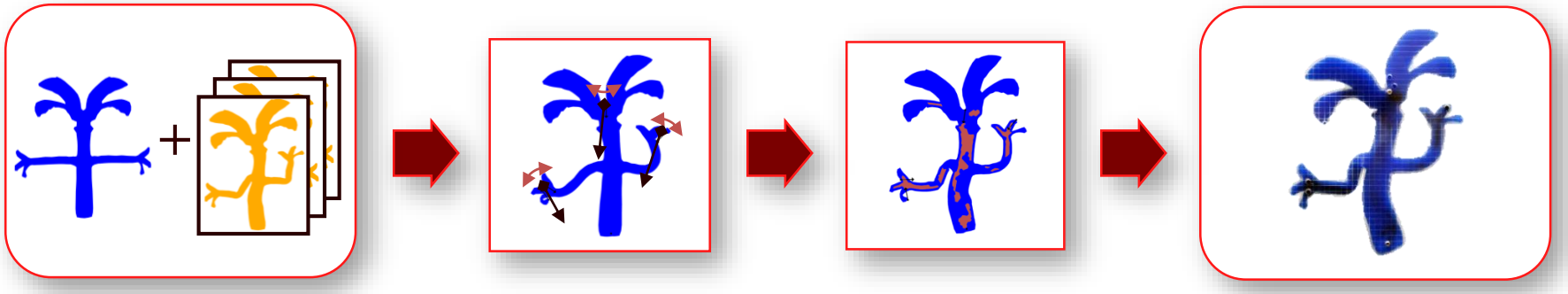




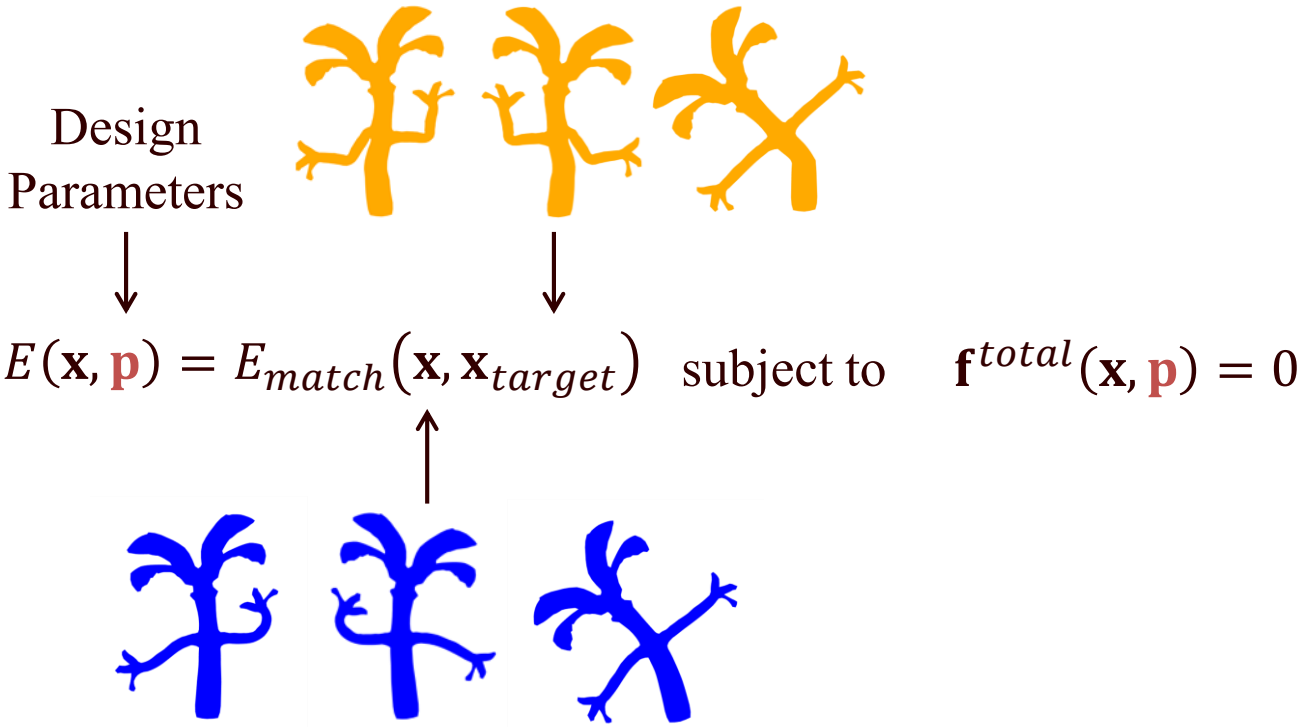
Input Animation

Fabricated Prototype

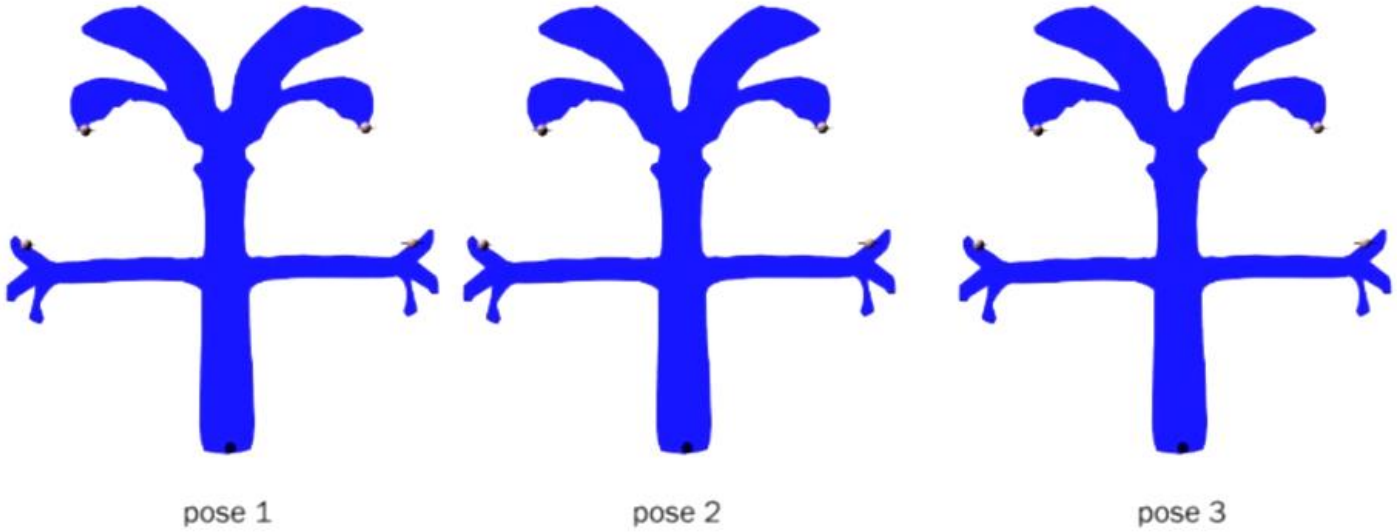
Pipeline


Input and Target Shapes
Actuator Location Optimization
Material Optimization
Fabricated Deformable Model

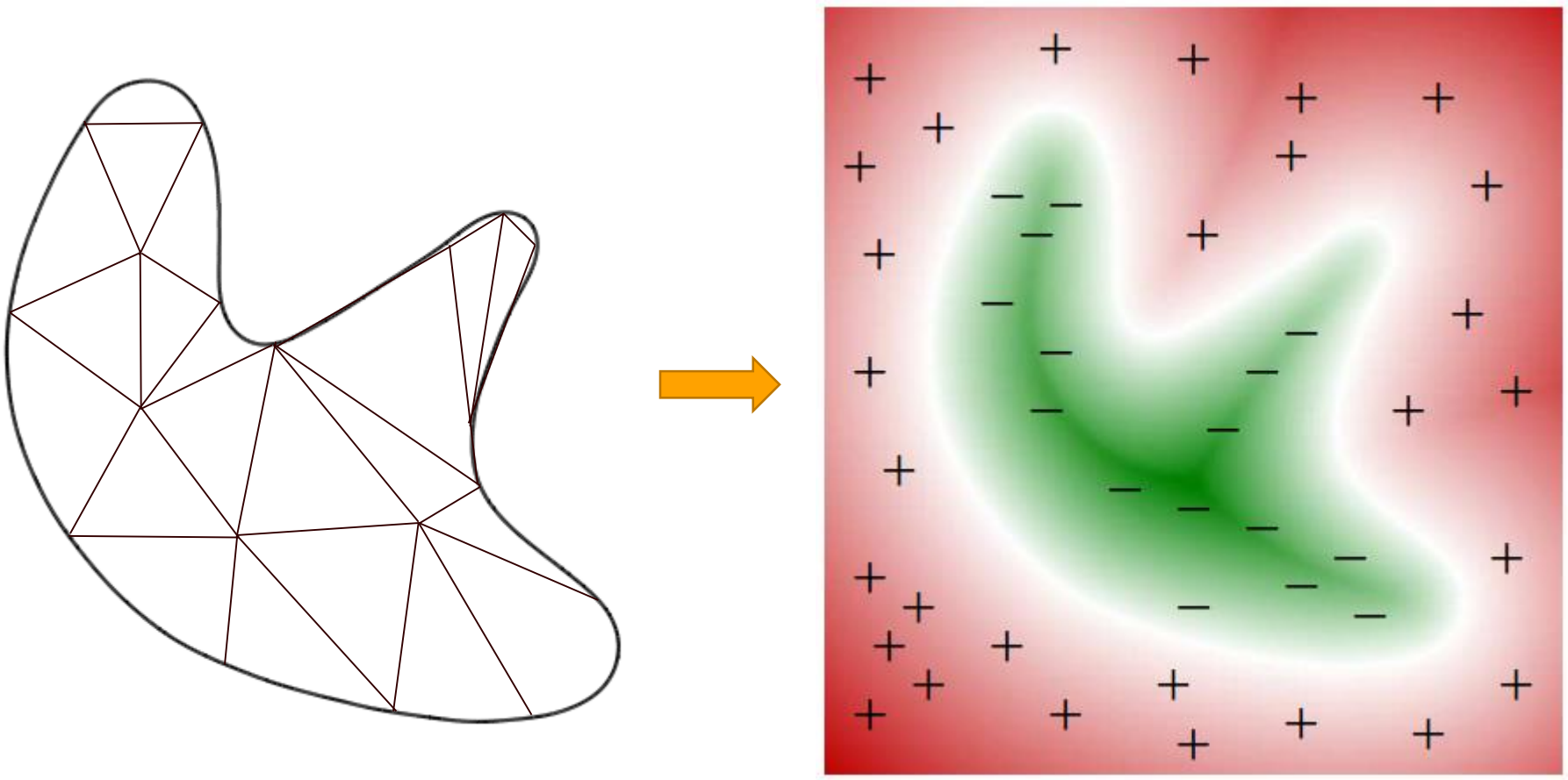
Mathematical Formulation



Actuator Location Optimization

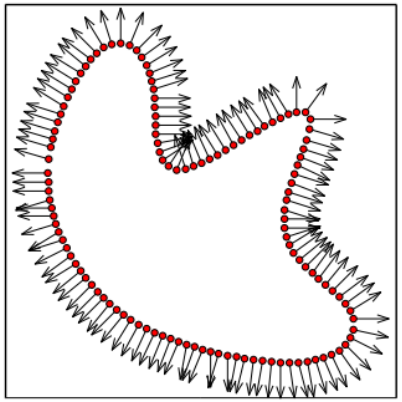


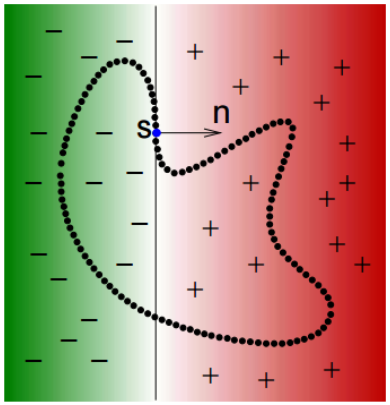
Actuator Location Optimization



Implicit Moving Least Squares

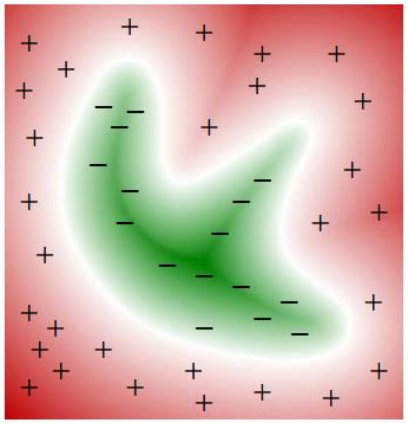
Input: Samples with normal vectors.





Build point function for each sample.

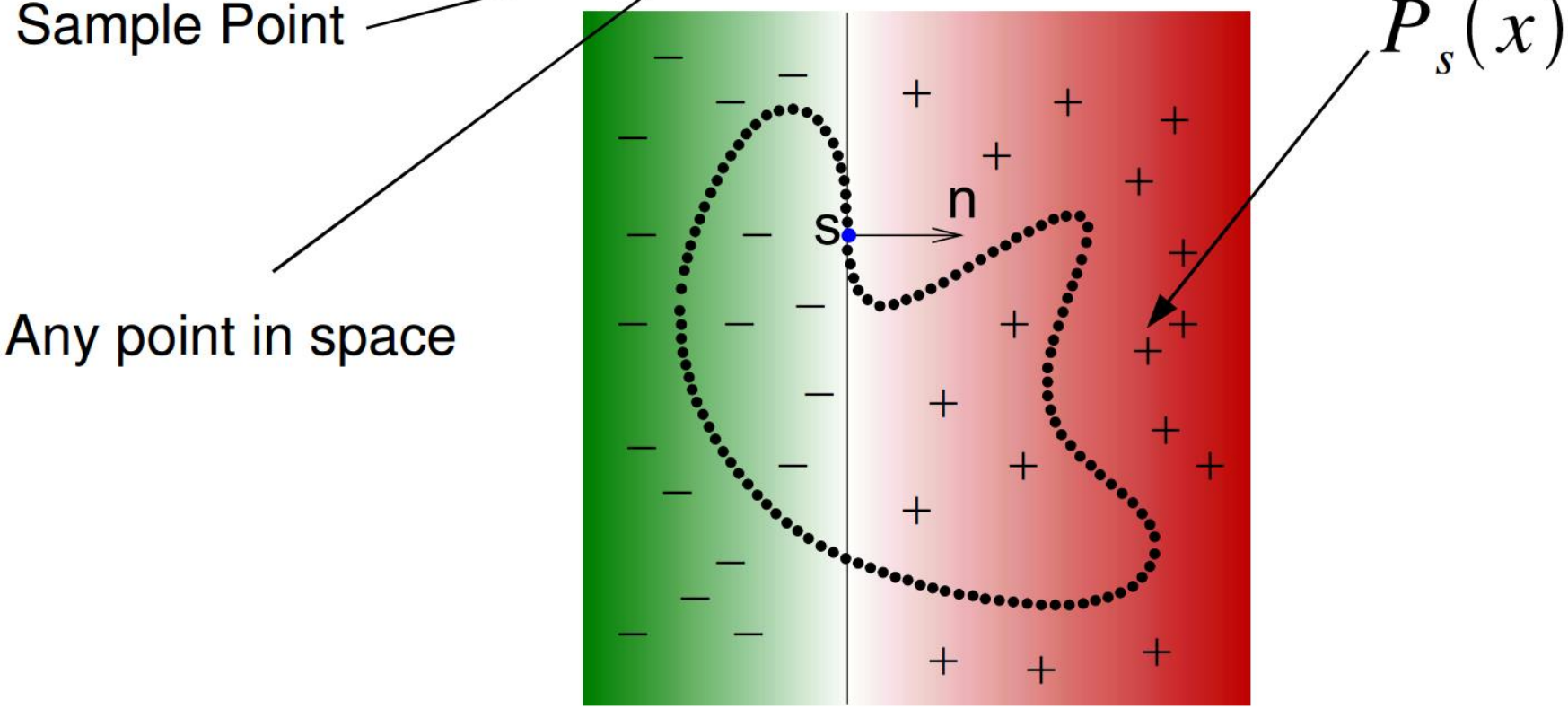
Weighted average of point functions (Gaussian weight functions).



Implicit Moving Least Squares

- Point functions: Signed distance to tangent plane

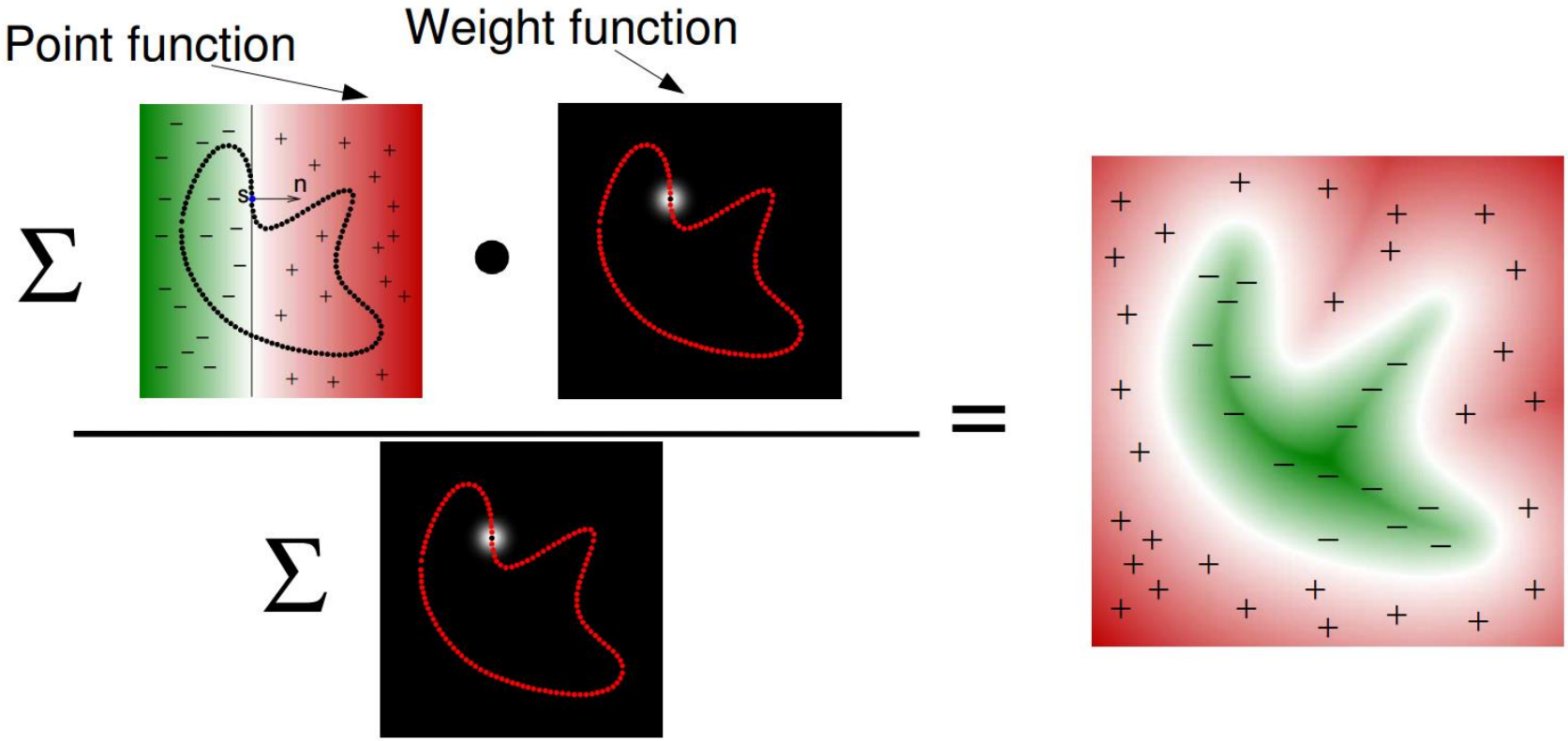
$$P_s(x) = (x - s) \cdot n$$



Implicit Moving Least Squares

Weighted average of point functions.

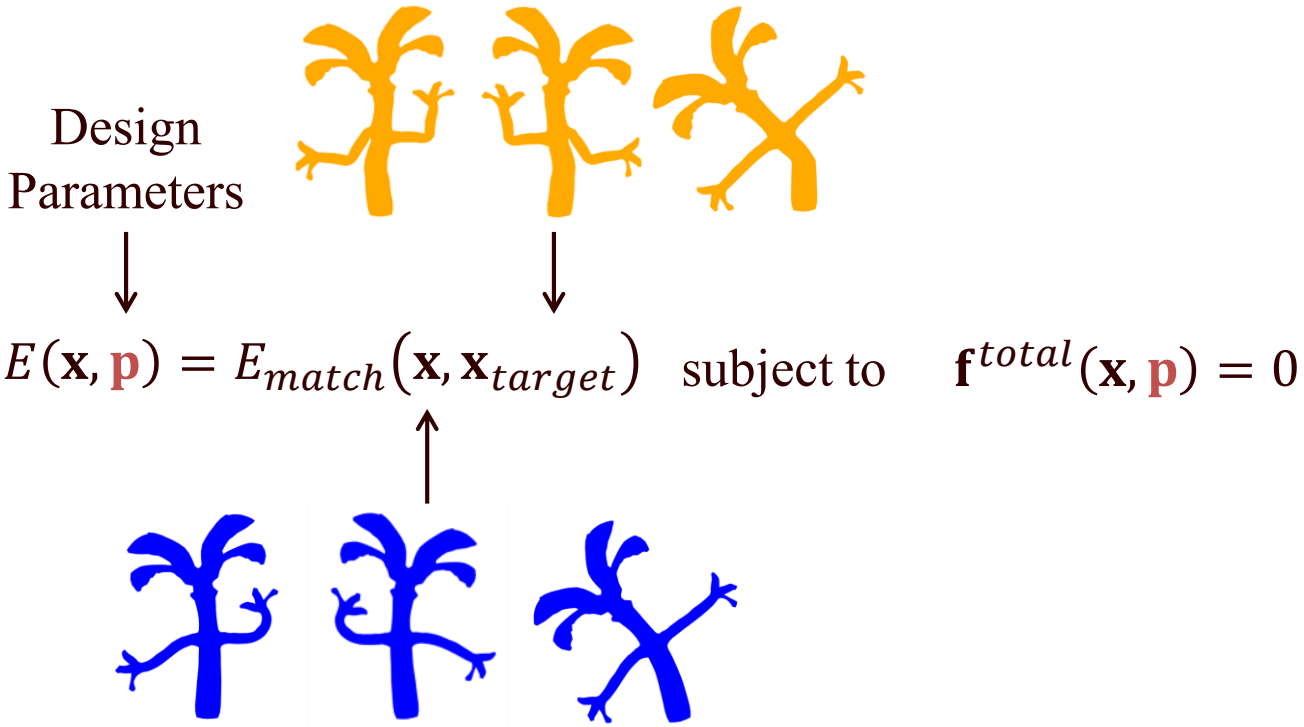
$$I(x) = \frac{\sum_s P_s(x) W_s(x)}{\sum_t W_t(x)}$$



Implicit Moving Least Squares

- Shen, O'Brien, and Shewchuk, “Interpolating and Approximating Surfaces from Polygon Soup,” SIGGRAPH 2004.

Actuator Location Optimization



What are the design parameters?

Actuator Optimization

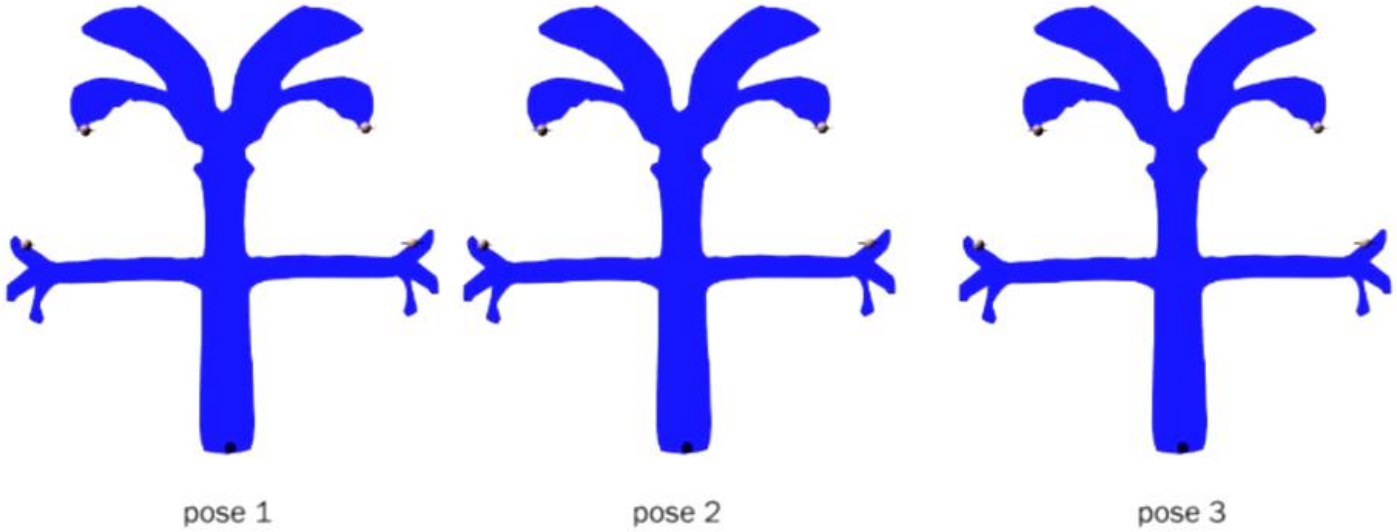
- For every actuation point:

$$\mathcal{Q} = (\mathbf{l}, \mathbf{q}^1, \ldots, \mathbf{q}^{n_p})$$

- $\mathbf{l}$: actuator location (in rest space!)
  - if it needs to lie on border, add new objective: $\mathrm{I}(\mathbf{l})^2$
- q: actuator force (in world space!)
  - must distribute it to nodes using weighting functions

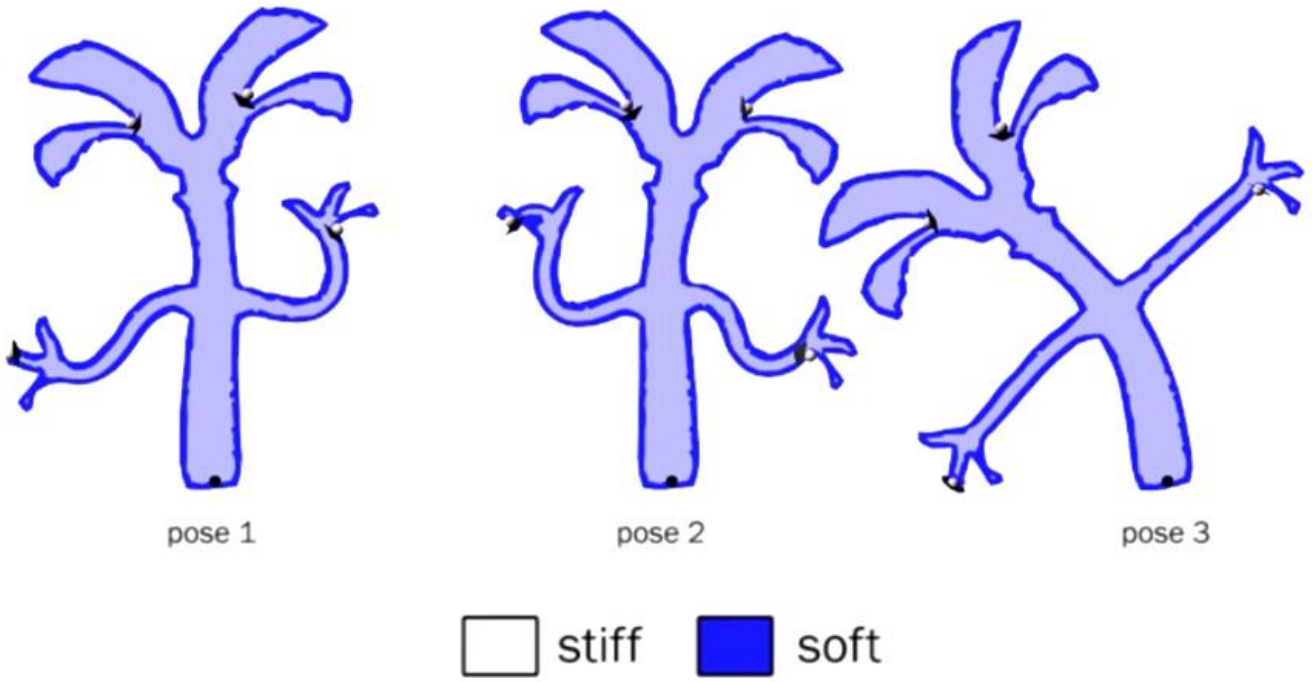
$$\mathbf{f}_k^i = \frac{\mathbf{q}_j^i \phi_k(\mathbf{l}_j)}{\sum_{l \in \mathcal{S}_j} \phi_l(\mathbf{l}_j)}$$

Actuator Optimization

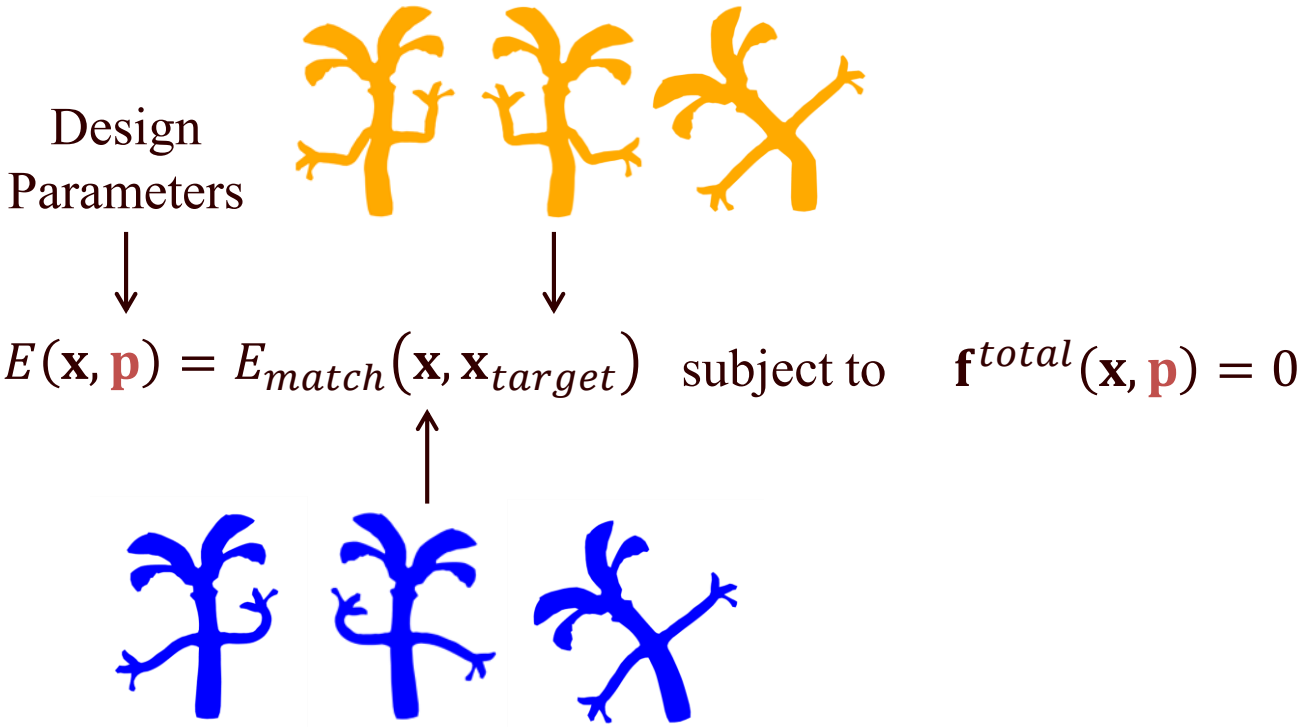


Results

Material Distribution Optimization



Material Distribution Optimization



What are the design parameters?

Material Distribution Optimization

- Interpolation between two materials, on an element-by-element basis:

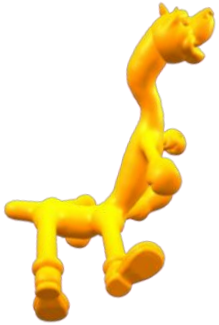
$$W(\mathbf{F}^e, \rho^e) = \rho^e \, W^1(\mathbf{F}^e) + (1 - \rho^e) \, W^2(\mathbf{F}^e)$$

- ρ^e is interpolation weight to be optimized
- must drive interpolation weights to 0 or 1

$$R_{\text{mat}} = k_{\text{mat}} \sum_e \rho^e (1 - \rho^e)$$

Results

Rest Pose



Target Pose

Stiff Soft

Other fun applications

- "Sag-free" physical objects
  - "An Asymptotic Numerical Method for Inverse Elastic Shape Design", Chen et al, 2014
- Designing rubber balloons
  - "Computational Design of Rubber Balloons", Skouras et al, 2012
- Copy+paste real-world objects
  - "Design and Fabrication of Materials with Desired Deformation Behavior", Bickel et al, 2010

Elastic models can be used for many other things

- Analyzing/optimizing structural properties
- Explore relationship between Form and Function (soft robots, compliant mechanisms, etc)

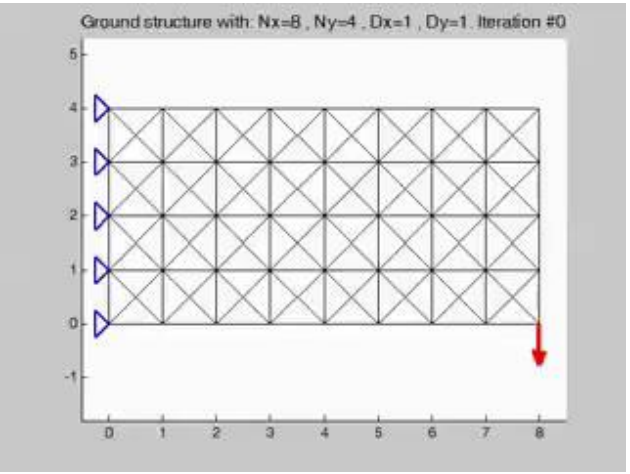
Topology Optimization

- Main idea:
  - Start with initial object
  - Iteratively remove unneeded material
  - Strike a balance between weight and strength
  - Concepts useful for anything from 3D printed objects to bridge design



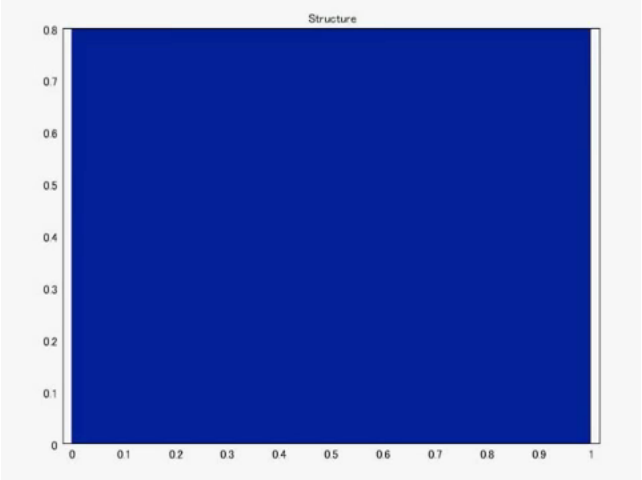
Topology Optimization

- The ground structure approach
  - model object using network of truss/beam elements
  - vary cross-section (stiffness) of every element

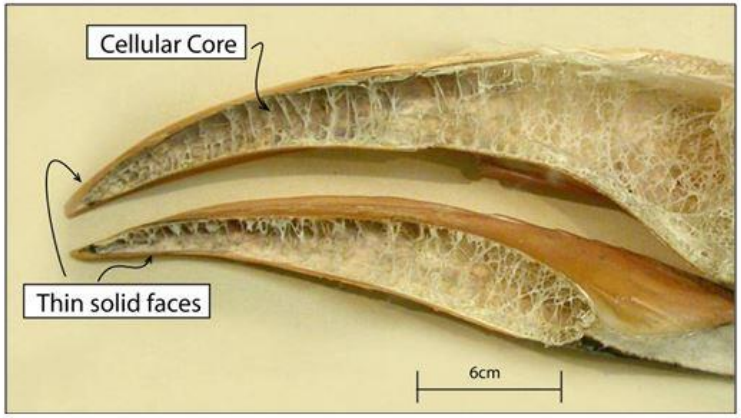


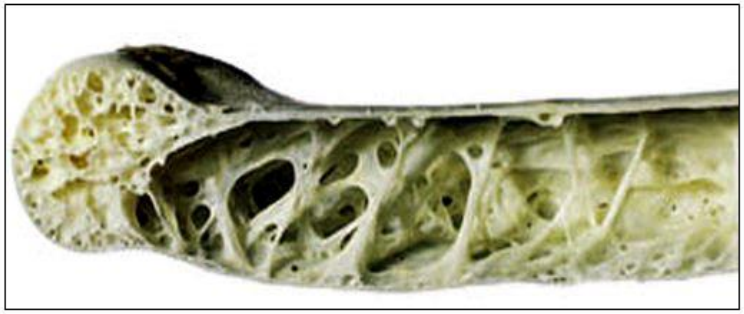
Topology Optimization

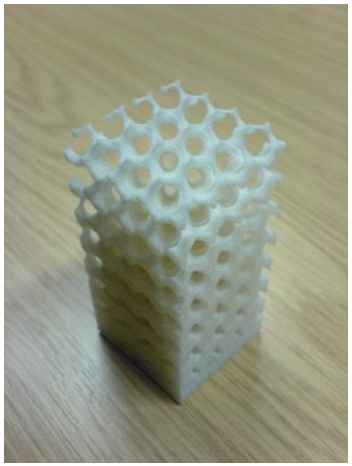
- Continuous material density approach
  - more general – vary material density anywhere in simulation domain
  - sometimes leads to emergence of interesting micro-structures

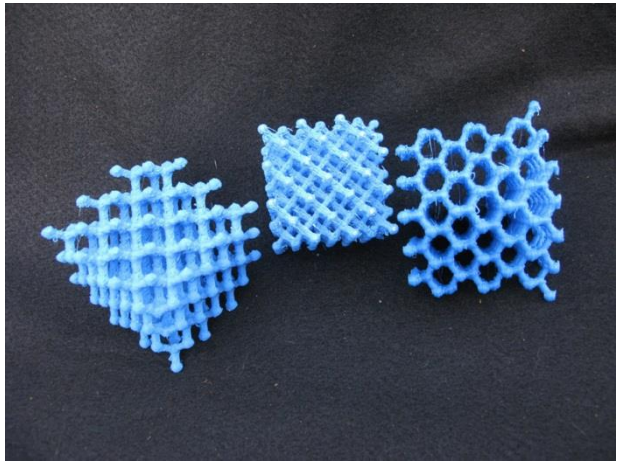


Microstructures



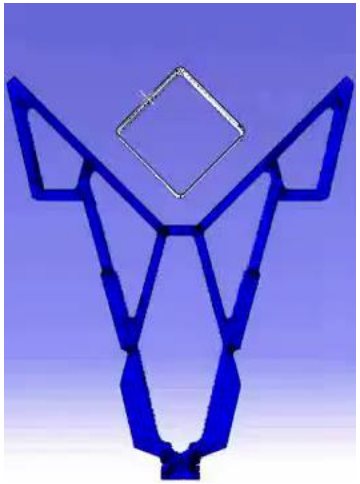


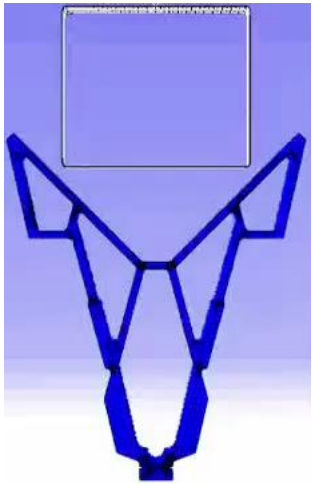




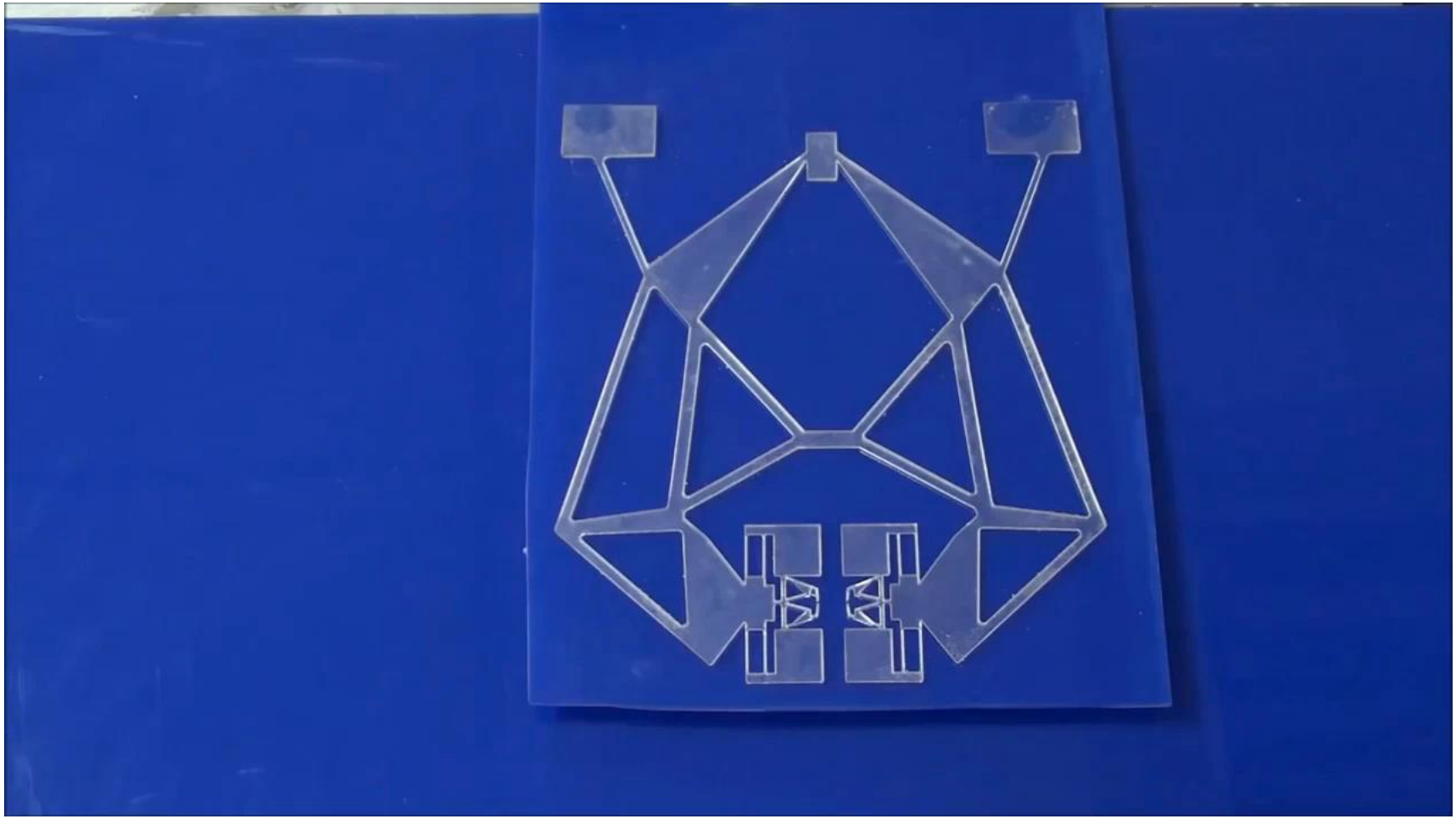
Beyond static structures

Compliant mechanisms





Compliant Mechanism to Clamp and Stretch Soft Objects



Modeling Soft Robots

That’s it for today

- Questions?
- Have a nice break!