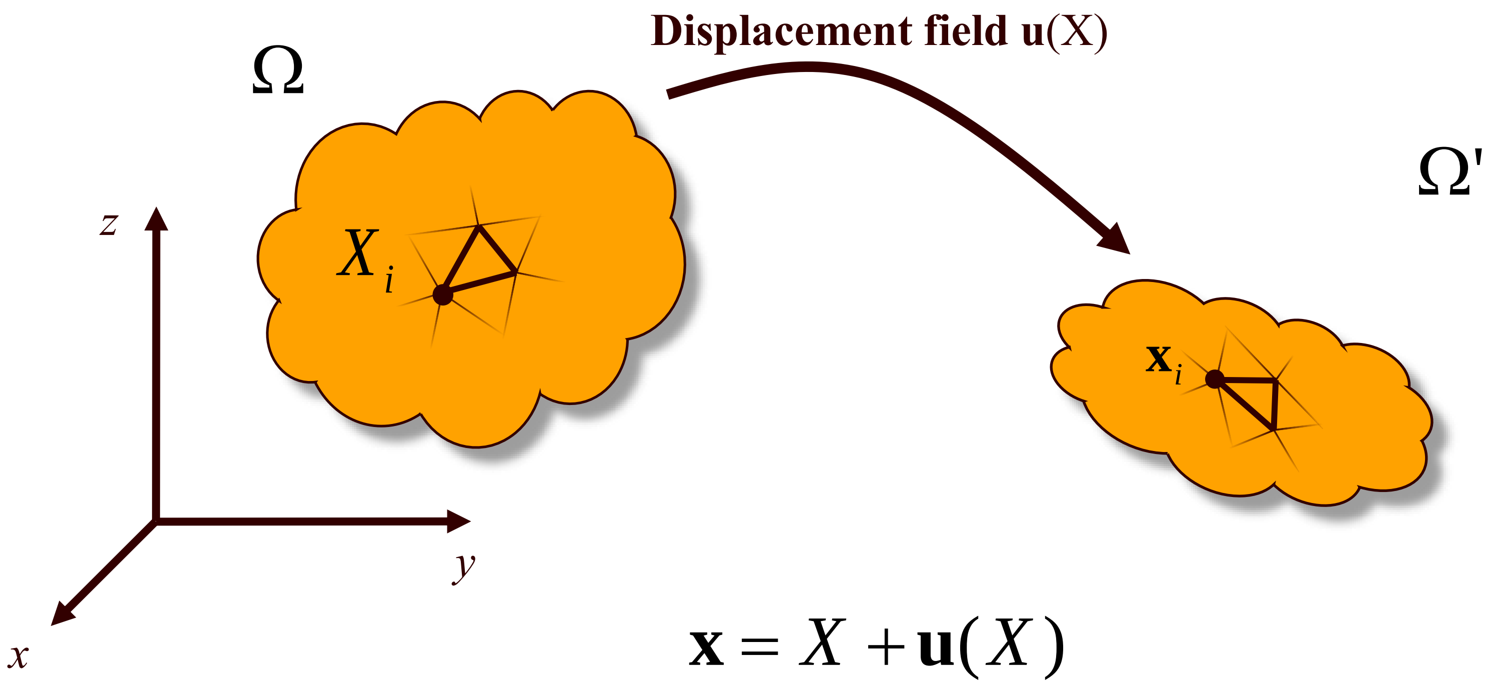
Fun With Elastica

Assignment 2

- Due on March ~~2nd~~ 10th @ midnight

Continuum Mechanics And the Finite Element Method



$$\mathbf{x} = X + \mathbf{u}(X)$$

Deformation gradient: $\mathbf{F} = \dfrac{\partial \mathbf{x}}{\partial X} = (\mathbf{I} + \nabla \mathbf{u})$

If strain assumed constant per element: $\mathbf{F} = \boldsymbol{e}\boldsymbol{E}^{-1}$

FEM recipe

St. Venant-Kirchhoff material

$$\mathbf{E} = \frac{1}{2}(\mathbf{F}^{\mathsf{T}}\mathbf{F} - \mathbf{I})$$

$$\Psi = \mu \|\mathbf{E}\|_{\mathrm{F}} + \frac{\lambda}{2}\mathrm{tr}^2(\mathbf{E})$$

$$\mathbf{P} = \mathbf{F}\,[2\mu\mathbf{E} + \lambda\mathrm{tr}(\mathbf{E})\mathbf{I}]$$

Neohookean elasticity

$$\mathrm{I}_1 = \|\mathbf{F}\|_{\mathrm{F}}^2, \quad \mathrm{J} = \det \mathbf{F}$$

$$\Psi = \frac{\mu}{2}(\mathrm{I}_1 - 3) - \mu \log(\mathrm{J}) + \frac{\lambda}{2}\log^2(\mathrm{J})$$

$$\mathbf{P} = \mu(\mathbf{F} - \mathbf{F}^{-\mathsf{T}}) + \lambda \log(\mathrm{J})\mathbf{F}^{-\mathsf{T}}$$

Area/volume of element

$$\boldsymbol{f} = -\frac{\partial W}{\partial \boldsymbol{x}} = -V\frac{\partial \Psi}{\partial \boldsymbol{F}}\frac{\partial \boldsymbol{F}}{\partial \boldsymbol{x}}$$

First Piola-Kirchhoff stress tensor **P**

FEM recipe

St. Venant-Kirchhoff material

$$\mathbf{E} = \frac{1}{2}(\mathbf{F}^T\mathbf{F} - \mathbf{I})$$

$$\Psi = \mu\|\mathbf{E}\|_F + \frac{\lambda}{2}\mathrm{tr}^2(\mathbf{E})$$

$$\mathbf{P} = \mathbf{F}\,[2\mu\mathbf{E} + \lambda\mathrm{tr}(\mathbf{E})\mathbf{I}]$$

Neohookean elasticity

$$I_1 = \|\mathbf{F}\|_F^2, \quad J = \det\mathbf{F}$$

$$\Psi = \frac{\mu}{2}(I_1 - 3) - \mu\log(J) + \frac{\lambda}{2}\log^2(J)$$

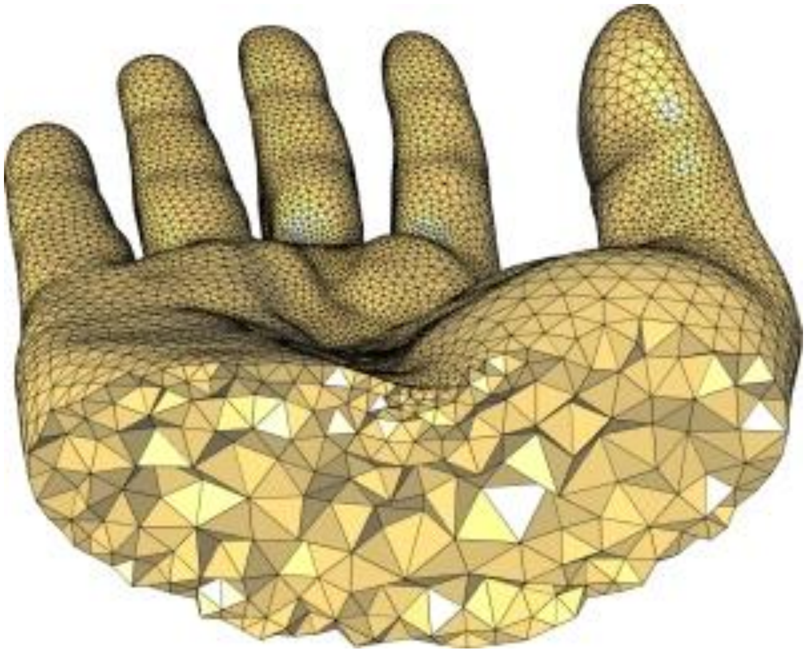
$$\mathbf{P} = \mu(\mathbf{F} - \mathbf{F}^{-T}) + \lambda\log(J)\mathbf{F}^{-T}$$

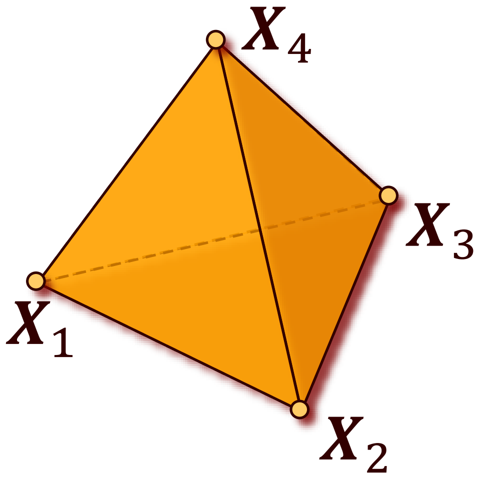
For a tetrahedron, this works out to:

$$[\boldsymbol{f}_1 \;\; \boldsymbol{f}_2 \;\; \boldsymbol{f}_3] = -V\boldsymbol{P}\boldsymbol{E}^{-T}; \boldsymbol{f}_4 = -\boldsymbol{f}_1 - \boldsymbol{f}_2 - \boldsymbol{f}_3$$

Additional reading: http://www.femdefo.org/

Tetrahedral Meshes





Statics Vs Dynamics

Principle of minimum potential energy

A mechanical system in static equilibrium will assume a state of minimum potential energy: find **x** to minimize W(**x**), or equivalently, such that $f_{int}(\mathbf{x}) + f_{ext} = 0$

Statics

Goal: find equilibrium configuration i.e., $\mathbf{f}_i = 0\ \forall i$

Given $\mathbf{x}$ with $\mathbf{f}(\mathbf{x}) \neq 0$, find $\Delta\mathbf{x}$ such that $\mathbf{f}(\mathbf{x}+\Delta\mathbf{x}) = \mathbf{0}$

$$\mathbf{f}(\mathbf{x}+\Delta\mathbf{x}) = \mathbf{f}(\mathbf{x}) + \mathbf{K}\Delta\mathbf{x} + O(\Delta\mathbf{x}^2)$$

⇨ Solve $\mathbf{K}\Delta\mathbf{x} = -\mathbf{f}(\mathbf{x})$ for $\Delta\mathbf{x}$

Stiffness matrix

$$\mathbf{K} = \frac{\partial \mathbf{f}}{\partial \mathbf{x}}$$

Statics

$$\min_x W(x) + x^T f$$

Dynamics – a variational formulation

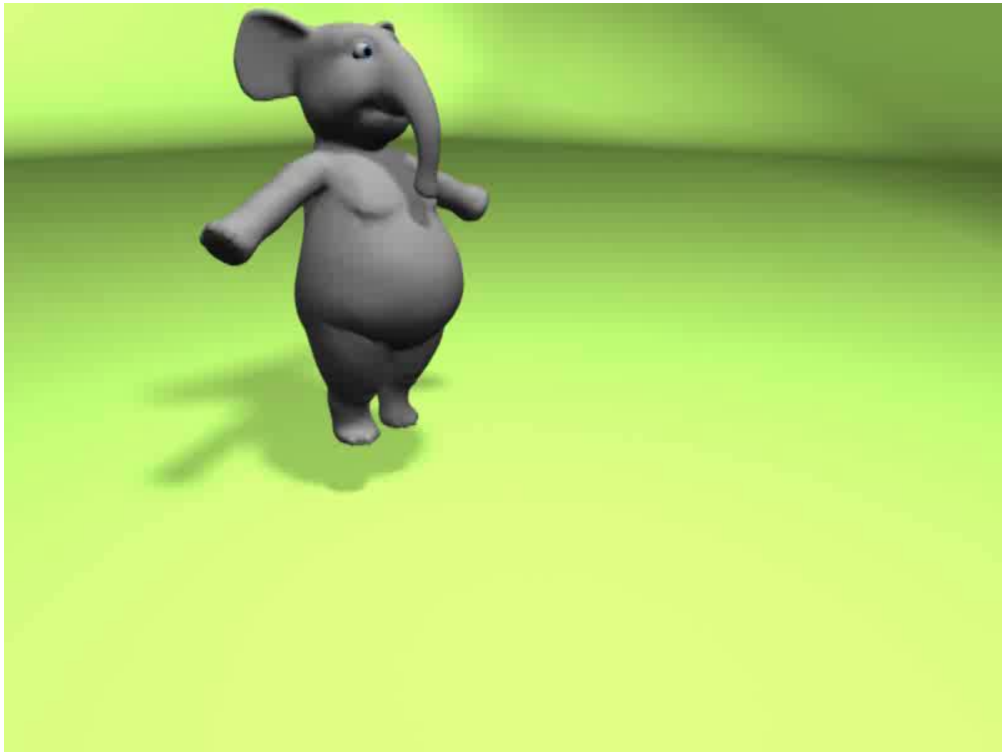
$$\min_{x} \frac{h^2}{2} a^T M a + W(x) + x^T f$$

$$a = \frac{(x - x_{old})}{h^2} - \frac{v_{old}}{h}$$

Fun things to do with simulation

Editing Simulation Results

Relatively easy to run physics-based simulations, but…



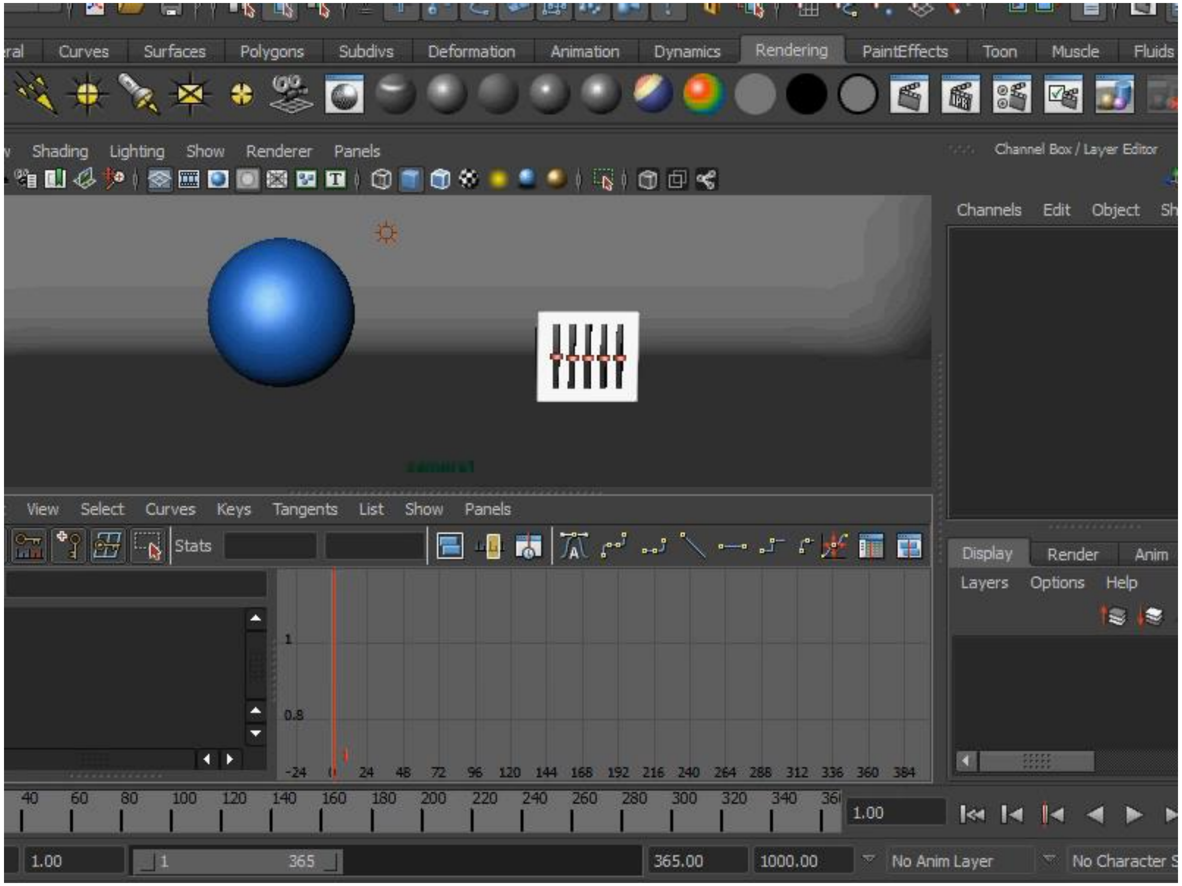
hard to edit

no artist control

different workflow

Editing Simulation Results

Animation rig

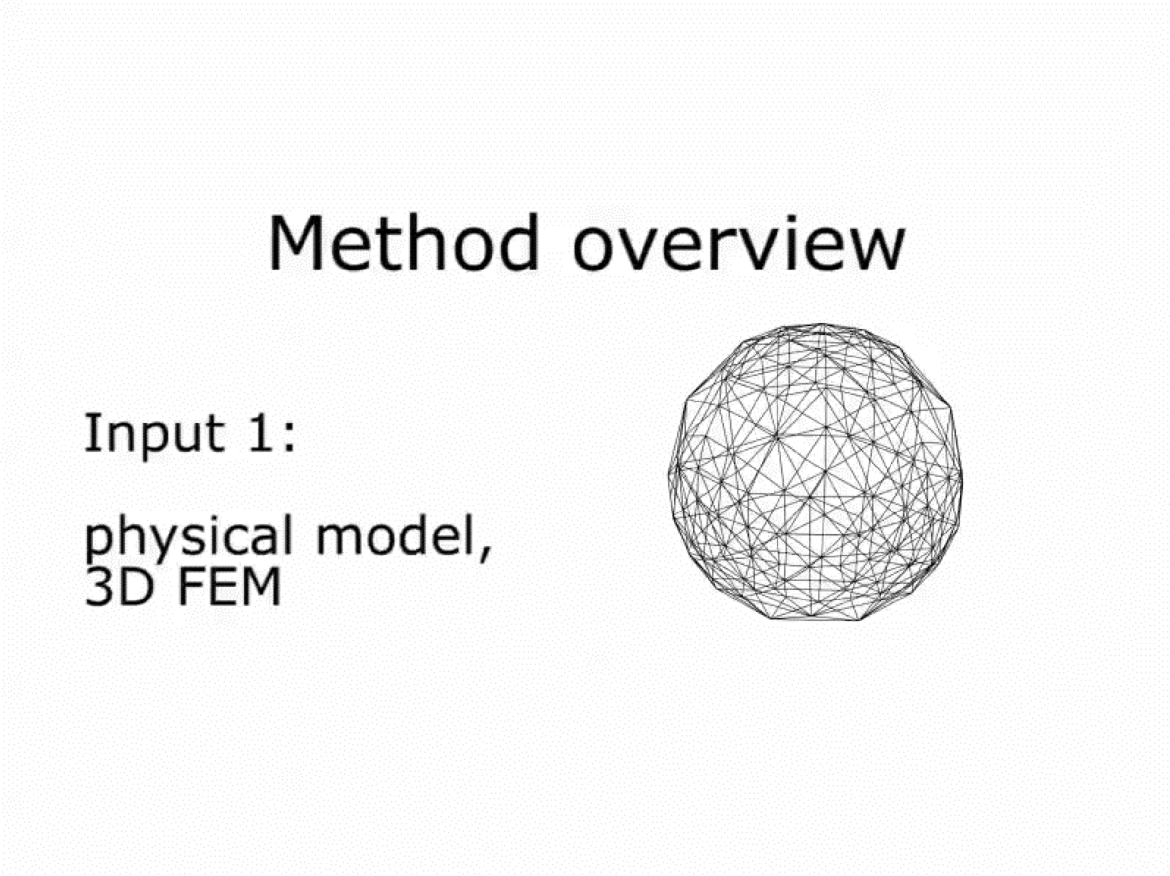


Editing Simulation Results

Goal: Bring benefits of physical simulation to traditional animation pipeline

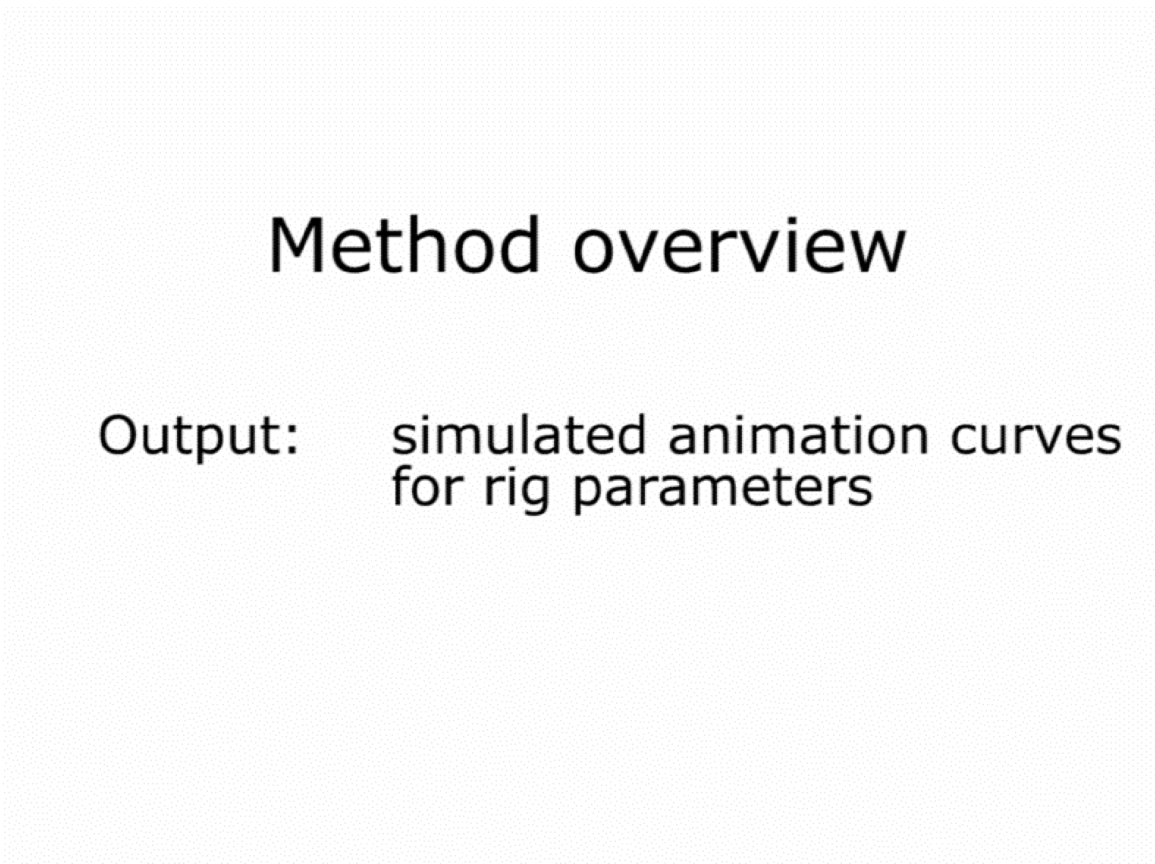
Editing Simulation Results

Rig-Space Physics, Hahn et al., 2012



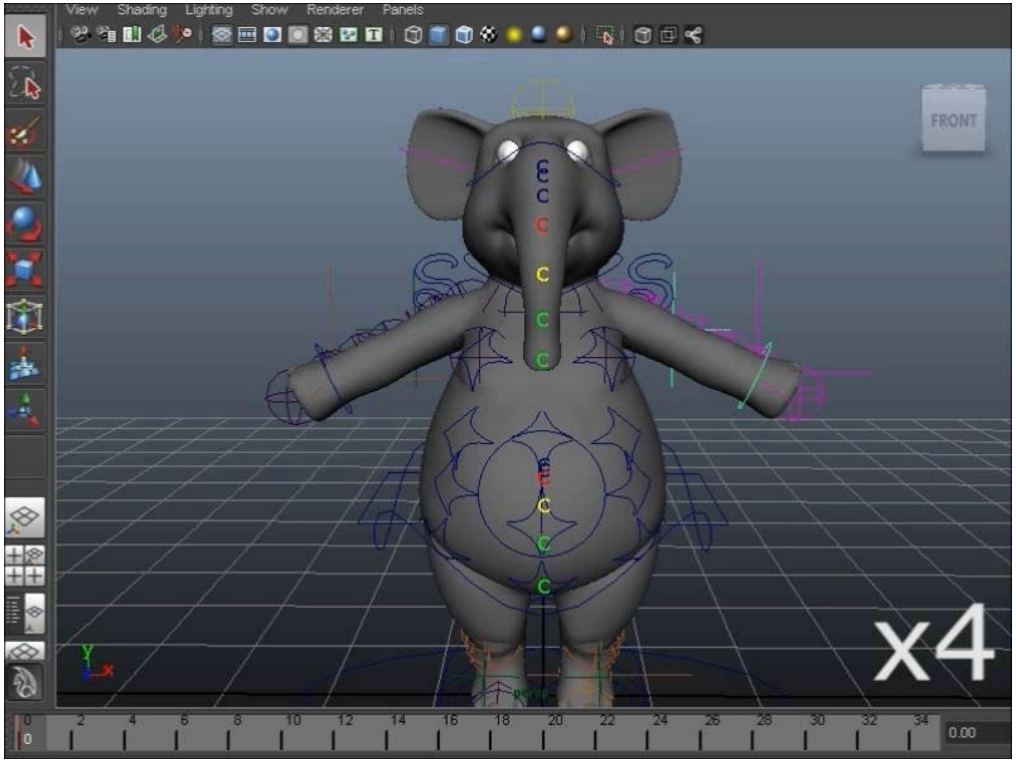
Editing Simulation Results

Rig-Space Physics, Hahn et al., 2012



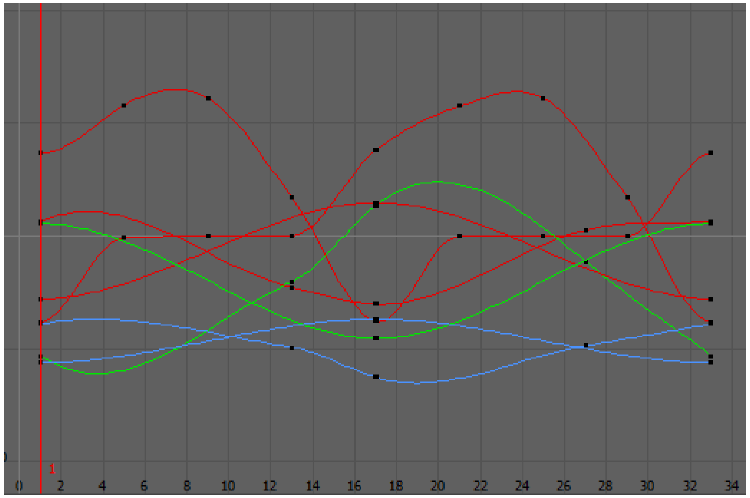
Editing Simulation Results

A more complex animation rig

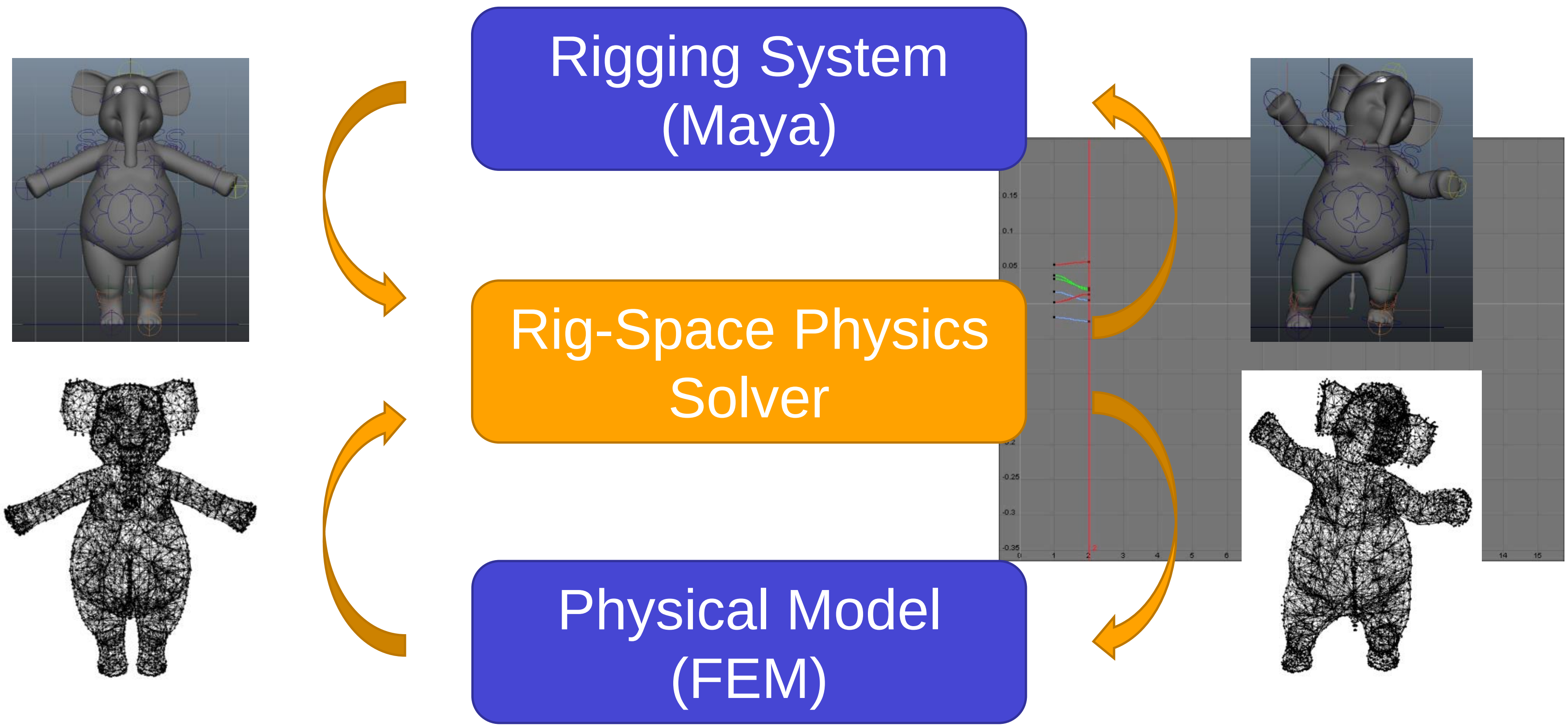


Editing Simulation Results

Animations created by keyframing rig parameters



Physics in Rig-Space

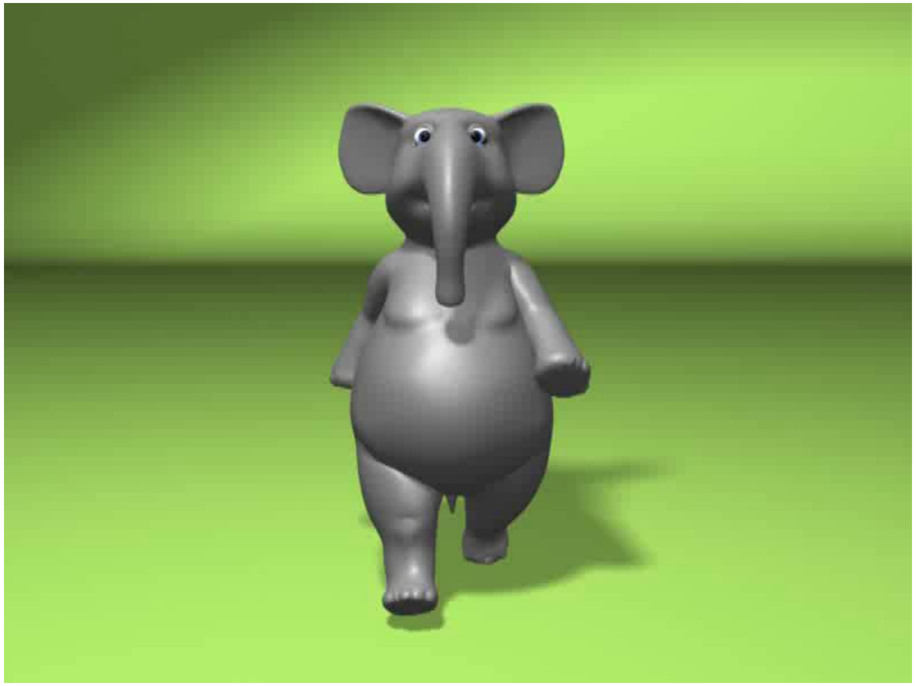


Subspace Simulation

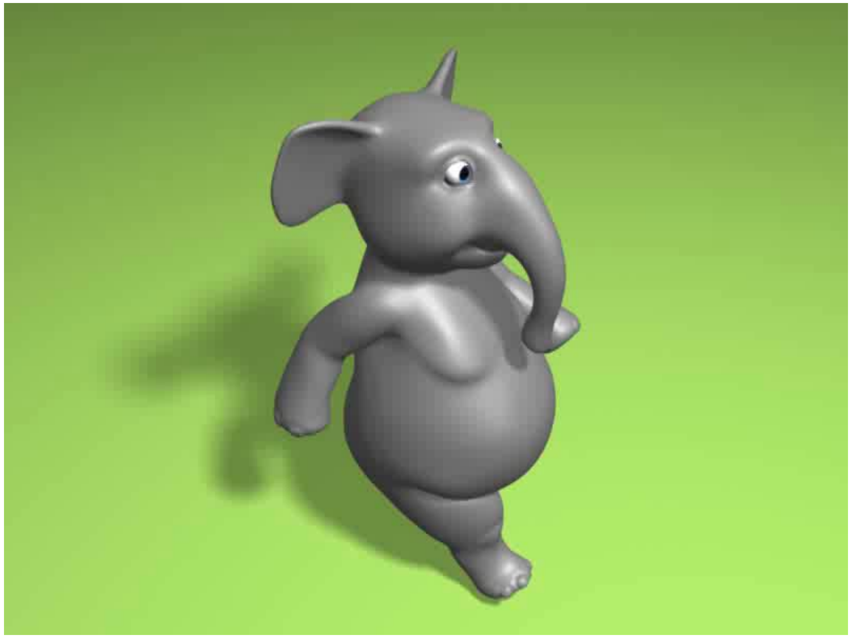
Find ~~$\mathbf{x}$~~ $\mathbf{p}$ $s.t.$

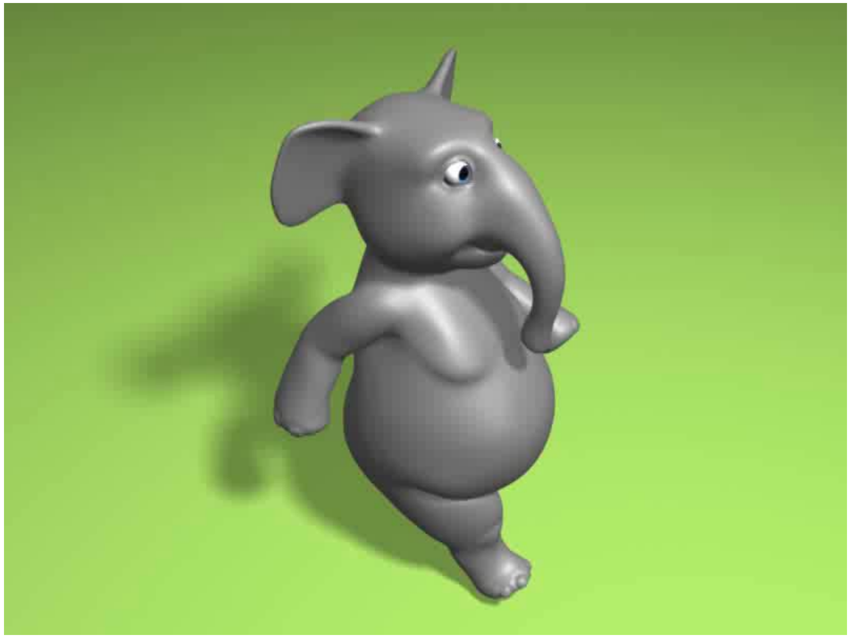
$$\mathrm{M}\left(\frac{\cancel{\mathbf{x}}\;\mathbf{x}(\mathbf{p})}{h^2} - \frac{\mathbf{x}_n}{h^2} - \frac{\mathbf{v}_n}{h}\right) = \mathbf{f}_{\mathrm{int}}(\mathbf{X}, \cancel{\mathbf{x}}\;\mathbf{x}(\mathbf{p})) + \mathbf{f}_{\mathrm{ext}}(\cancel{\mathbf{x}}\;\mathbf{x}(\mathbf{p}))$$

Physics in Rig-Space



Editing Simulation Results

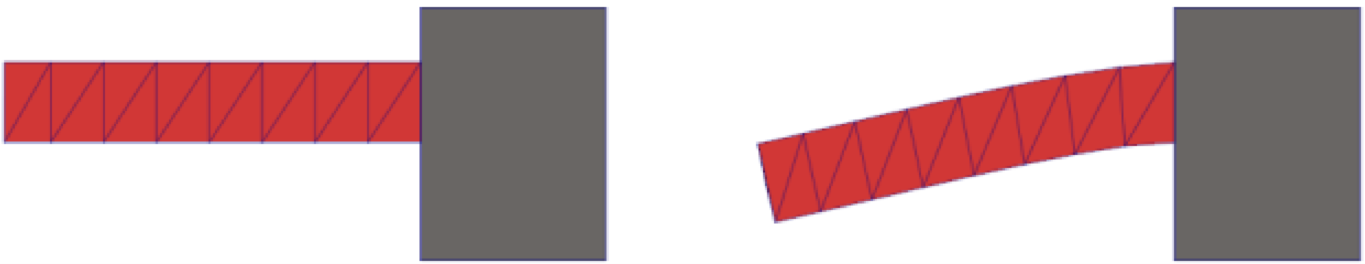




Fun things to do with simulation

Sag-free simulations

Optimization for sag-free simulations, Twigg and Kačić-Alesić, 2011



Sag-free simulations

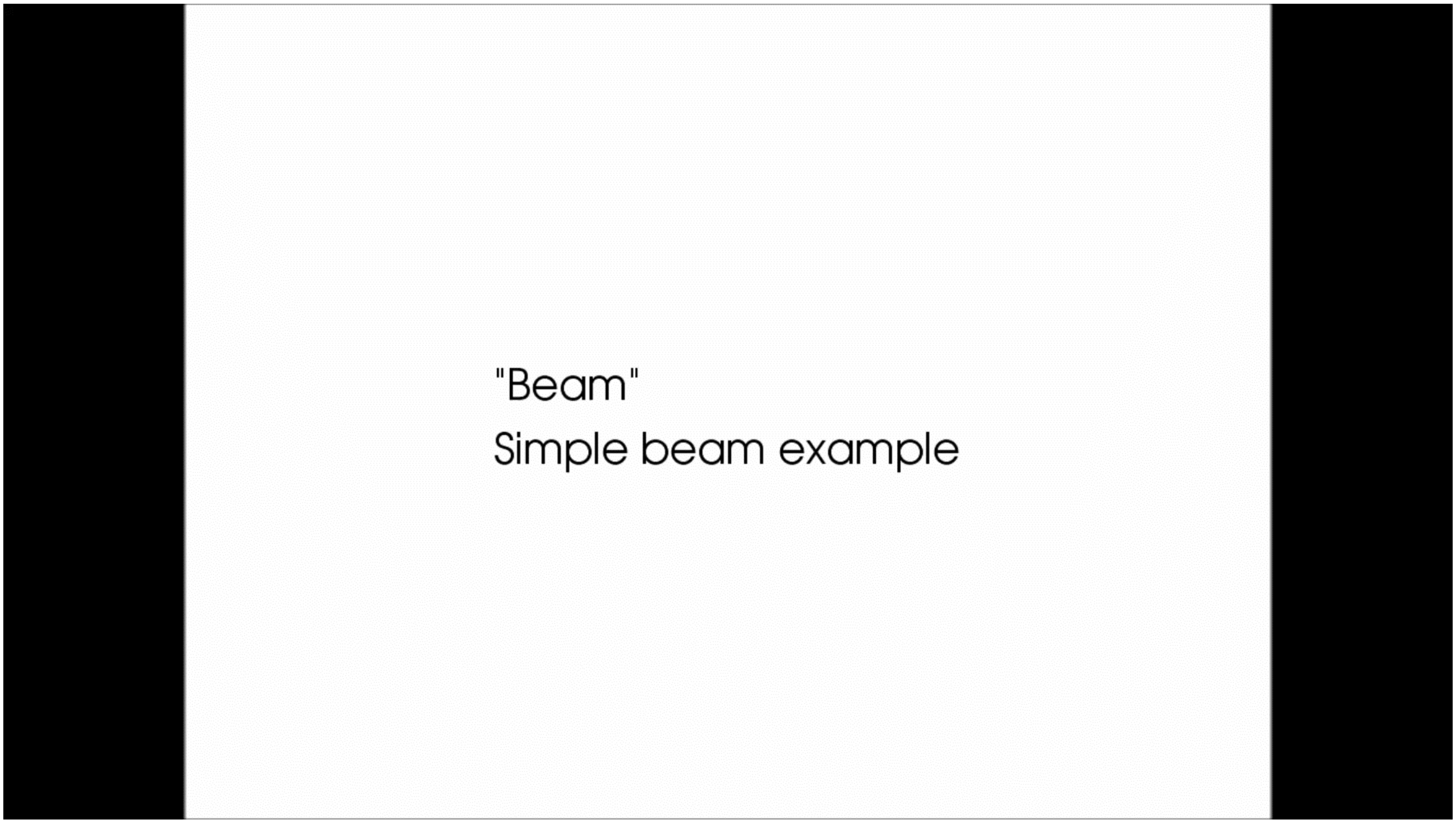
- Want:

$$\min_{X} | \mathbf{x}' - \arg\min_{\mathbf{x}} W(\mathbf{x}, X) - \mathbf{x}^T f |_2^2$$

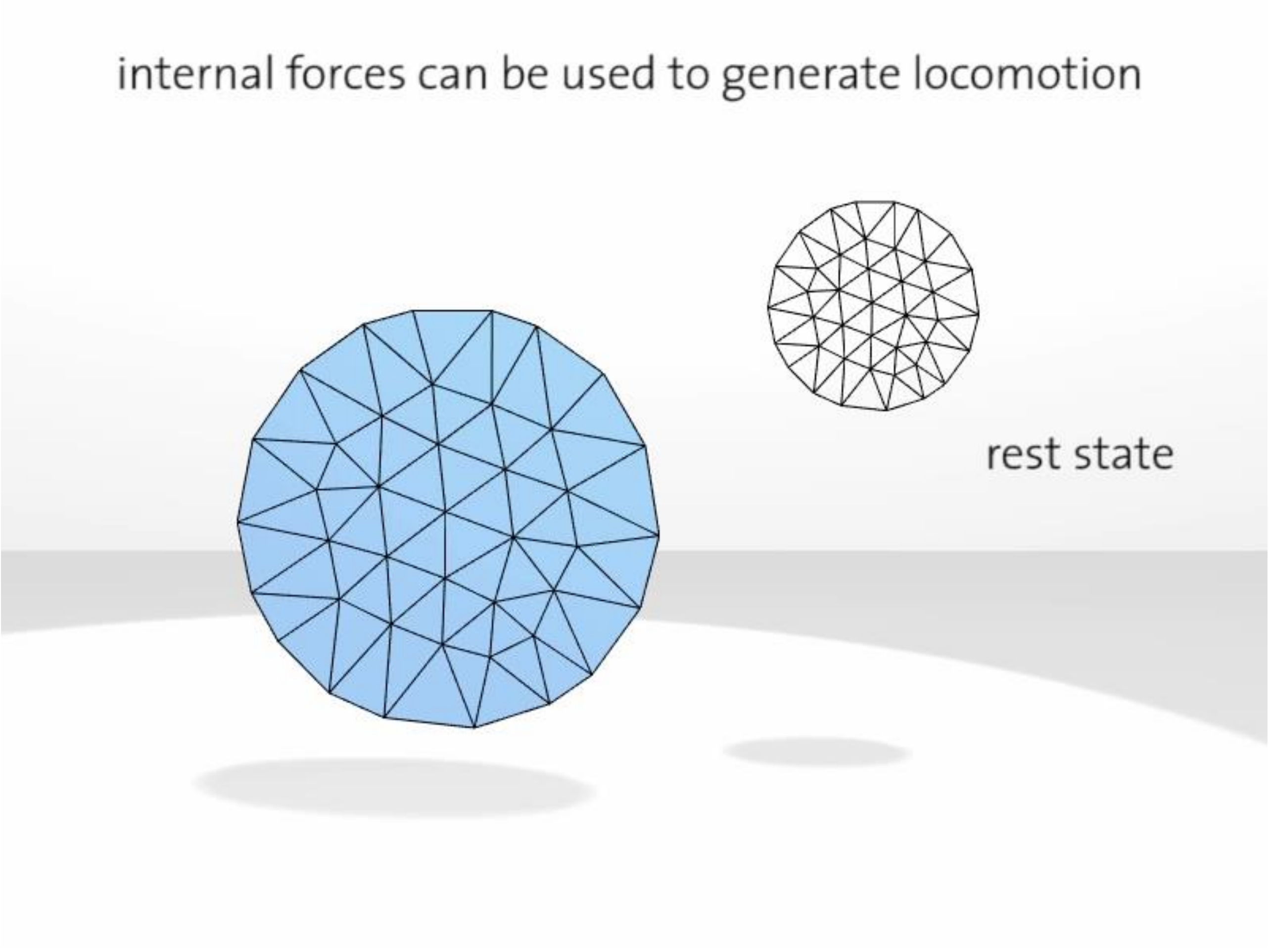
- But solve:

$$\min_{X} | \mathbf{f}(\mathbf{x}', X) |_2^2$$

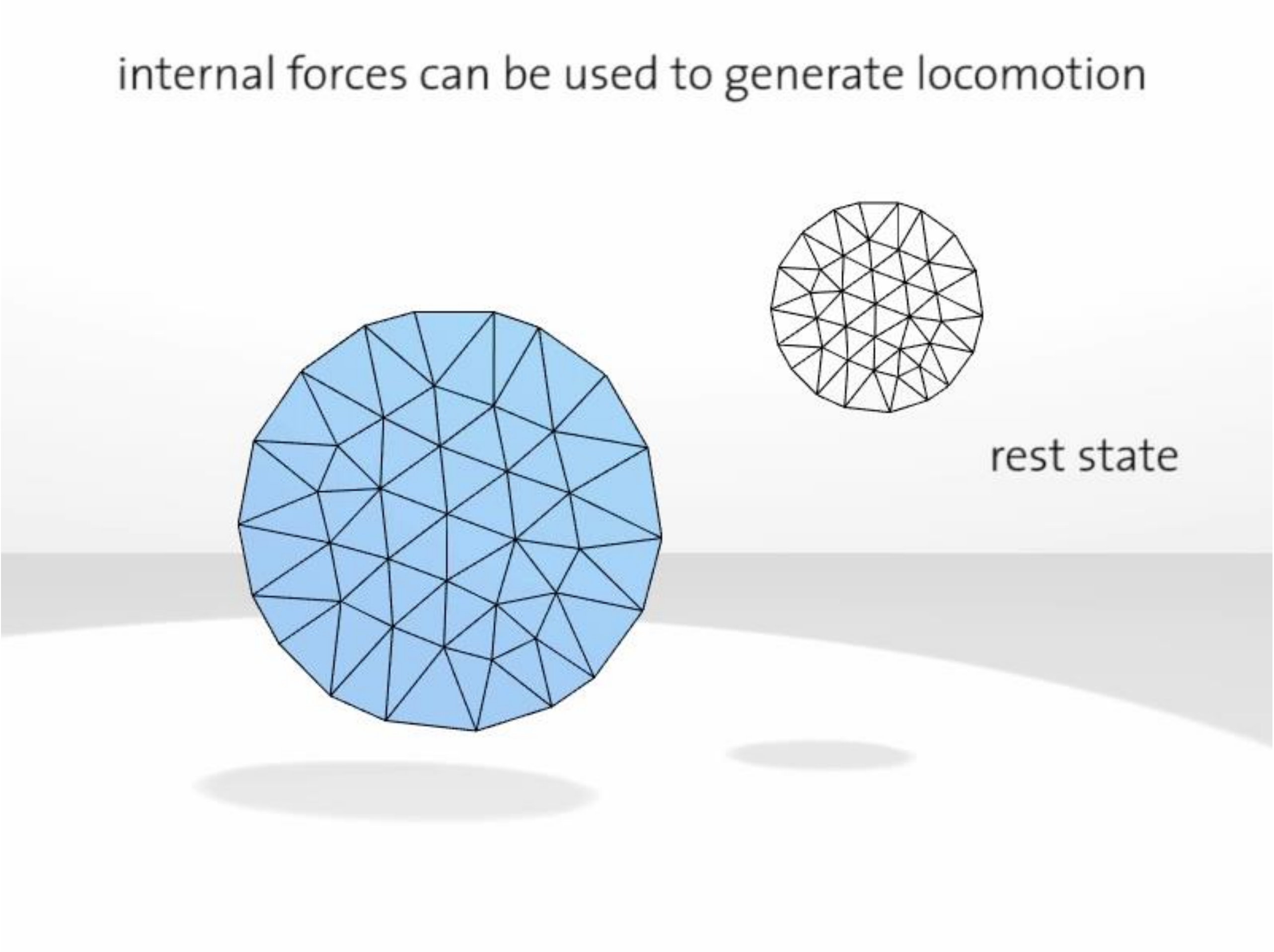
Sag-free simulations



Why stop here?

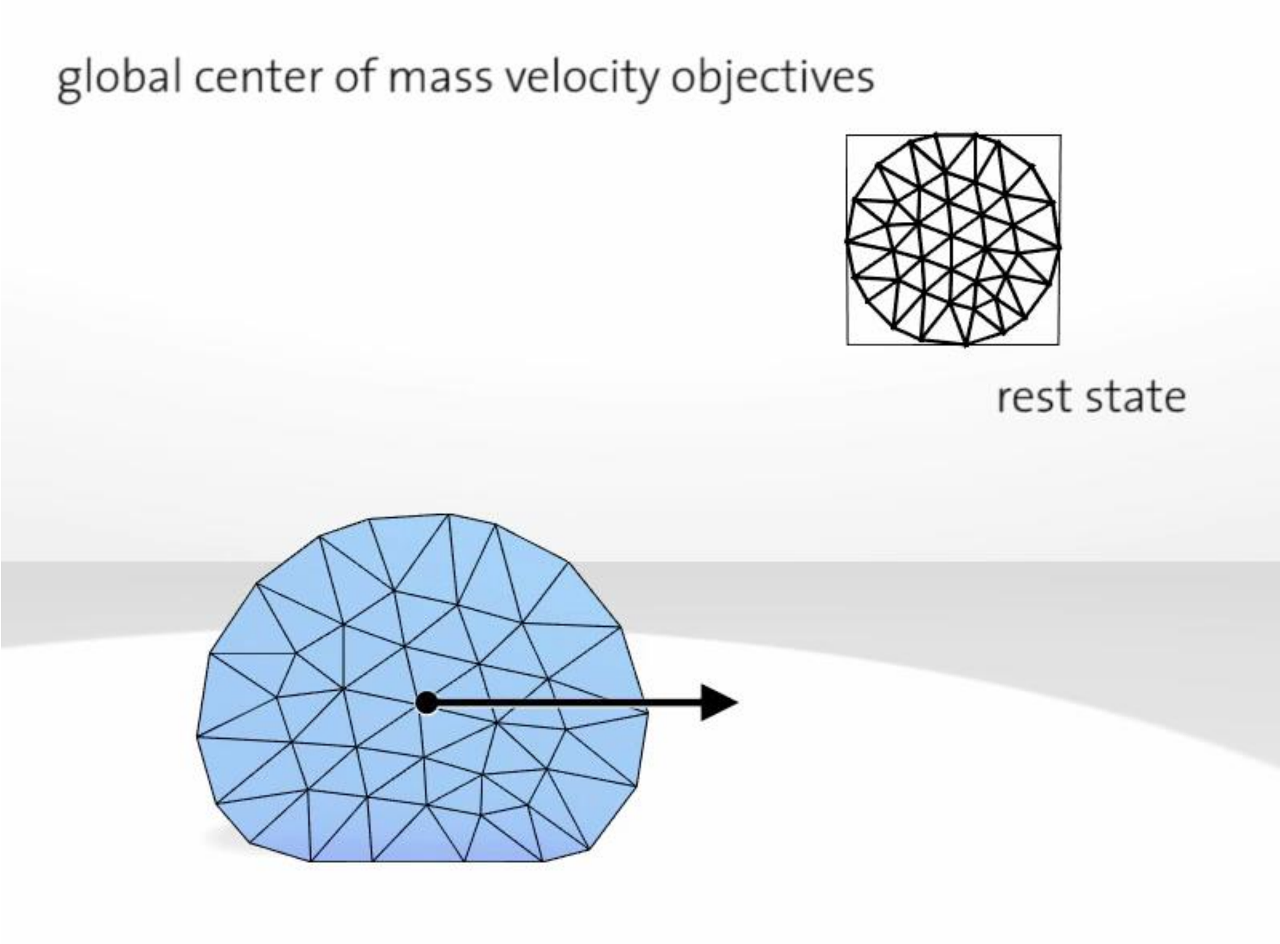


Dynamically changing rest pose leads to interesting behaviors



Dynamically changing rest pose leads to interesting behaviors

Deformable Objects Alive!, Coros et al., 2012



Problem Formulation

Find $\mathbf{x}$ $s.t.$

$$\mathbf{M}\left(\frac{\mathbf{x}}{h^2} - \frac{\mathbf{x}_n}{h^2} - \frac{\mathbf{v}_n}{h}\right) = \mathbf{f}_{\text{int}}(\mathbf{X}, \mathbf{x}) + \mathbf{f}_{\text{ext}}(\mathbf{x})$$

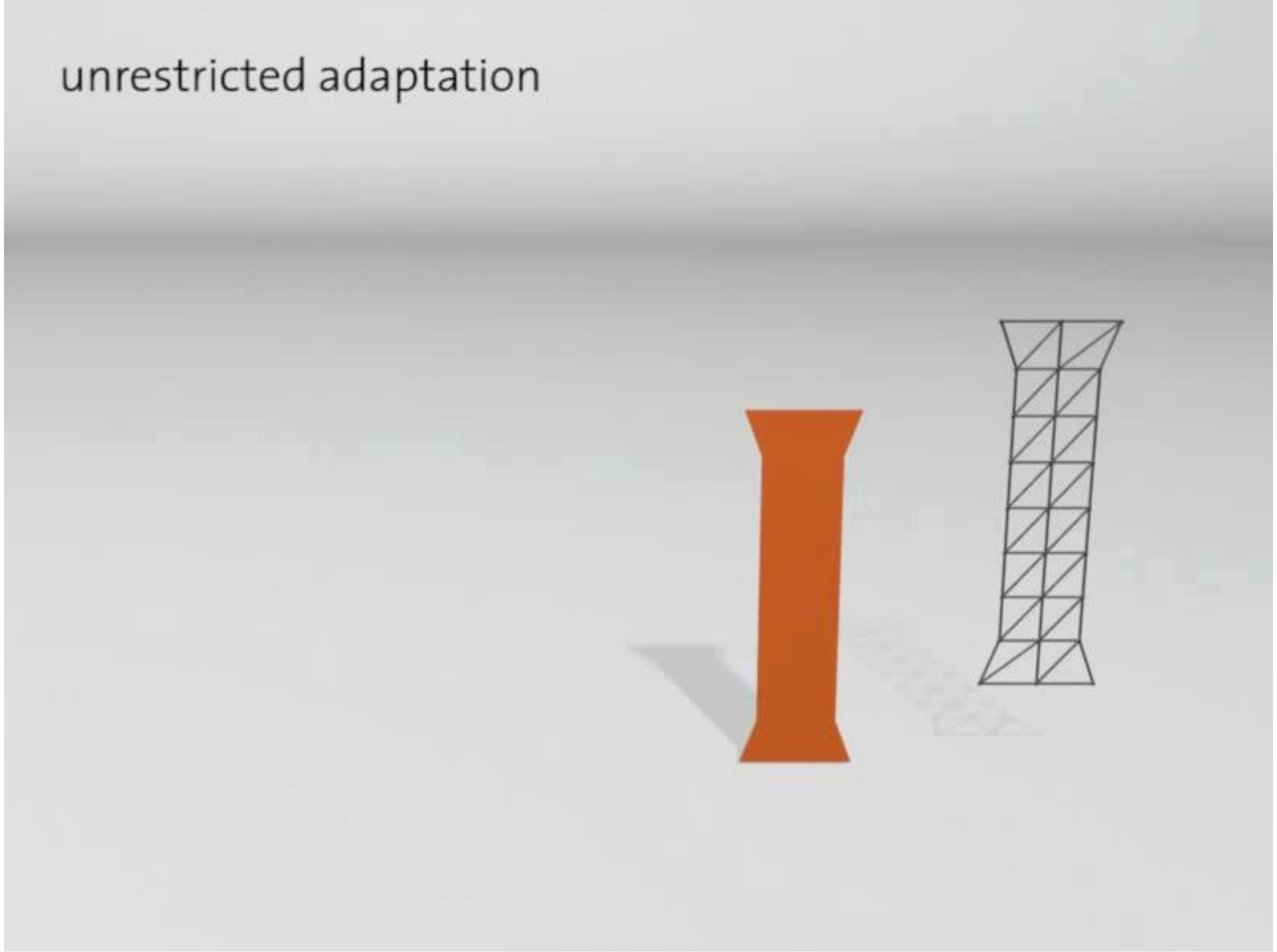
Problem Formulation

Find $\mathbf{x}, \mathbf{X}$ to minimize $\mathbf{g}(\mathbf{x}, t)$

$$s.t.$$

$$\mathbf{M}\left(\frac{\mathbf{x}}{h^2} - \frac{\mathbf{x}_n}{h^2} - \frac{\mathbf{v}_n}{h}\right) = \mathbf{f}_{\text{int}}(\mathbf{X}, \mathbf{x}) + \mathbf{f}_{\text{ext}}(\mathbf{x})$$

Rest shape adaptation



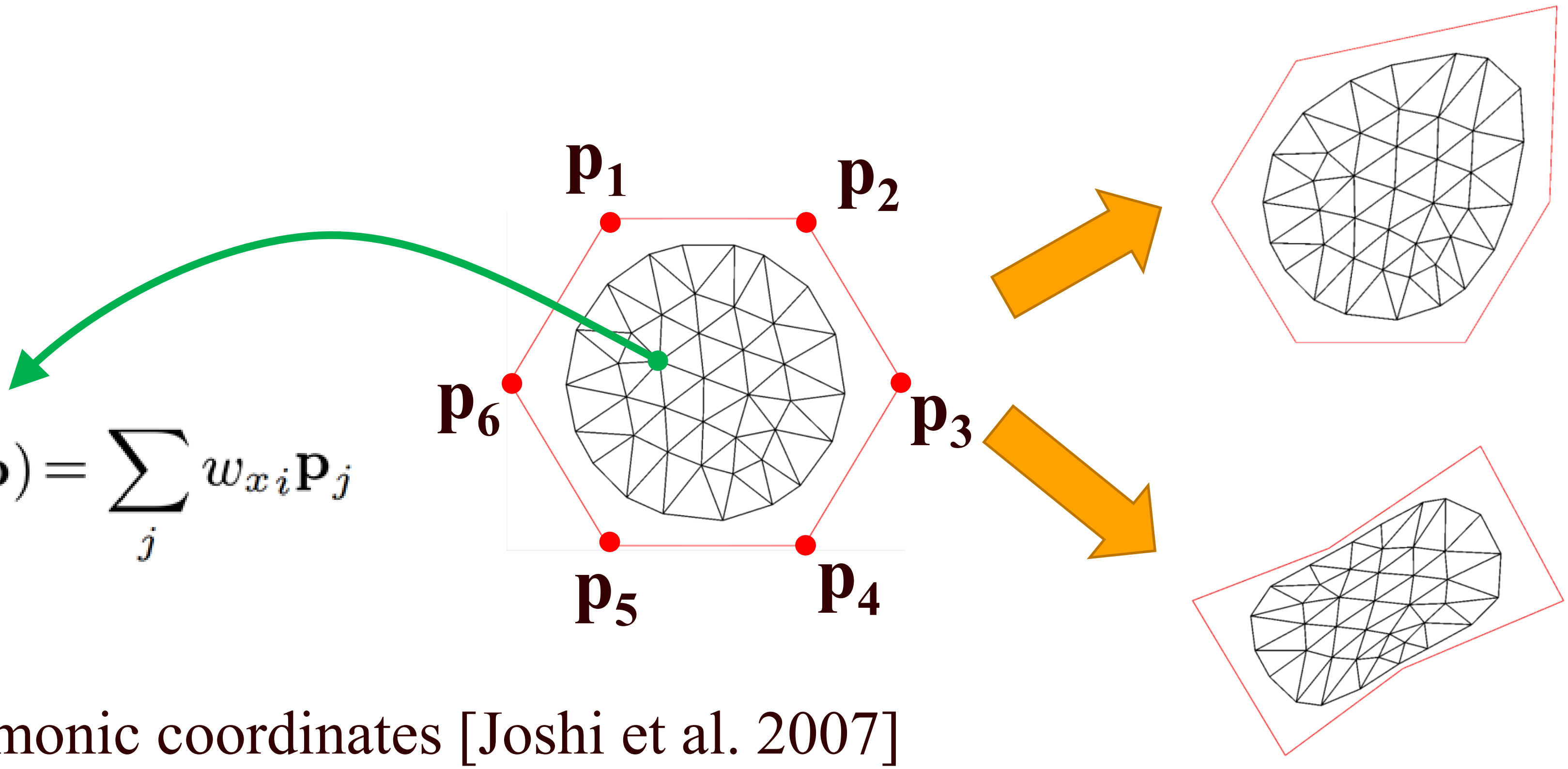
Rest shape adaptation

Find $\mathbf{x}$, $\cancel{\mathbf{X}}$ $\mathbf{p}$ to minimize $\mathbf{g}(\mathbf{x}, t)$

$$s.t.$$

$$\mathbf{M}\left(\frac{\mathbf{x}}{h^2} - \frac{\mathbf{x}_n}{h^2} - \frac{\mathbf{v}_n}{h}\right) = \mathbf{f}_{\text{int}}(\cancel{\mathbf{X}}\ \mathbf{X}(\mathbf{p}), \mathbf{x}) + \mathbf{f}_{\text{ext}}(\mathbf{x})$$

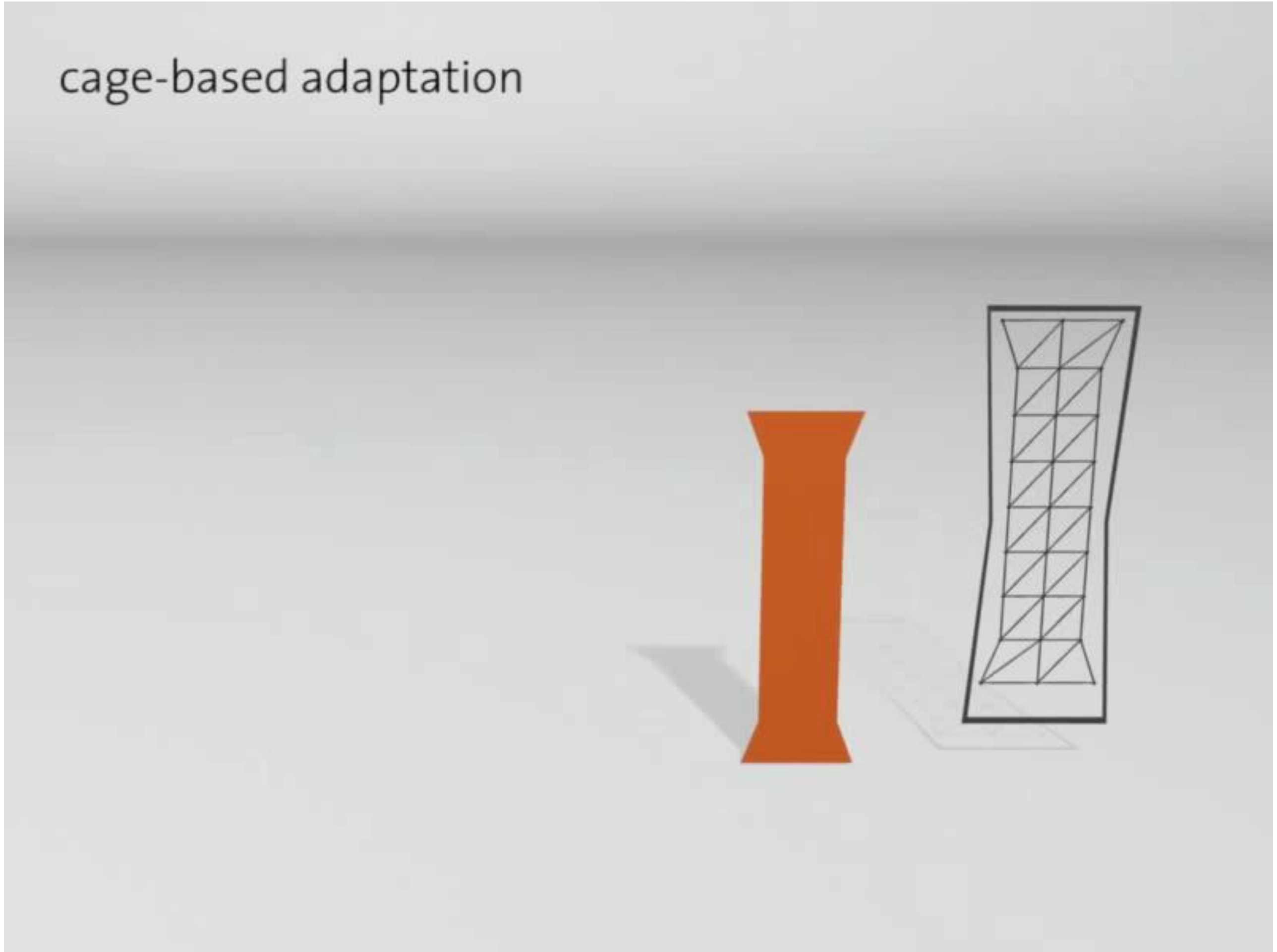
Cage-based rest shape adaptation



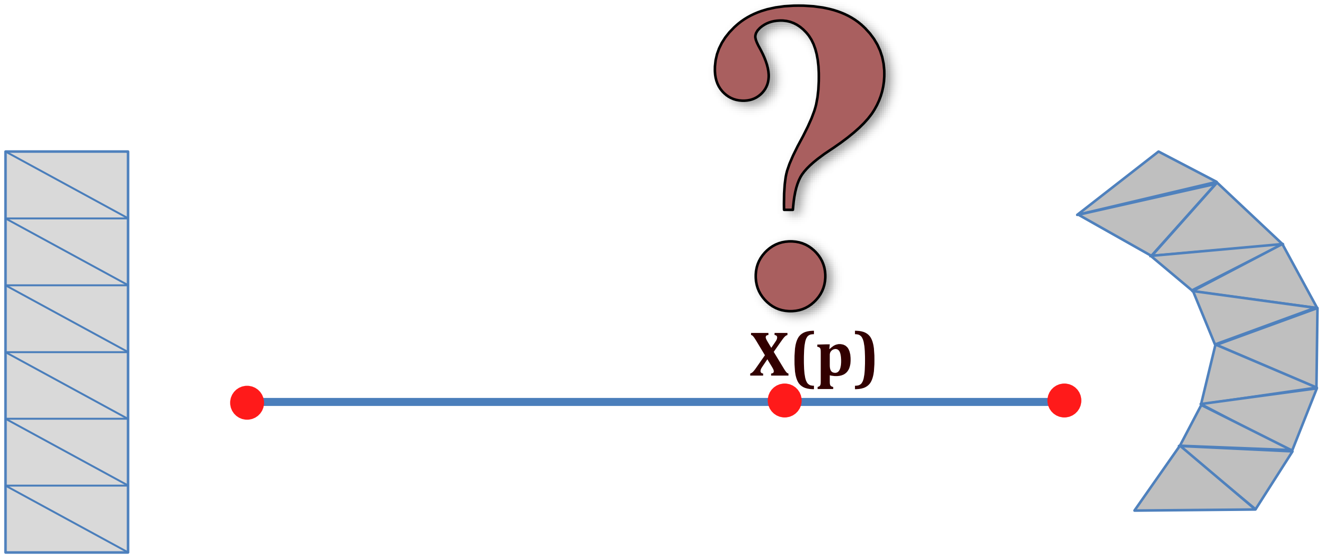
$$\mathbf{X}_i(\mathbf{p}) = \sum_j w_{x\,i}\mathbf{p}_j$$

Harmonic coordinates [Joshi et al. 2007]

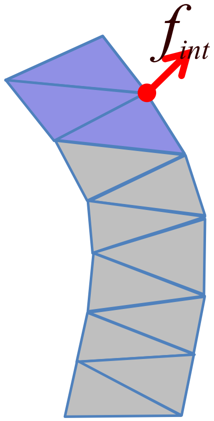
Cage-based rest shape adaptation



Example-based rest shape adaptation

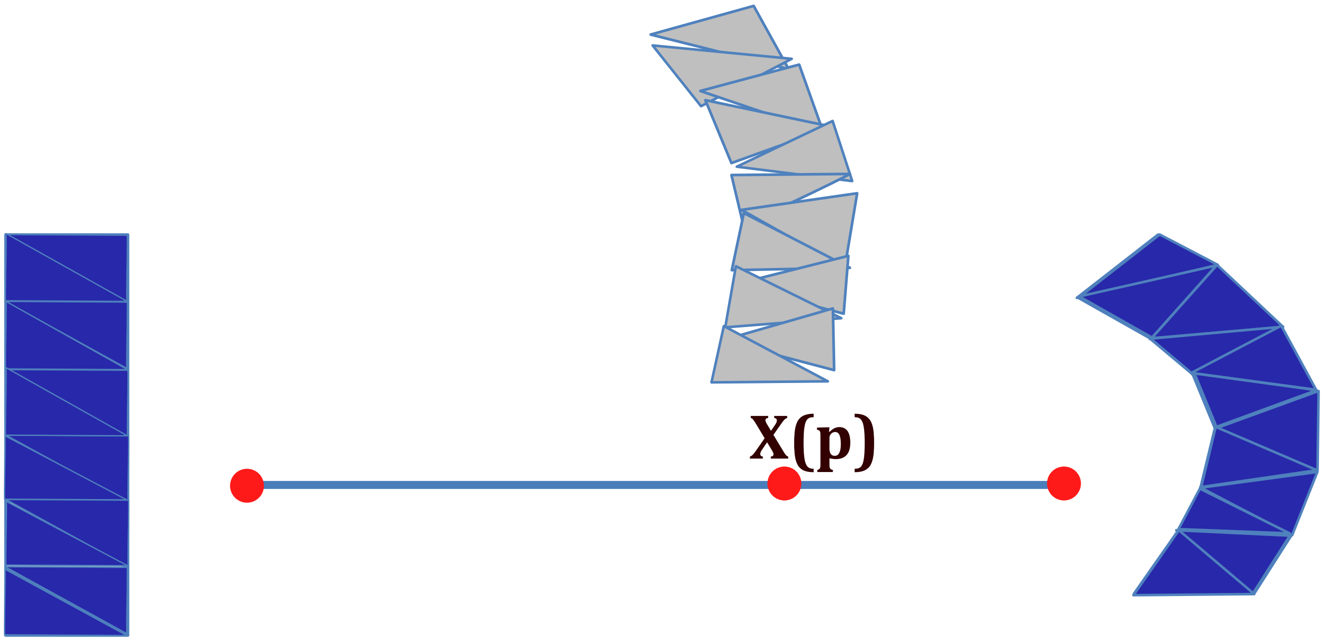


Example-based rest shape adaptation



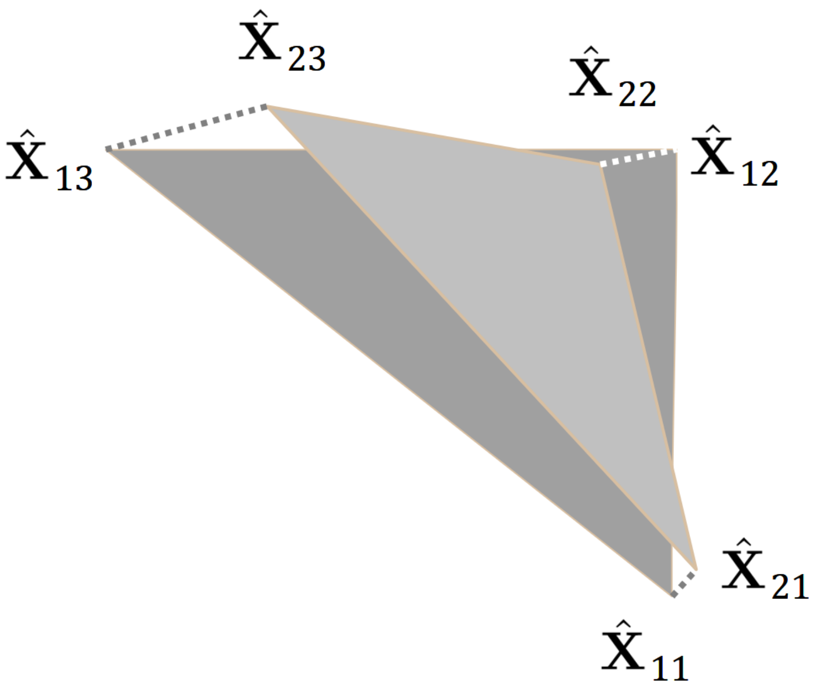
Internal potential energy $W(\mathbf{X}, \mathbf{x})$

Example-based rest shape adaptation



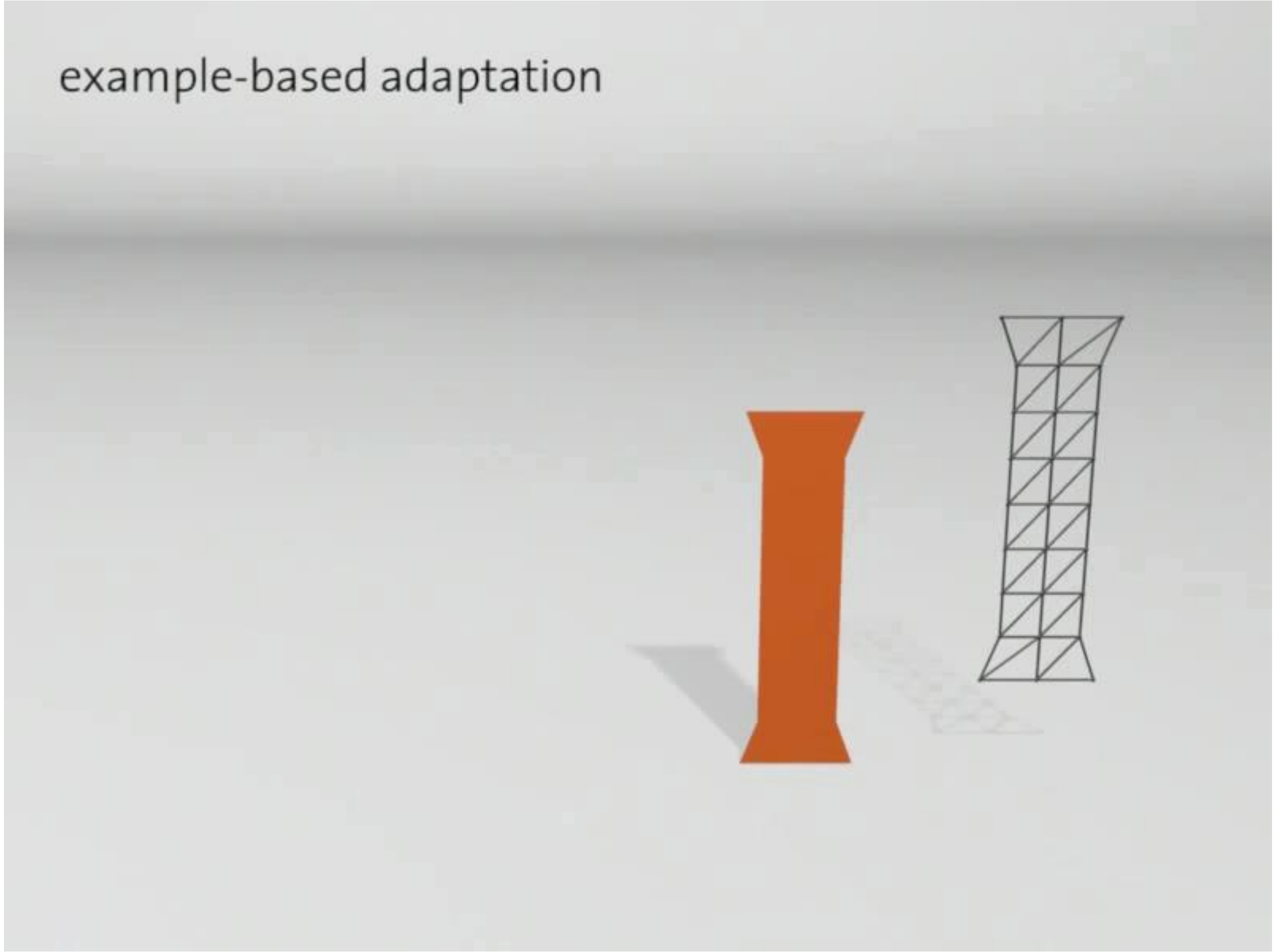
Example-based rest shape adaptation

Example-based rest shape adaptation

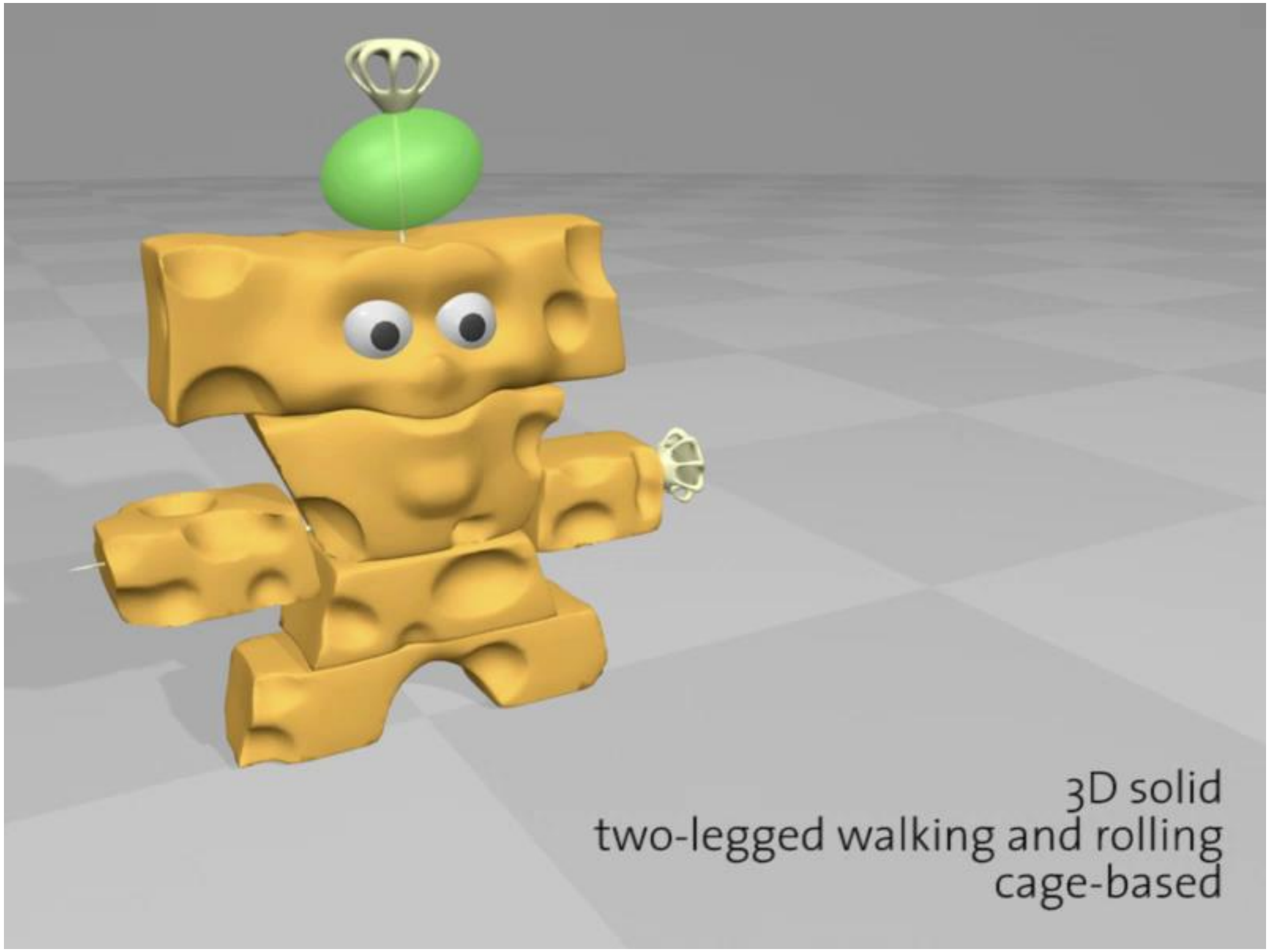


$$\mathbf{X}_i(\mathbf{p}) = \sum_j \mathbf{p}_j \hat{\mathbf{X}}_{ji}$$

Example-based rest shape adaptation

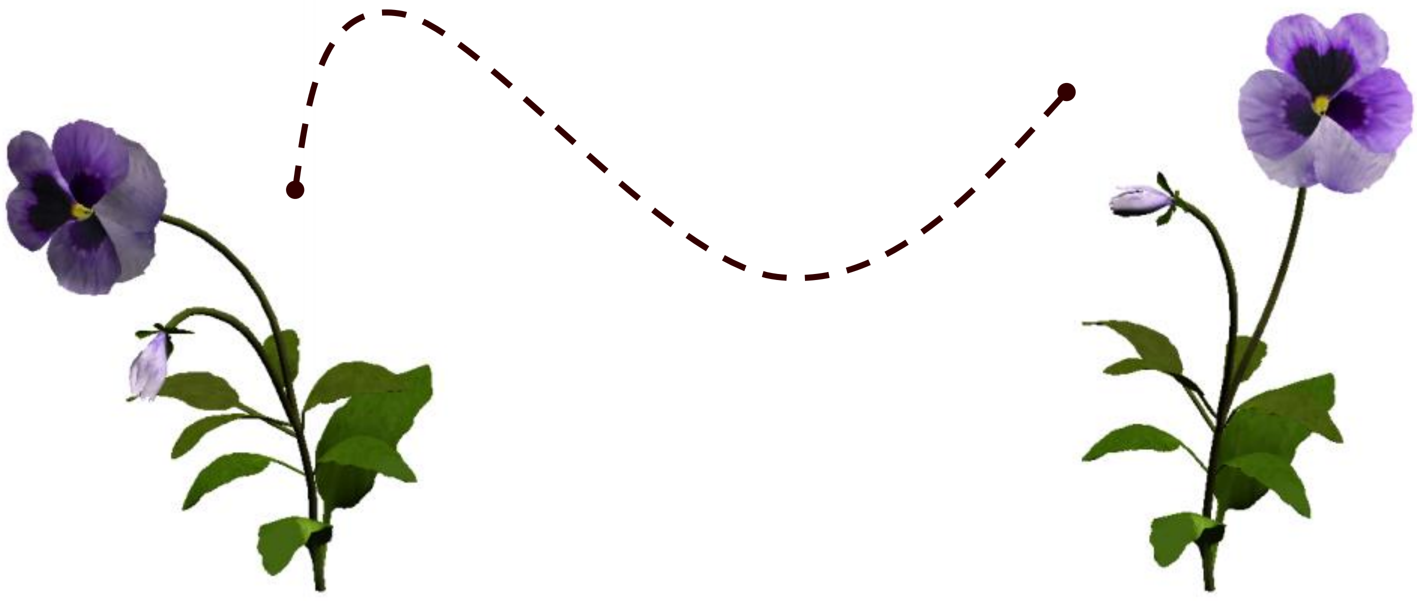


Example

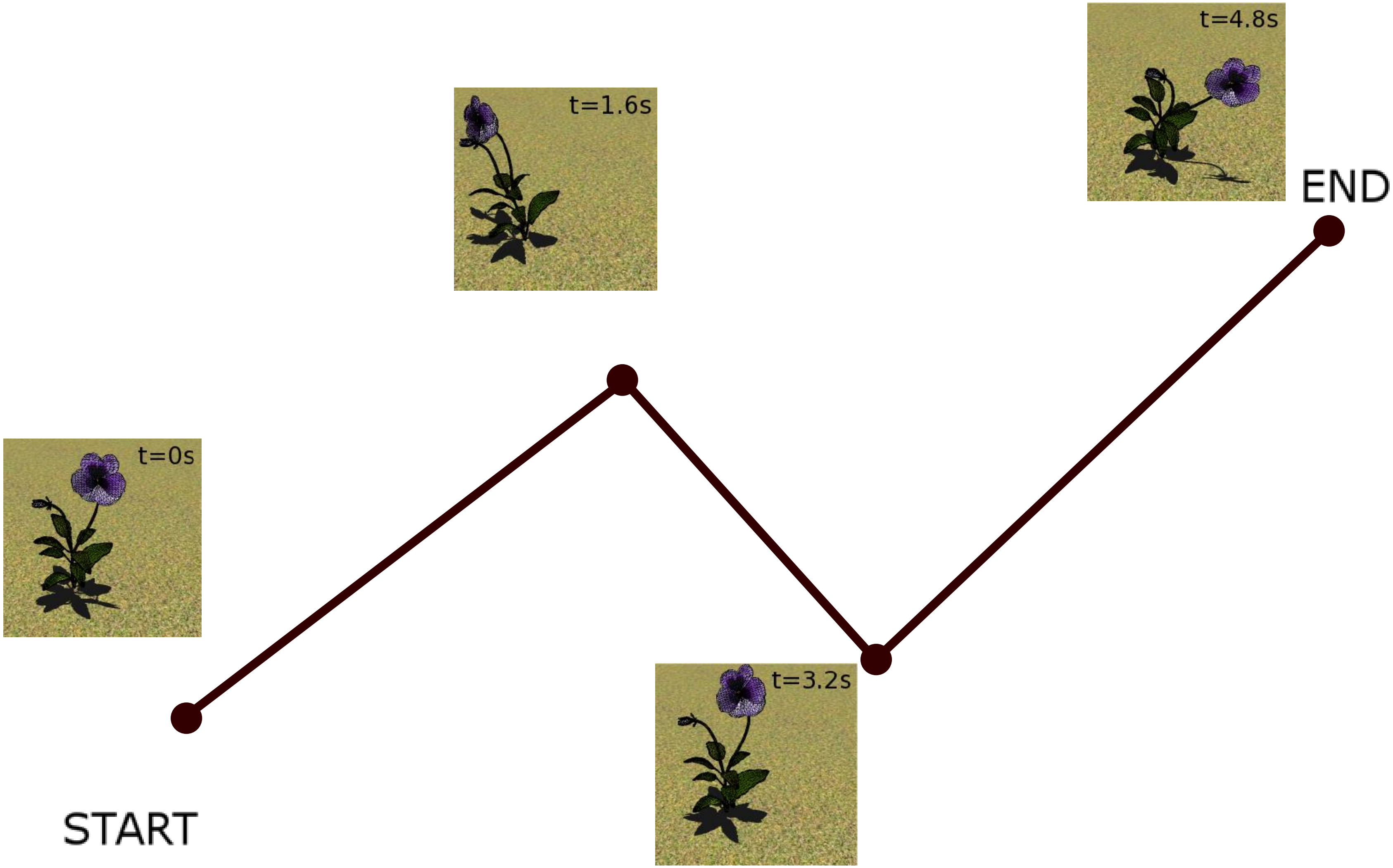


Increasing planning horizon

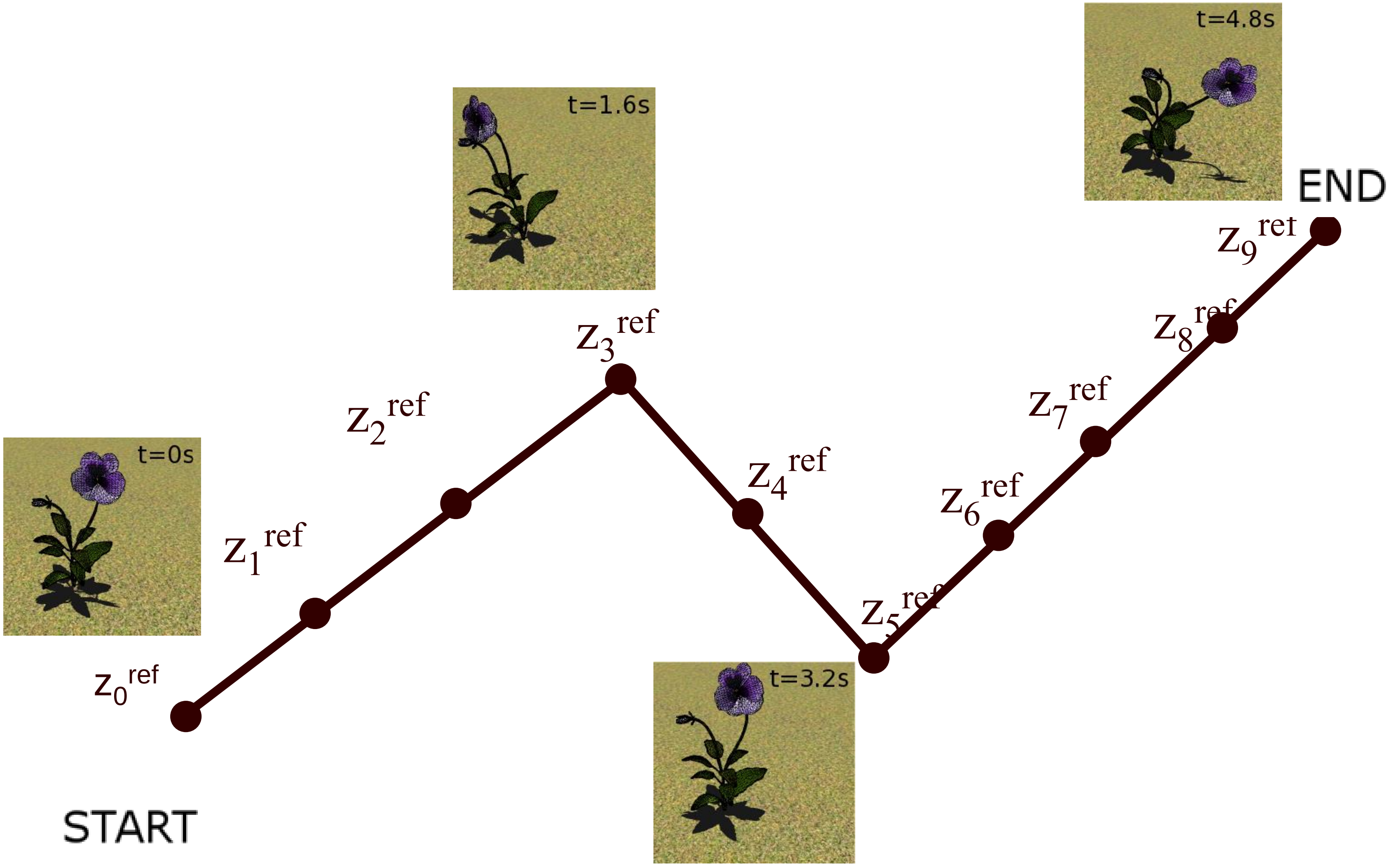
Deformable Object Animation Using Reduced Optimal Control, Barbic et al., 2009



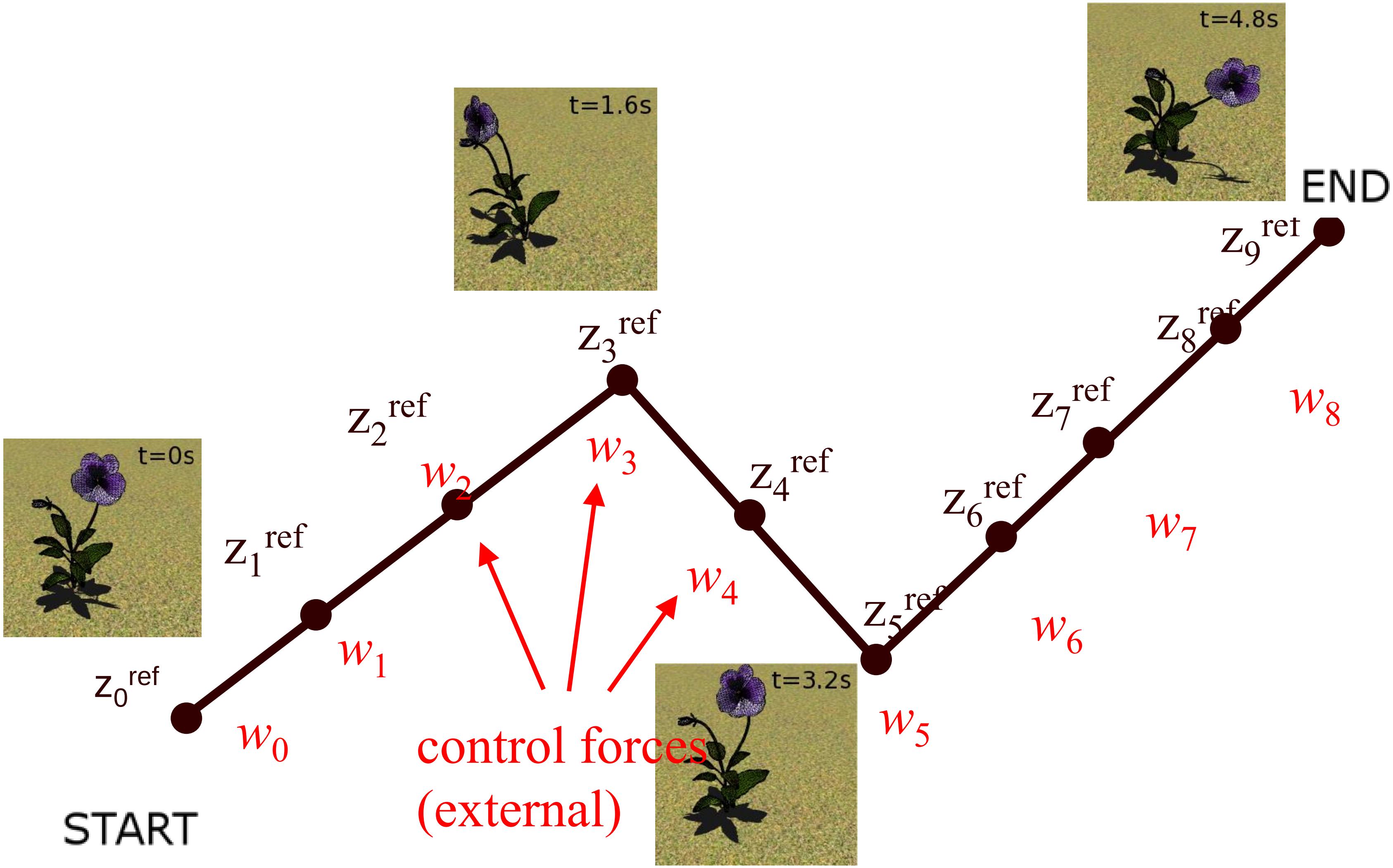
Animations: trajectories in space of deformations



Animations: trajectories in space of deformations



Animations: trajectories in space of deformations



Spacetime optimization formulation:

Find $z(t)$ and w_i that minimizes

$$\sum_{k=1}^{K} ||z(t_k) - \overline{z_k}||^2 + \sum_{i}^{T} ||w_i||^2$$

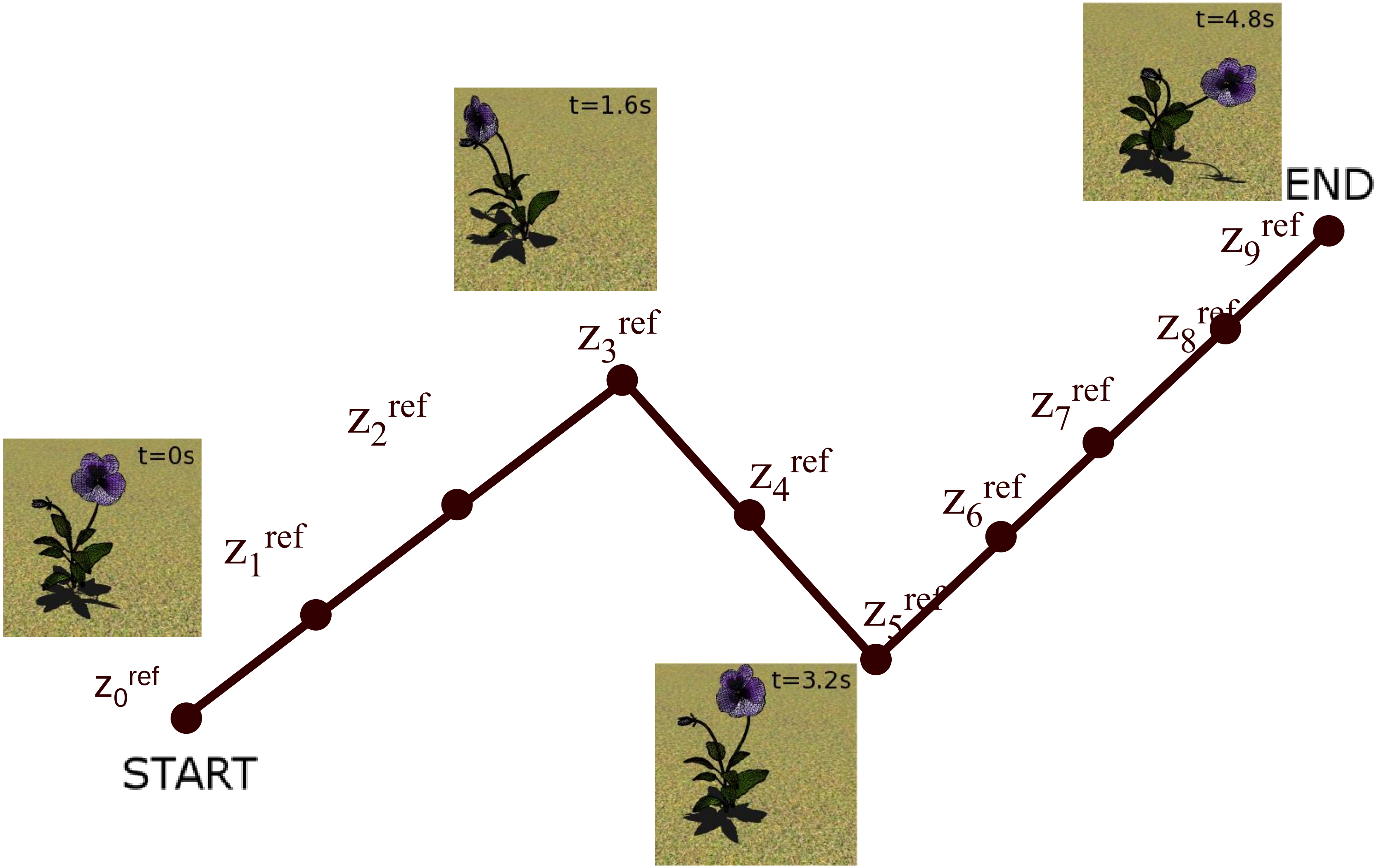
Keyframe deviation

External forces that explain current trajectory (effort)

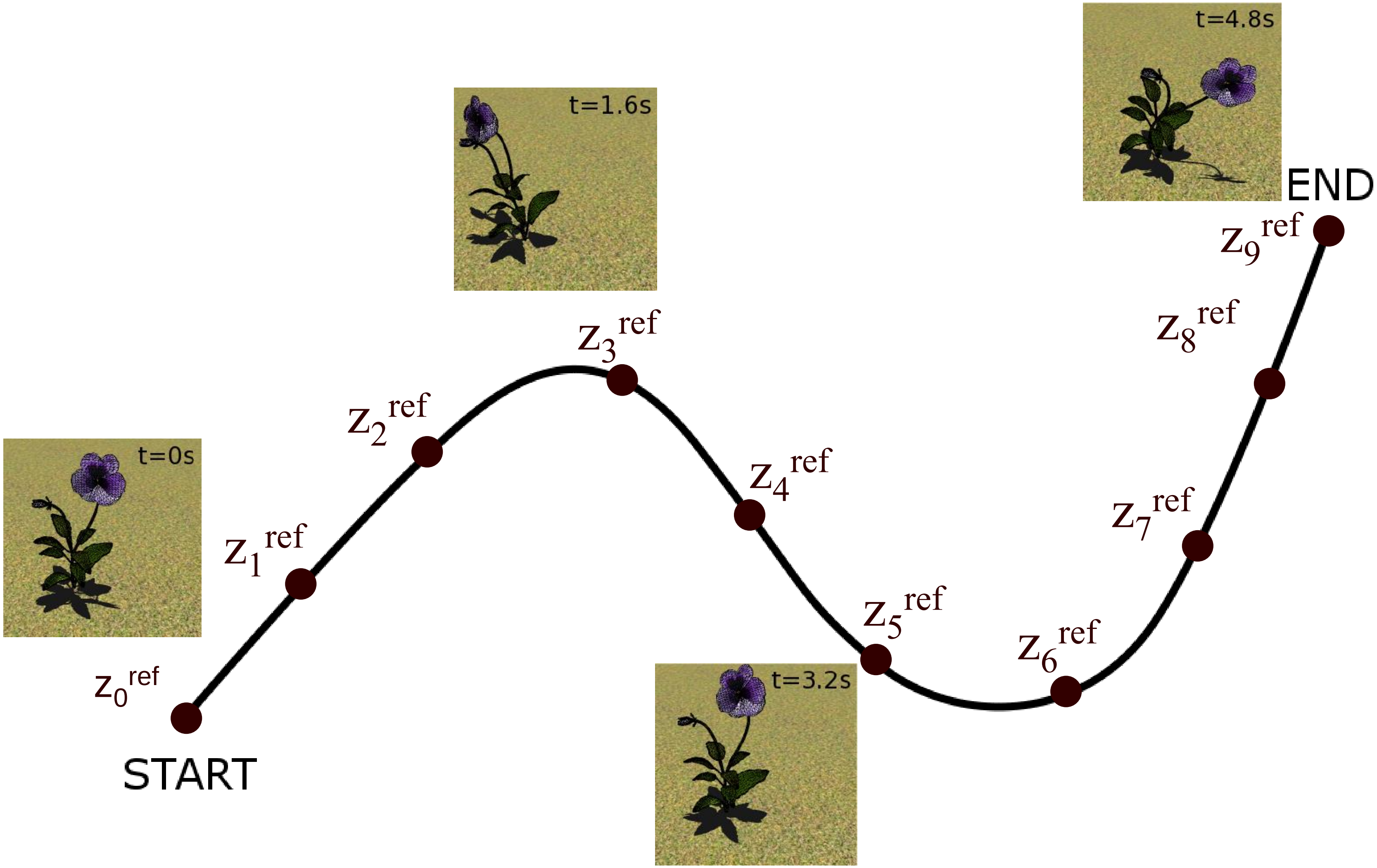
subject to

F=Ma

Animations: trajectories in space of deformations

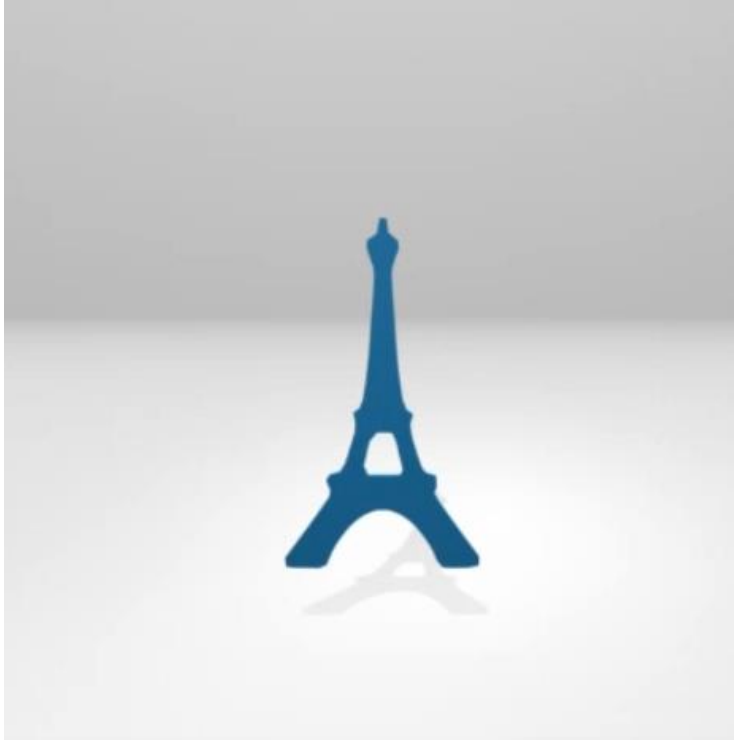


Animations: trajectories in space of deformations



Example

On to the real world…

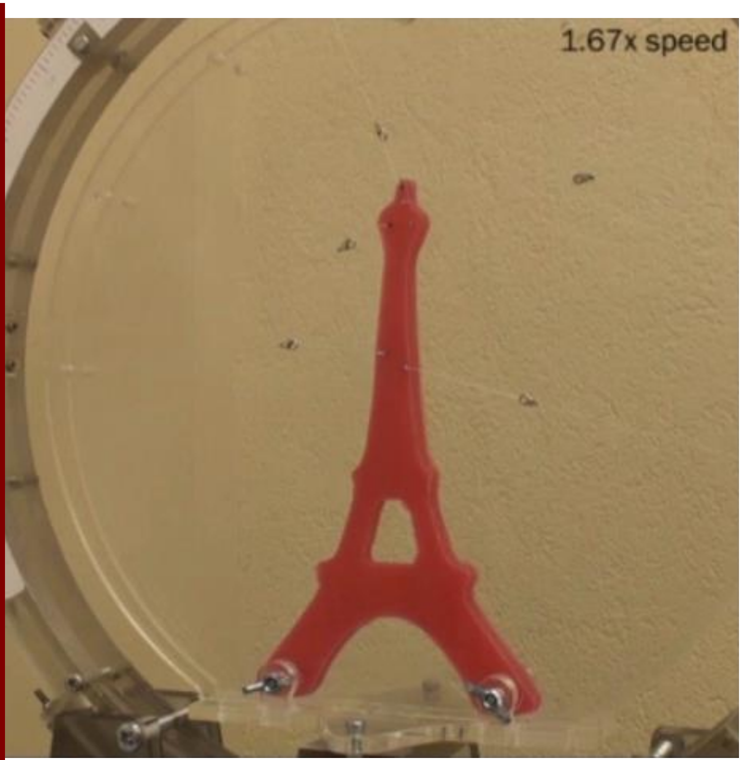


Why?

How would you get here?

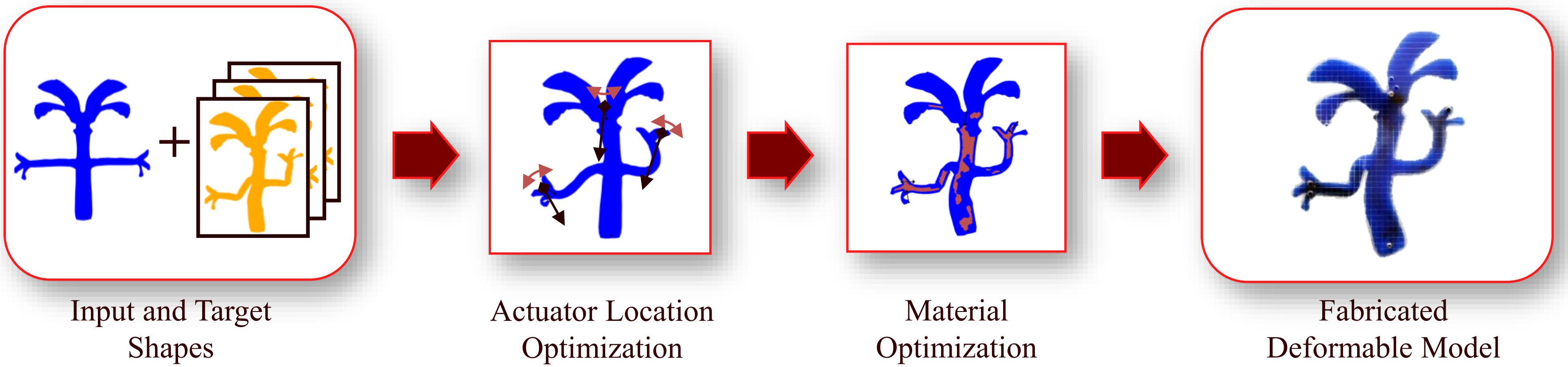
Computational Design of Actuated Deformable Characters, Skouras et al., 2013

Input Animation



Fabricated Prototype

Pipeline

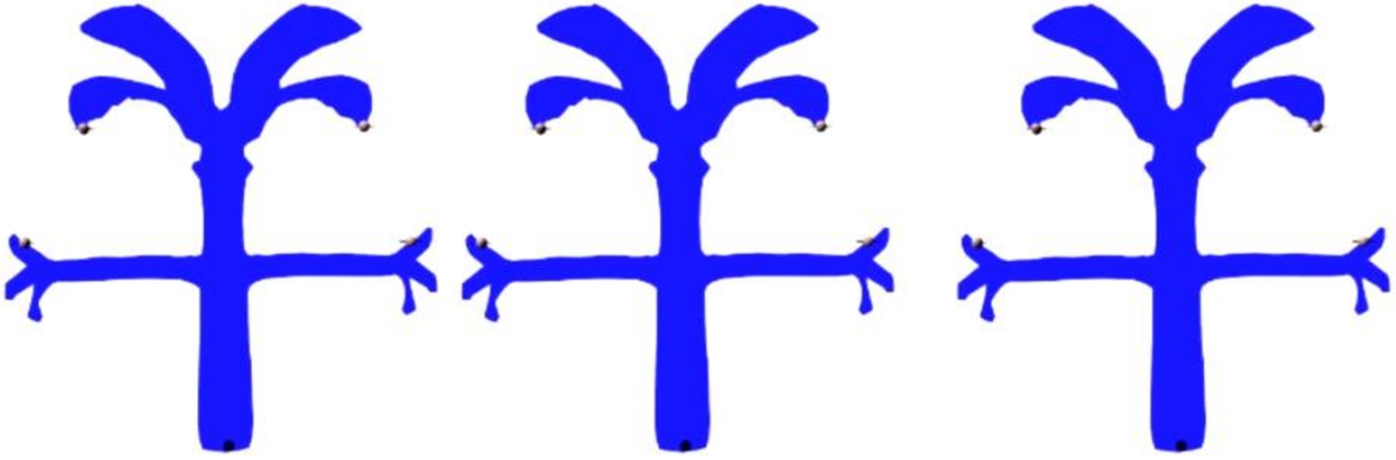

Input and Target Shapes
Actuator Location Optimization
Material Optimization
Fabricated Deformable Model

Mathematical Formulation

Design Parameters

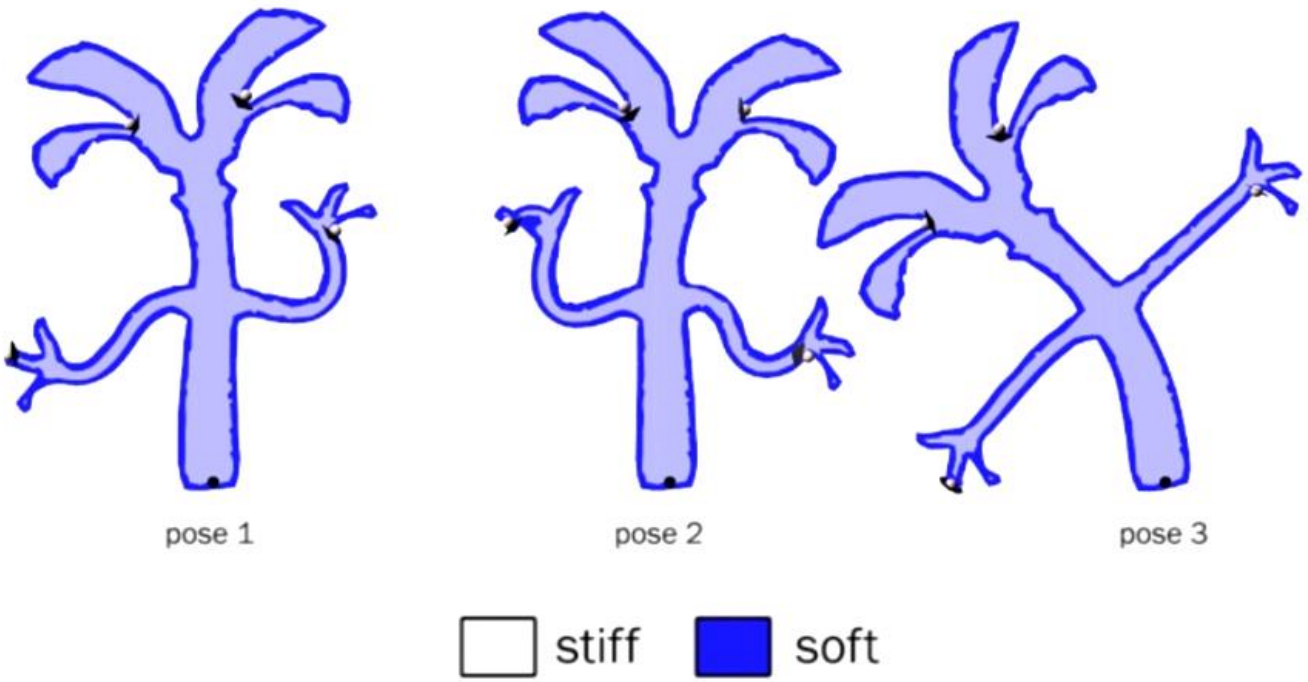
$$E(\mathbf{x}, \mathbf{p}) = E_{match}\left(\mathbf{x}, \mathbf{x}_{target}\right) \quad \text{subject to} \quad \mathbf{f}^{total}(\mathbf{x}, \mathbf{p}) = 0$$

Actuator Location Optimization



pose 1 pose 2 pose 3

Material Distribution Optimization



Results

Rest Pose

Target Pose

Stiff Soft

That’s it for today – questions?