Sharing the Cost of Multicast Transmissions

[Preliminary Version]

Joan Feigenbaum
AT&T Labs – Research
180 Park Ave., C203
Florham Park, NJ 07932
jf@research.att.com

Christos Papadimitriou*
Computer Science Dept.
U. C. Berkeley
Berkeley, CA 94704
christos@cs.berkeley.edu

Scott Shenker
ACIRI/ICSI
1947 Center Street, Suite 600
Berkeley, CA 94704
shenker@icsi.berkeley.edu

ABSTRACT

We investigate cost-sharing algorithms for multicast transmission. Economic considerations point to two distinct mechanisms, *marginal cost* and *Shapley value*, as the two solutions most appropriate in this context. We prove that the former has a natural algorithm that uses only two messages per link of the multicast tree, while we give evidence that the latter requires a quadratic total number of messages. We also show that the welfare value achieved by an optimal multicast tree is NP-hard to approximate within any constant factor, even for bounded-degree networks. The lower-bound proof for the Shapley value uses a novel algebraic technique for bounding from below the number of messages exchanged in a distributed computation; this technique may prove useful in other contexts as well.

1. INTRODUCTION

The traditional form of routing provided by the Internet is *unicast* routing; each packet sent from a source is delivered to a single receiver. To send the same packet to multiple sites, a source has to send a separate copy of this packet to each receiver. This results in many identical packets traversing the network links close to the source. To avoid this waste of bandwidth, a different form of routing, called *multicast* routing [4], has been developed. Multicast routing creates a directed tree connecting the source to all the receivers; when a packet reaches a branch point in the tree, the router duplicates the packet and then sends a copy over each downstream link. Thus, a source can reach multiple receivers without sending duplicate copies of the packet over any link. Because of this increased transmission efficiency, multicast routing will greatly facilitate the transmission of highly popular content such as live concerts and movies.

Such applications are likely to be significant sources of revenue. However, multicasting high quality audio and video to a large population is likely to incur significant costs. Unlike the unicast case, the bandwidth used by a multicast transmission is not directly attributable to a single receiver.[1] Thus, one must find a way to distribute the cost among the various receivers. A *cost-sharing* mechanism determines which users receive the multicast transmission and how much they are charged. We let $x_i \geq 0$ denote how much user i is charged and σ_i denote whether user i receives the transmission; $\sigma_i = 1$ if the user receives the multicast transmission, and $\sigma_i = 0$ otherwise.

We consider a particular multicast transmission of, say, a movie and assume each user i derives utility u_i from the movie. A user's individual *welfare*, her overall happiness after seeing the movie (or not) and paying the resulting cost, is given by $w_i = \sigma_i u_i - x_i$. The vectors σ and x are functions of the utility vector u; a cost-sharing mechanism is defined by the functions $x(u)$ and $\sigma(u)$. However, the network does not *a priori* know the u_i (as they are properties of the users, not of the network); it must rely on the users to report these values. In the applications we are considering, such as the transmission of a movie, the users are independent and have little reason to cooperate with each other or with the network. Therefore, we cannot assume that users will truthfully report their utilities to the network; instead, we assume that users are selfish and will report the values that maximize their individual welfare w_i. The network can discourage such deception by using *strategyproof* cost-sharing mechanisms in which each user maximizes her welfare by truthfully revealing u_i; that is, $w_i(u) \geq w_i(u|^i v_i)$ for any v_i and all i (where we are using the notation that $(u|^i v_i)_j = u_j$ for $j \neq i$ and $(u|^i v_i)_i = v_i$). Strategyproofness is a very strong condition; strategyproofness, plus a few additional constraints described in Section 2, leads us to two natural strategyproof cost-sharing mechanisms: Marginal Cost (MC) and Shapley Value (SH) (see [22]).

A cost-sharing *mechanism* is merely a pair of formulas for $x(u)$ and $\sigma(u)$. However, computing these quantities is not a trivial task, because the u_i (and the network costs, as we describe in Section 2) are spread throughout the network and the results of the computation, the $x_i(u)$ and $\sigma_i(u)$,

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[1] There are many contexts in which it would be natural for the source to foot the bill (*e.g.*, a sales presentation) or for the bill to be shared in some fashion between source and receiver (*e.g.*, a teleconference). However, we are restricting our attention here to cases in which the receiver is responsible for paying for the transmission. This is the most challenging, if perhaps not the most common, case. It may also be the case most appropriate for mass-market content.

must be delivered to the individual users. Therefore, once given a cost-sharing *mechanism*, we need to define the distributed cost-sharing *algorithm* or *protocol* that implements that mechanism. We do not require that the algorithm end with some network router knowing the results of the entire computation, merely that all users are informed of their individual allocations x_i and σ_i. Because these cost-sharing algorithms might deal with extremely large user populations, it is crucial that they not impose significant computational and communication burdens on the network. In particular, we care about how these computational and communication burdens grow with the size of the user population. If the computational burden on each network router is reasonable, as it is for the algorithms we discuss, then there is little motivation to reduce the computational complexity further, because the computing resources of a router are not fungible. However, there is intense competition for link bandwidth; thus, minimizing the communication burden of these distributed cost-sharing algorithms is of great interest. In this paper, we focus on the communication complexity of algorithms for implementing the MC and SH cost-sharing mechanisms. We prove a lower bound suggesting that any cost-sharing algorithm implementing the Shapley Value mechanism must send a linear number of messages over each of a linear number of links, a quadratic number of messages overall. By contrast, we give a natural algorithm that implements the Marginal Cost mechanism using only two messages traversing each link (one in each direction).

This paper brings together two separate concerns. The incentive considerations of strategyproofness restrict the class of cost-sharing mechanisms that can be employed. Similarly, complexity considerations constrain the set of algorithms that can be used to implement these mechanisms. One must find mechanisms that both have the desired incentive properties and also can be feasibly implemented. Our results suggest that, although both the Shapley Value and Marginal Cost mechanisms have the proper incentive properties (and are, in a well defined sense made clear in the next section, the two most appropriate mechanisms for this problem), only the Marginal Cost mechanism can be feasibly implemented.

This remainder of this paper is organized as follows. Section 2 contains some technical preliminaries and a brief discussion of related work. We present our results on the Marginal Cost mechanism in Section 3 and show that the welfare value achieved by an optimal multicast tree is NP-hard to approximate within any constant in Section 4. Section 5 contains our results on a lower bound (in a restricted computational model) for algorithms implementing the Shapley Value mechanism. We conclude in Section 6 with a brief summary and discussion of open problems.

2. TECHNICAL PRELIMINARIES

2.1 Network Model

Consider a user population P, a set of network nodes N, and a set of bidirectional network links L. Each user $i \in P$ resides[2] at some network location $\alpha \in N$. In this paper we restrict our attention to a particular multicast flow emanating from a source located on network node $\alpha_s \in N$. The multicast routing infrastructure, given any set of receivers $R \subseteq P$, constructs a tree (the *multicast tree*) $T(R) \subseteq L$ rooted at α_s and connecting α_s to the nodes where users in R reside. For most of this paper, we shall assume that each user i has a fixed path $T(i)$ from the source to it, determined by the multicast routing infrastructure. Then, for every set R of receivers, the delivery tree is merely the union of these fixed paths: $T(R) = \cup_{i \in R} T(i)$. This approach is relatively simple to implement and makes the multicast tree quite stable as receivers join and leave, both of which are extremely important in practice; however, this approach precludes the use of more theoretically familiar structures like Steiner trees. We make this assumption because it represents the design philosophy embedded in essentially all current multicast routing proposals (see, for example, [1; 4; 5; 13; 26]). Note that this form of multicast routing may lead to suboptimal trees; in our class of trees, the data path from the source to a particular user is independent of which other users are present, whereas Steiner trees do not have this property.

Each link $l \in L$ has an associated cost $c(l) \geq 0$, which is known by the nodes on each end. The cost of the tree $T(R)$ reaching a set of receivers R is $c(T(R))$, and the total welfare, or net worth, is $NW(R) = u_R - c(T(R))$, where $u_R = \sum_{i \in R} u_i$ and $c(T(R)) = \sum_{l \in T(R)} c(l)$. Note that the overall welfare is not the sum of the individual welfares and does not depend on the cost shares. The overall welfare measures the total benefit of providing the multicast transmission (the sum of the utilities minus the cost); the cost shares are merely transfers between the receivers and the network but do not change the overall level of welfare. The fact that $T(R) = \cup_{i \in R} T(i)$ ensures that the cost $c(T(R))$ is a nondecreasing and submodular function of the set R: $c(T(R + i)) \geq c(T(R))$ and $c(T(R_1)) + c(T(R_2)) \geq c(T(R_1 \cup R_2)) + c(T(R_1 \cap R_2))$, for all $R, R_1, R_2 \subseteq P$. In Section 4 below, we examine the effect of dropping the restriction $T(R) = \cup_{i \in R} T(i)$.

2.2 Cost-Sharing Mechanisms

Cost-sharing mechanisms are defined by the functions $x_i(u)$ and $\sigma_i(u)$. For a given cost-sharing mechanism, define $R(u) = \{i \in P | \sigma_i(u) = 1\}$, the *receiver set*, to be the set of users selected to receive the transmission at a given utility profile u. Similarly, let $W(u) \equiv NW(R(u))$ be the welfare resulting from the mechanism at utility vector u. We consider only strategyproof cost-sharing mechanisms, and so we must have $w_i(u) \geq w_i(u|^i v_i)$, for all u, i, and v_i. We also impose several additional basic requirements:

- No Positive Transfers (NPT): $x_i(u) \geq 0$. This precludes *paying* receivers to receive the transmission. We are only considering situations in which the users pay for the right to participate.
- Voluntary Participation (VP): $w_i(u) \geq 0$. Users are always free to not receive the transmission and not be charged, which would result in an individual welfare of zero; the network can't force a user to be worse off than this baseline option. Note that this implies that $x_i = 0$ whenever $\sigma_i = 0$, *i.e.*, that only the receivers of the data have to pay.
- Consumer Sovereignty (CS): For all u, $\sigma(u|^i v_i) = 1$ for sufficiently large v_i. The cost-sharing algorithm

[2]The nodes represent routers, and a user's being *resident* at a node means that the node is the first-hop router for that user.

cannot arbitrarily exclude any users; the network has to allow users to receive the transmission if they are willing to pay a sufficiently high cost.

There are many strategyproof cost-sharing mechanisms that satisfy the above requirements, some of them quite impractical. For instance, one could send to the highest bidder (breaking ties arbitrarily), deny all other users, and charge the chosen user the second highest reported utility; this would result in all users but one having zero welfare and could result in negative overall welfare if the network costs were sufficiently high. To exclude such alternatives, we find it useful to consider two other requirements:

- Budget-balance: $\sum_{i \in P} x_i = c(T(R(u)))$. This requires that the money raised from the receivers cover the cost of the transmission exactly.
- Efficiency: $NW(R(u)) \geq NW(R)$ for all $R \subseteq P$. This requires that the receiver set maximize the overall benefit of the network. A set that maximizes $NW(R)$ is called an *efficient set*.

It is a classical result in game theory [9; 27] that, in this context, these two requirements are mutually exclusive; there are no strategyproof cost-sharing mechanisms that are both budget-balanced and efficient. It is easy to see, and is shown in [22], that there is essentially only one strategyproof cost-sharing mechanism that both satisfies NPT and VP and is efficient: the Marginal Cost mechanism.[3] The MC mechanism is a special case of a more general class of mechanisms, the Vickrey-Clarke-Groves [2; 10; 11; 35] mechanisms, and is defined as follows. Let $R^*(u)$ denote the largest efficient set; this is well defined, because submodularity of the cost function guarantees that the union of two efficient sets is also an efficient set. We set $\sigma_i(u) = 1$ for all $i \in R^*(u)$ and $\sigma_i(u) = 0$ for all $i \notin R^*(u)$, and so $W(u) = NW(R^*(u))$. The cost shares are given by:

$$x_i = u_i \sigma_i(u) - \Big(W(u) - W(u|^i 0) \Big) \qquad (1)$$

This mechanism gives each user a welfare $w_i = W(u) - W(u|^i 0)$ that represents the marginal contribution to the overall welfare provided by user i's having nonzero utility for the transmission. Unfortunately, this mechanism is not budget-balanced. It never runs a budget surplus but can run a budget deficit; in fact, in many cases it raises no revenue at all [22].

While the requirement of efficiency picked out a single natural cost-sharing mechanism, the requirement of budget-balance leaves many possible mechanisms. By imposing the stronger condition of group strategyproofness (no group of users can increase their welfares by lying about their utilities), one can completely characterize the class of possible cost-sharing mechanisms [21; 22]. Each mechanism is defined by a function $f : 2^P \mapsto \Re_{\geq 0}^{|P|}$ with the properties that $\sum_i f_i(R) = c(T(R))$ and $f_i(R + j) \leq f_i(R)$ for all $i, j \in P$. One can use the function f to define $x(u)$ and $\sigma(u)$ in an iterative fashion. First, set $\sigma_i^{(1)}(u) = 1$. At each step k, let $R^{(k)}(u) = \{i | \sigma_i^{(k-1)}(u) = 1\}$, $x_i^{(k)}(u) = f_i(R^{(k)}(u))$, and $\sigma_i^{(k)}(u) = 1$ if $u_i \geq x_i^{(k)}(u)$ with $\sigma_i^{(k)}(u) = 0$ if $u_i < x_i^{(k)}(u)$. The sets $R^{(k)}$ form a monotonic sequence, $R^{(k)} \subseteq R^{(k-1)}$, and thus converge after a finite number of steps to some set $\hat{R}(u)$; the resulting values for $x(u)$ and $\sigma(u)$ define the cost-sharing mechanism, and $\hat{R}(u)$ is the receiver set. The Shapley Value mechanism uses the Shapley Value [30; 34] for the function f; the Shapley Value for the network cost-sharing problem is defined as follows:

$$f_i(R) = \sum_{\tilde{R} \subseteq R - i} \frac{|\tilde{R}|!(|R| - |\tilde{R}| - 1)!}{|R|!} [c(T(\tilde{R} \cup i)) - c(T(\tilde{R}))]$$

This formula is quite forbidding, but it has a simple intuitive explanation: The cost of a link l is shared equally by all those receivers who are downstream of the link.

None of these budget-balanced mechanisms maximizes the overall welfare. Out of all of them, the Shapley Value mechanism minimizes the worst-case welfare loss; the quantity $Max_u[NW(R^*(u)) - NW(R(u))]$ produced by the Shapley value mechanism is strictly smaller than the corresponding quantity produced by other mechanisms in this class [22]. If we insist on budget-balance as a requirement, it seems natural to choose the mechanism that minimizes the resulting loss of efficiency; the Shapley Value is thus a natural choice in this class of budget-balanced cost-sharing mechanisms.

2.3 Computational Model and Costs

For a cost-sharing algorithm to be practical, both communication and local computation should require only modest resources, because cost-sharing is not the *raison d'être* of the network —it is done only to support the primary goal of transmitting content to receivers in a financially viable manner. Hence, analyzing the complexity of algorithms for implementing cost-sharing, especially in terms of their communication costs, is important.

An instance of a cost-sharing problem has size $n + p + m$, where $n = |T(P)|$, $p = |P|$, and m is the total size of the numerical input $\{c(l)\}_{l \in L} \cup \{u_i\}_{i \in P}$. Ideally, the *total* number of messages sent by a cost-sharing algorithm should be $O(n)$, and the *maximum*, over all $l \in L$, of the number of messages sent over l should be $O(1)$. Note that we care about "hot spots" in bandwidth utilization as well as total bandwidth utilization; for example, an algorithm in which the total number of messages is $O(n)$ but $\Omega(n)$ of the messages "bunch up" on one or a few links is unsatisfactory.

In this paper, our primary focus is communication cost, rather than local computation cost, but we do not completely disregard local computation cost. For example, a mechanism that is NP-hard even to approximate closely is obviously impractical; it could not be implemented efficiently even if communication costs associated with the distributed nature of the inputs and outputs were zero and *a fortiori* cannot be implemented efficiently if these costs are nonzero, as they are in our model. Finally, we require that message size be reasonable. It would not be satisfactory, for example, to implement a polynomial-time mechanism as follows: Each node, after receiving a message from each of its children, concatenates all of the received messages, all of the utilities of players resident at it, and the cost of the link connecting it to its parent and then sends the result to its parent; when the root α_s of $T(P)$ has received all of the util-

[3] More precisely, all such strategyproof mechanisms are welfare-equivalent to the MC mechanism in that the resulting individual welfares w_i are the same. The only degree of freedom in deviating from the MC mechanism is that one can exclude users who don't change the overall welfare and whose own individual welfare is zero.

ities and link costs, it computes σ and x and sends them to its children so that the needed values can be "peeled off" as they reach the appropriate nodes, and the remaining values can be sent further down the tree. This trivial algorithm would achieve the goal of $O(1)$ messages per link, but the maximum size of a message could be $\Omega(m)$, which is too big. Message size is not a major issue in this paper, but we revisit it in context where appropriate in what follows.

We do not address any details of message transmission in this model. We assume that messages arrive reliably, within a small bounded time, and arrive in order. It is an open question whether one would obtain fundamentally different computational and communication costs for the mechanisms considered here if one assumed an unreliable network.

2.4 Related Work

This work lies in the intersection of game theory, theoretical computer science, and networking. Questions of incentives have long been central to game theory, and there is a vast literature on the *mechanism design* or *implementation* paradigm in which resource-allocation mechanisms are designed to achieve the socially desirable outcomes in spite of user selfishness. (See [12] for a review and [25] for an introduction.) Many of these approaches use Nash equilibria (or other notions of noncooperative behavior) rather than strategyproofness: That is, they assume that simultaneous selfish play leads to a self-consistent equilibrium, called a Nash equilibrium, in which no agent can improve her lot by deviating. The *Nash implementation* approach [3; 19] involves designing resource allocation mechanisms with Nash equilibria that yield the socially desirable outcome (such as an efficient and/or budget-balanced and/or fair allocation). In contrast, strategyproofness ensures that no matter how other agents behave – whether selfish, spiteful, or stupid – truthful revelation is the optimal (dominant) strategy for each user. Strategyproofness is much more exacting than Nash implementation, and so Nash implementation can achieve a much wider variety of outcomes. However, [7] argues that the strategyproofness may be the only viable approach in the Internet, because one cannot ensure that simultaneous selfish play will reach the traditionally defined notions of equilibrium.

There is a long history of applying game-theoretic techniques to networking problems. Some use incentives as a metaphor for distributed protocol design (*e.g.*, [17]); the elements in the protocol are not selfish, but microeconomic principles are used to achieve coordination between the elements. Others analyze the impact of selfish users on current designs (*e.g.*, [14]), determining the resource allocations achieved at Nash equilibrium. Still others adopt the mechanism design approach and modify network designs to cope with user selfishness (*e.g.*, [6; 16; 29; 31; 32]). There is also significant use of mechanism design in distributed artificial intelligence (*e.g.*, [28; 33]) and in market-based computation (*e.g.*, [36]) Despite their increasing role in some of the more applied areas in computer science, incentives have rarely been an important consideration in traditional theoretical computer science. Typically, users are assumed either to be *cooperative* (*i.e.*, to follow the prescribed algorithm) or to be *adversaries* who attempt to harm other users. In contrast, the selfish users of game theory are neither cooperative nor spiteful; instead they attempt to optimize their own individual welfare. While one cannot assume selfish users will cooperate, one can assume that they will respond to incentives. Thus, one need not cope with the worst-case behavior of a byzantive adversary – only with predictably selfish behavior.

In short, the game-theoretic literature stresses incentives with (usually) little regard for computational or communication complexity, while the theoretical computer literature (usually) focuses on complexity without much consideration of incentives. In one of the first works to merge these two concerns, Nisan and Ronen [23; 24] pose noncooperative allocation problems (routing and load balancing) and analyze the computational complexity of the relevant strategyproof mechanisms. Our work is very much in the same spirit as theirs, except that they consider a centralized setting for the computation, and we address the communication complexity in a distributed setting, for the concrete application of multicast cost sharing. The mechanisms we consider would not be interesting in Nisan and Ronen's model. Because their model allows all of the users to send their data to one node, the receiving node could just compute SH or MC cost shares, both of which are polynomial-time computable. SH and MC computations are interesting only when the computational model is distributed and communication costs matter.

3. MARGINAL COST MECHANISM

THEOREM 3.1. *MC cost sharing requires exactly two messages per link. There is an algorithm that computes the cost shares by performing one bottom-up traversal of $T(P)$, followed by one top-down traversal, and this algorithm is optimal with respect to number of messages sent.*

PROOF. In order to describe the algorithm we need the following notation. Let u^α denote the sum of the utilities of the users located at node α, c^α the cost of the link immediately above α in the tree $T(P)$, $Ch(\alpha)$ all the child nodes of α in the tree $T(P)$, $V(P)$ all nodes in the tree $T(P)$, $res(\alpha)$ the set of users at node α, and $T^\alpha(P)$ the union of the subtree rooted at α and the link from α to its parent $p(\alpha)$. With this, we can compute $W^\alpha(u)$, which is the welfare – utilities minus cost – of $T^\alpha(P)$, as follows:

$$W^\alpha(u) = u^\alpha + \left(\sum_{\beta \in Ch(\alpha) | W^\beta(u) \geq 0} W^\beta(u) \right) - c^\alpha.$$

Note that these values can be computed by the bottom-up traversal given in Figure 1 below.[4] Naturally, $\sigma(i) = 1$ (that is, user i is included in the multicast) if $W^\alpha(u) \geq 0$ for all nodes α in the path from user i to the root.

Once the $W^\alpha(u)$'s have been computed, the values of the $\sigma_i(u)$'s – that is, the bits that indicate whether a user i is a member of the efficient set $R^*(u)$ – can be propagated in a top-down traversal.

The cost share $x_i(u)$ for user $i \in R^*(u)$, given by Equation 1, does not require a from-scratch recomputation of $W(u|^i 0)$. For each node α and user $i \in R^*(u)$ at α, let $y_i(u)$ be the smallest $W^\beta(u)$ of any node β in the path from α to the root. (This minimum welfare value might occur at α.) Then there are two cases:

[4] Johnson, Minkoff, and Phillips [15] use essentially the same formula in their (independently discovered) strong-pruning algorithm for the prize collecting Steiner tree problem.

Figure 1: Bottom-Up Traversal: Computing Welfare Values

At node $\alpha \in V(P)$
After receiving a message A^β from each child $\beta \in Ch(\alpha)$
$W^\alpha \leftarrow u^\alpha + (\sum_{\beta \in Ch(\alpha)} A^\beta) - c^\alpha$
If $W^\alpha \geq 0$ then
{
$\sigma_i \leftarrow 1$ for all $i \in res(\alpha)$
Send W^α to parent $p(\alpha)$
}
Else
{
$\sigma_i \leftarrow 0$ for all $i \in res(\alpha)$
Send 0 to parent $p(\alpha)$
}

- If $u_i \leq y_i(u)$, then, without user i, the efficient set is the same as it is with user i. That is, $R^*(u) = R^*(u|^i 0))$, and the difference $W(u) - W(u|^i 0)$ is u_i. Therefore, user i must pay $x_i(u) \equiv u_i - (W(u) - W(u|^i 0)) = 0$.

- If, however, $u_i > y_i(u)$, then dropping user i results in the elimination from $R^*(u)$ of a subtree of total welfare $y_i(u)$, and thus user i must pay exactly $x_i(u) = u_i - y_i(u)$.

To see this, note that dropping user i decreases $W^\alpha(u)$ by u_i. If $u_i > y_i(u)$, then there is some lowest (furthest from the source) ancestor α' of α for which dropping user i causes $W^{\alpha'}(u)$ to become negative and $T^{\alpha'}(P)$ to be dropped from $R^*(u)$. Dropping $T^{\alpha'}(P)$ may in turn result in a negative welfare value for some ancestor α'' of α', which would cause $T^{\alpha''}(P)$ to be dropped, and so forth. This "chain reaction" stops after the removal of a minimum-welfare subtree $T^\beta(P)$. On the other hand, if $u_i < y_i(u)$, then the tree structure of $R^*(u|^i 0)$ is the same as that of $R^*(u)$.

Observe that this propagation of $y_i(u)$ can be combined with the propagation of $\sigma_i(u)$, as shown in Figure 2 below.

In the top-down traversal given in Figure 2, we assume that each node α has the "state" from the (earlier) execution of the bottom-up traversal; this consists of the messages that it received from its children, the message that it sent to its parent, and the values σ_i for users i at α (some of which were erroneously, but temporarily, set to 1 and will be corrected in the top-down traversal). The top-down traversal has to convey enough information to allow nodes to compute cost shares using Equation 1 and to correct erroneous σ_i values. Finally, note that two messages must in fact travel over each link in $T(P)$ if cost shares are to be computed correctly. There are instances in which, for all $\alpha \in V(P)$, cost shares at α depend on utilities at every descendant of α and on at least one utility or link cost between α and the root α_s of $T(P)$. In our model, α can only compute such a share after receiving *some* information from its parent and each of its children (although perhaps not as many bits of information as our algorithm sends). Examples of such instances include those in which $R^*(u) = P$ but setting $u_i = 0$ for any i would cause all users in a subtree rooted at some $\beta \in Ch(\alpha_s)$ not

Figure 2: Top-Down Traversal: Computing Membership Bits and Cost Shares

Initialize: Root α_s sends W^{α_s} to each of its children.
For each $\alpha \in V(P) - \{\alpha_s\}$
After receiving message A from parent $p(\alpha)$
//Case 1: $T^\alpha(P) \cap T(R^*(u)) = \emptyset$.
//Set σ_i's properly at α and propagate
//non-membership downward.
If $\sigma_i = 0$, for all $i \in res(\alpha)$, or $A < 0$ then
{
$x_i \leftarrow 0$ and $\sigma_i \leftarrow 0$ for all $i \in res(\alpha)$
send -1 to β for all $\beta \in Ch(\alpha)$
}
//Case 2: $T^\alpha(P) \cap T(R^*(u)) \neq \emptyset$.
//Compute cost shares and propagate
//minimum welfare value downward.
Else
{
$A \leftarrow \min(A, W^\alpha)$
For each $i \in res(\alpha)$
If $u_i \leq A$, then $x_i \leftarrow 0$, else $x_i \leftarrow u_i - A$
For each $\beta \in Ch(\alpha)$
Send A to β
}

to receive the transmission (because each link joining α_s to one of its children has a very high cost). □

4. WELFARE-MAXIMIZING MULTICAST TREE

The form of routing used in this paper requires that all trees are merely subsets of a given tree that connects the source to all users. While this accurately models the current design philosophy of multicast routing, which emphasizes simplicity and stability over optimization, there are other contexts involving tree-like delivery structures in which optimality would become the dominant consideration. One such example is that of installing communication lines connecting several institutions; the situation is static once installed, and thus stability is not an issue, but minimizing cost can be quite important. A natural generalization of the MC mechanism would be to apply it to the case in which the routing infrastructure, for a given receiver set, chooses the tree that minimizes the cost. In this context, there is no notion of a "player" that is resident at a node: Each institution corresponds to a single node, and it has a single utility (or "prize") value; either this node will be connected to the delivery tree by the routing infrastructure, or it won't be.

More formally, let $\mathcal{T}(R)$ denote the set of all trees connecting the source to a receiver set R. Given a receiver set R, we choose the tree $T \in \mathcal{T}(R)$ that minimizes $c(T)$; define $C(R) = Min_{T \in \mathcal{T}(R)} c(T)$. Then, an efficient set $R^* \subseteq N$ is one that maximizes $[u_R - C(R)]$. There is no longer a unique largest efficient set, because the cost function $C(R)$ is not submodular (unlike $c(T(R))$ in our original problem, which is submodular). Let $W(N, L, c, u, r) = Max_{R \subseteq N - \{r\}}[u_R - C(R)]$ be the optimal efficiency. For a given selection of ef-

ficient sets $R^*(u)$, Equation 1 defines the cost shares for an MC-like strategyproof mechanism satisfying NPT, VP, and CS. The question is whether we can find a feasible algorithm to implement this mechanism.

Because computing $W()$ is equivalent to the net-worth maximization version of the *Prize Collecting Steiner Tree* problem [15], it is NP-hard. We now show that $W()$ is NP-hard to approximate within any constant factor, even if (N, L) is a bounded-degree graph.[5]

THEOREM 4.1. *Let $G = (N, L)$ be a graph, $r \in N$ a distinguished root node, $c : L \to \Re^{\geq 0}$ a cost function on links, and $u : N \to \Re^{\geq 0}$ a utility function on nodes. For $R \subseteq N - \{r\}$, let $C(R)$ be the cost of a minimum-cost tree, the node-set of which contains r and R. Then, for any constant ϵ, $0 < \epsilon < 1$, the function*

$$W(N, L, c, u, r) = \max_{R \subseteq N - \{r\}} (u_R - C(R))$$

is NP-hard to approximate[6] within ratio ϵ. Moreover, this hardness result holds even if (N, L) is a bounded-degree graph.

PROOF. Let $(x_1, \ldots, x_n, C_1, \ldots, C_m)$ be a SAT instance. Assume without loss of generality that the formula contains a clause $(x_i \vee \bar{x}_i)$, for all $1 \leq i \leq n$.

The equivalent graph has four levels:

- The first level contains the root r, with utility ϵ^3.
- The second level contains a node r', with utility zero, connected to r via a link of cost $m \cdot K - n - 1$, where $K \gg m$.
- The third level contains $2n$ nodes, one for each x_i and $\bar{x}_i$, all with utility zero, and all connected to r' via unit-cost links.
- Finally, the fourth level contains m nodes, one for each clause, each of utility K, and each connected via zero-cost links to the level-3 nodes corresponding to the literals it contains.

Any graph of this form has a trivial solution of welfare ϵ^3, containing only the root node r.

To get a solution of welfare greater than ϵ^3, we must include the link (r, r'), which in turn forces the inclusion of all of the level-4 nodes. At most n links from r' to level-3 nodes can be included, because $n + 1$ such links would put us back to welfare ϵ^3. For each i, one of the x_i-node or the $\bar{x}_i$-node must be included, in order to reach the level-4 node corresponding to clause $(x_i \vee \bar{x}_i)$. Therefore, a solution of welfare greater than ϵ^3 corresponds to a satisfying assignment; in fact, such a solution will have welfare exactly $1 + \epsilon^3$.

Unsatisfiable formulas correspond to graphs of maximum welfare ϵ^3, on which an ϵ-approximation algorithm for $W()$ would return a value less than or equal to ϵ^2. Satisfiable formulas correspond to graphs of maximum welfare $1 + \epsilon^3$, on which an ϵ-approximation algorithm would return a value greater than or equal to $\epsilon(1+\epsilon^3) > \epsilon$. Thus ϵ-approximating $W()$ is NP-hard.

[5]This easy result had been absent from the extensive literature on this problem.

[6]Recall that algorithm A "approximates f within ratio ϵ, $0 < \epsilon < 1$," if, for all x, $\epsilon \leq A(x)/f(x) \leq 1/\epsilon$.

To obtain a hardness result for bounded-degree graphs, we modify the reduction as follows. First, replace the $2n$ links that connect r' to the level-3 "literal nodes" with a binary tree whose root is r' and whose leaves are the $2n$ literal nodes. All of the nodes in this binary tree have utility zero, and all of the links have cost zero, except for the links that connect the literal nodes to their parents; each of these $2n$ links has cost one. Next, assume without loss of generality that each clause C_j either is the disjunction of three literals or is of the form $(x_i \vee \bar{x}_i)$. For each literal y, use a binary tree to connect the literal node corresponding to y to all of the nodes corresponding to clauses in which y appears. All of the links in these trees have cost zero, and all of the nodes have utility zero, except for the "clause nodes" themselves, each of which has utility K, as it did in the original reduction. □

5. SHAPLEY VALUE MECHANISM

SH cost shares can be computed by a brute-force algorithm involving, in the worst case, the exchange of $\Omega(n \cdot p)$ messages, with at least p messages over certain links in the worst case. We conjecture that this complexity is the best possible and present some evidence for this below. We first, however, explain the brute-force algorithm.

The simplest case of the SH cost-share problem is the one in which all u_i are sufficiently large to guarantee that all of P receives the transmission. (For example, $u_i > c(T(P))$, for all i, would suffice.) For this case the SH cost-shares can be computed as follows.[7] Do a bottom-up traversal of the tree that determines, for each node α, the number p_α of users in the subtree rooted at α. Then, do a top-down traversal, which the root initiates by sending the number $md = 0$ to its children. After receiving message md, node α computes $md' \equiv \left(\frac{c(l)}{p_\alpha}\right) + md$, where l is the network link between α and its parent, assigns the cost share md' to each of its resident users, and sends md' to each child. Thus, each user ends up paying a fraction of the cost of each link in its path from the source, where the fraction is determined by the number of users sharing this link.

To apply the brute-force algorithm to the general case, we must proceed in an iterative fashion. We initially start, as before, with $R = P$ and compute the cost shares as above. However, in the general case we cannot assume that $u_i \geq md'$ for all i, and so some users may prefer not to receive the transmission. So, after an iteration, we update R by omitting all users i such that $u_i < md'$, and repeat. The algorithm terminates when no more users need to be dropped.

THEOREM 5.1. *The brute-force algorithm implements the SH mechanism with $O(n \cdot p)$ message exchanges.*

We next show a result strongly suggesting that the brute-force algorithm is essentially optimal, and that such quadratic dependence is inherent.

Lower Bound

We prove our lower bound in the context of a particular class of distributed algorithms. These algorithms start with real variables stored locally at the nodes and proceed by exchanging messages between nodes, interleaved with local

[7]This simple case is essentially a distributed version of the linear-time algorithm given in [20].

computations. Each message is a real number, a *linear combination of the variables at the sending site and the messages received at the sending site at previous steps.*[8] If the coefficients involved in these linear combinations are constant and fixed in advance, then the algorithm is called an *oblivious linear distributed algorithm.*

A more general class of algorithms is the one in which the computation of the next message to be sent from a node is a decision tree with comparisons of linear combinations of the local variables and the messages received; the leaves of the tree then compute linear combinations of the local variables and the previously received messages. We call these the *non-oblivious linear distributed algorithms*, or simply *linear distributed algorithms*, a class that includes all algorithms described in this paper. In such algorithms, the space of all possible variable values of the initial variables is subdivided into convex polytopes, and particular values for the constant coefficients of the linear combinations prevail at each polytope.

We can show the following result, suggesting that the $O(n \cdot p)$ upper bound in the previous theorem is optimal:

THEOREM 5.2. *There is an infinite family of multicast computations, with n nodes and $O(n)$ users, such that any linear distributed algorithm that implements the SH mechanism requires in the worst case $\Omega(n^2)$ message exchanges.*

Proof: Consider a tree that is a path with nodes α_0 through α_n. The source is α_0, the cost of the link $[\alpha_i, \alpha_{i+1}]$ is c_{i+1}, each of the nodes $\alpha_i, i = 1, \ldots, n-1$ has an agent with utility u_i, while there are n agents at α_n, with utilities $u_{nj}, j = 1, \ldots, n$. For reasons of purely notational simplicity, we assume here that the cost c_i is known only by node α_{i-1} (and not by node α_i); this only increases the number of messages that need to be exchanged across each link by at most one. We shall show that $\Omega(n^2)$ messages are necessary. In particular, we shall show that i messages must be exchanged across link $[\alpha_i, \alpha_{i+1}], i = 0, \ldots, n-1$.

Consider the last link $[\alpha_{n-1}, \alpha_n]$ (the general case is very similar, and is explained later). Suppose that the utilities u_i are very large, while all n agents at α_n are excluded one by one at each iteration of the brute-force algorithm. Consider the task of actually *checking* that the link costs and the utilities u_{nj} fall into this case. When the u_{nj} are in decreasing order, this is tantamount to checking the inequalities

$$u_{nj} < \sum_{i=1}^{n} \frac{c_i}{n+j-i} \qquad (2)$$

Because we are only interested in the messages across link $[\alpha_{n-1}, \alpha_n]$, we can regard the network as two nodes, α_n and another node α that is formed by merging all the rest. The variables $u_{nj}, j = 1, \ldots, n$ are local to node α_n, while the variables $c_i, i = 1, \ldots, n$ are known to the node α. The assertion of the Theorem follows from the following general result about distributed computation:

LEMMA 5.3. *Any linear distributed algorithm by two nodes for checking the inequalities $Ax + By > a$ (assumed to have a feasible solution), where x is an n-vector known to node I, y is an n-vector known to II, A, B are $n \times n$ nonsingular matrices known to both, and a is an n-vector known to both, requires n message exchanges.*

Proof of the Lemma: We shall first prove the oblivious case, which contains the basic mathematical idea, and then discuss the necessary extensions for the general case. Consider an oblivious algorithm with $d < n$ message exchanges. It is easy to see that the d message exchanges have the effect of I and II agreeing on the value of a d-vector $Cx + Dy$. Assume without loss of generality that $Ax + By > a$. After all of the messages have been exchanged, one of the nodes will decide that $Ax + By > a$, and we assume that without loss of generality that this is node I.

Since $\text{rank}(D) < n = \text{rank}(B)$, there is a vector $\hat{y} \neq 0$ in the null space of D, that is, $D\hat{y} = 0$ but $B\hat{y} \neq 0$. By adding to y an appropriate multiple of $\hat{y}$, an adversary can change y so that the values $Cx + D(y + \hat{y})$ of the exchanged messages stay the same (and hence agent I must make the same decision), and still $Ax + B(y + \hat{y})$ has a nonpositive component, contradicting the correctness of the algorithm. This concludes the proof for the oblivious case.

To deal with the non-oblivious case (where the matrices C and D can be functions $C(x, y)$ and $D(x, y)$ of the particular subdivision of the x, y space), we consider the affine space AS satisfying $Ax + By = a$.

We also take a particular full-dimensional polytopal subdivision which (a) intersects the feasible region $Ax + By > 0$, and (b) intersects AS (it is easy to see that, since we have assumed that fewer than n messages were exchanged, such a subdivision must exist). Let the $d \times n$ matrices for this subdivision be C and D, and consider, as before, a vector $\hat{y}$ in the null space of D that is not in the null space of B. $B\hat{y}$ has a non-zero element in one of the rows in the maximal independent set used to define AS, and so, an appropriate multiple of $\hat{y}$, added to a particular feasible y within this subdivision, invalidates one inequality of $Ax + By > a$. □

Notice that the inequalities (2), with x standing for the u_{nj}'s and y for the c_i's, are within the scope of the Lemma, with $A = -I$ and B the nonsingular Toeplitz matrix $T_{ij} = \frac{1}{n+j-i}$. Hence it follows from the Lemma that at least n messages must be exchanged across link $[\alpha_{n-1}, \alpha_n]$. However, the same lemma shows that $n-1$ messages must be exchanged across link $[\alpha_{n-2}, \alpha_{n-1}]$, and in general $n-j$ messages are needed across link $[\alpha_{n-j-1}, \alpha_{n-j}]$. It follows that $\Omega(n^2)$ messages are needed in toto.

We would like to prove two extensions of this lower bound: The first deals with non-linear algebraic operations and the second with the case in which the costs c_i are known in advance. The first extension should be provable by exploiting some algebraic-geometric understanding of the local structure of the high-dimensional variety that would now be the analogue of $Cx + Dy = b$. The second seems true but may require more sophisticated lower bound techniques.

[8] Synchronization is, of course, crucial in such algorithms. However, as it will soon become apparent, we shall be interested in distributed algorithms over *two* sites, in which message exchange can be assumed to proceed in an orderly alternating fashion. Notice also that we assume that each message is a real number with no bounds on size, precision, or other computational attributes, in the spirit of algebraic complexity —a framework which excludes encoding tricks but includes all algorithms presented in this paper and enables our algebraic lower-bounding argument.

6. SUMMARY AND OPEN PROBLEMS

In this paper, we considered the problem of sharing the cost of a multicast transmission. We assumed that multicast transmissions incurred per-link network costs and that the strategyproof cost-sharing mechanisms had to satisfy the NPT, CS, and VP conditions described in Section 2.2. Based on the characterization in [22] of such mechanisms that are either budget-balanced or efficient, we focused on two cost-sharing mechanisms: Marginal Cost and Shapley Value. The treatment in [22] focused solely on the game-theoretic aspects of these two mechanisms, and our goal here was to determine whether they could be feasibly implemented on a network. We were thus led to analyze their communication complexity, and the apparent gap between the communication complexity of MC and that of SH was the topic of our main results.

The MC mechanism is implementable with an algorithm that only requires a single message sent in each direction on each link in the tree $T(P)$. We call a mechanism *nonseparable*[9] if the cost shares of an agent can depend on those of every other agent with which it shares a link; that is, if, for every i, j with $T(i) \cap T(j) \neq \emptyset$, there exist utility vectors u and v, $u_k = v_k$ for all $k \neq j$, such that $x_i(u) \neq x_i(v)$ or $\sigma_i(u) \neq \sigma_i(v)$. Implementing a nonseparable mechanism requires that at least one message traverse each link in $T(P)$. Our results thus imply that, up to constant factors, the Marginal Cost mechanism is as easy to implement as any other nonseparable mechanism. Informally, we use the term "minimal" to describe mechanisms that can be implemented with $O(1)$ messages traversing each link.

By contrast, there is an infinite family of cases in which the SH mechanism requires (in the limited computational model we considered) a linear number of messages on a linear number of links. Note that this is roughly the same amount of communication used by the *centralized* approach of sending all the u_i and c_j values to a designated node, computing the resulting cost shares at that designated node, and then sending the x_i and σ_i values back to each node. This centralized approach can be applied to all polynomial-time cost-sharing mechanisms; thus, at least in the limited computational model we considered, no cost-sharing mechanisms are qualitatively harder to implement than the SH mechanism. Informally, we the term "maximal" to describe mechanisms that require a linear number of messages to traverse a linear number of links.

Therefore, the two mechanisms we considered are at the opposite ends of the feasibility spectrum. Although the purely game-theoretic analysis in [22] gave little guidance as to which might be more appropriate in practice, the communication-complexity analysis here leaves little doubt that only the MC cost-sharing mechanism is feasible for large user populations. This example suggests that communication complexity may be an important factor in evaluating possible cost-sharing mechanisms in distributed systems.

Our study leaves many open questions. We divide them into four categories.

First, we know very little about minimal and maximal mechanisms in general. Natural questions include:

- Are all nonseparable budget-balanced mechanisms (satisfying group strategyproofness, CS, VP, and NPT) maximal? That is, does the budget-balance requirement lead naturally, with a few additional restrictions, to the infeasibility of the mechanism?
- Are there any efficient and strategyproof mechanisms besides Marginal Cost that are minimal? Recall that MC was essentially the only efficient strategyproof mechanism satisfying VP and NPT; thus, this question asks whether relaxing the VP and/or NPT requirements would lead to mechanisms that were less easily implementable than MC.
- What are the game-theoretic properties of strategyproof, minimal mechanisms? Are there game-theoretic properties that are especially compatible with ease of implementation?

Second, the current model is, in many ways, unrealistic; we should move beyond the current model to see what factors significantly change the nature of the results. For example, it is doubtful that network providers would charge per-link for transmissions. While it is not clear what a more realistic charging model would be, it would be of interest to know how the results here would change as the charging model changed. In addition, the NPT requirement – that users are not paid for participating – does not apply to all real-world businesses, particularly e-businesses; as noted above, we would like to determine the communication-complexity properties of cost-sharing mechanisms that do not have the NPT requirement.

In a similar vein, we can broaden the class of mechanisms considered by addressing different goals. Our treatment, following [22], only considered budget-balance and efficiency as the two natural goals (in addition to our basic requirements of strategyproofness, CS, VP, and NPT). However, profit maximization is also an important goal for economic mechanisms. While budget-balance and efficiency are compatible with strategyproofness, the naive definition of maximal profit is not; all profit-maximizing mechanisms are equivalent to the one that charges each agent in the efficient coalition u_i and excludes all agents not in the efficient coalition; this is clearly not strategyproof. The open question is how to define a standard for profit maximization that is compatible with strategyproofness. Goldberg *et al.* [8] address this issue quite elegantly for the case in which there are no costs. Their approach does not appear to generalize easily to the case in which users incur different network costs.

Third, the basic question of feasibility can be applied to a much more general class of mechanism-design problems. For all of the Nash implementation and strategyproof mechanisms in the literature, one can ask what their communication complexity would be if the users were on a network. While in our current problem the network setting is intrisincally intertwined with the statement of the problem, there will doubtless be situations in which agents involved in some other mechanism interact via the network rather than being centrally located. The question is whether communication complexity is likely to be an important issue for these mechanisms, and that will depend greatly on the particular setting. An example of where this might be important is in *distributed auctions*; what class of strategyproof auctions can be implemented so that the number of messages traversing any link is $O(1)$?

[9] This definition of "nonseparable" is much stronger than necessary, but the more appropriate definition is significantly more complicated to express.

Fourth, and last, we can leave the game-theoretic considerations behind and focus solely on the communication complexity of various distributed functions. Viewing the u_i as general inputs and the x_i as general outputs, one can ask which functions $x_i(u)$ can be feasibly computed on a network. In particular, which other functions $x_i(u)$ are minimal, which are maximal, and which are in between? For example, the average function $x_i(u) = \frac{\sum_j u_j}{n}$ is minimal, but we do not yet know the communication complexity of the median function $x_i(u) = \texttt{med}\, u_j$.

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