CS 15-892 Foundations of Electronic Marketplaces
Homework 3

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Due November 25th in the beginning of class.

This homework is self-contained so you will not need any sources beyond the course materials. However, you may use any sources that you want. If you do so, you must cite the sources that you use. Teamwork is not allowed.

1. Review questions. Use at most 10 sentences for each answer. You do not need to write any formal proofs, but you should justify your answers. You are allowed to refer to known results in those justifications.

   - (5 pts) From a game theory perspective, is it possible to build a combinatorial exchange that leads to a Pareto efficient outcome in dominant strategy equilibrium (we require that participation is voluntary, and that the exchange is *ex post* budget-non-negative (that is, the auctioneer does not need to subsidize the exchange))? What about in Bayes-Nash equilibrium? [You can assume that each bidder knows his own valuations on all bundles, and that communication and computation are no problem.]
   - (5 pts) From a game theory perspective, is it possible to build a combinatorial reverse auction that leads to a Pareto efficient outcome in dominant strategy equilibrium (we require that participation is voluntary, and that the exchange is *ex post* budget-non-negative (that is, the auctioneer does not need to subsidize the exchange))? What about in Bayes-Nash equilibrium? [You can assume that each bidder knows his own valuations on all bundles, and that communication and computation are no problem.]
   - (5 pts) From a game theory perspective, is it possible to build a combinatorial auction that leads to a Pareto efficient outcome in dominant strategy equilibrium (we require that participation is voluntary, and that the exchange is *ex post* budget-non-negative (that is, the auctioneer does not need to subsidize the exchange))? What about in Bayes-Nash equilibrium? [You can assume that each bidder knows his own valuations on all bundles, and that communication and computation are no problem.]
   - (5 pts) What is the computational complexity of finding a feasible solution in a combinatorial auction, combinatorial reverse auction, and a combinatorial exchange? How, if at all, do these complexities change if the bidders can submit XOR-constraints between some of their bids?

- (5 pts) How well can the winners in a combinatorial auction, combinatorial reverse auction, and a combinatorial exchange be approximated in polynomial time? How, if at all, do these complexities change if the bidders can submit XOR-constraints between some of their bids?
- (5 pts) What is the computational complexity of finding an optimal solution in a combinatorial auction, combinatorial reverse auction, and a combinatorial exchange? How, if at all, do these complexities change if the bidders can submit XOR-constraints between some of their bids?

2. (30pts) In a combinatorial auction that uses the GVA (that is, Clarke tax mechanism), the payment that bidder i has to make is computed as follows. First, determine the winners. Call the sum of the winning bids of the other agents (except i) a. Then, determine the winners again without i's bids. Call the sum of the winning bids b. Now, agent i pays $b - a$.

   Another generalization of the Vickrey auction to the combinatorial auction setting would determine i's payment differently as follows. For each winning bid S of agent i, let a^i_S be the sum of the prices of the *other* winning bids (by agent i and by the other agents). Then, determine the winners again with bid S removed. Call the sum of the winning bids b^i_S. Now, the "price" of bid S is $b^i_S - a^i_S$. The amount that agent i has to pay overall is

$$\sum_{S \in \text{i's winning bids}} b^i_S - a^i_S$$

   Is this mechanism incentive compatible? If so, prove that. If not, show a manipulation.