CS 15-892 Foundations of Electronic Marketplaces
Homework 2

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Due November 4th in the beginning of class.

You may use any sources that you want. If you do so, you must cite the sources that you use. Teamwork is not allowed.

1. (20pts) Consider a private-values auction of one good when bidders have quasilinear utility functions and know their own valuations. Prove that bidding truthfully is a (weakly) dominant strategy in the Vickrey auction. [Prove this from first principles; do not use the fact that the Vickrey auction is a special case of the Groves mechanism.]

2. (40pts) Consider designing a mechanism (where participation is *ex post* individually rational) for the following setting. You have one company (*Profit & Gamble)* to sell. You don't care about keeping it or getting rid of it. There are two bidders with quasilinear utility functions. Bidder 1's valuation is drawn from a uniform distribution on [0,1] (1 = one billion dollars). Bidder 2's valuation is independently drawn from a uniform distribution on [1,4].

   (a) Design a mechanism that uses take-it-or-leave-it offers (at most one offer to each bidder) and attempts to maximize revenue subject to that. What is your expected revenue in your mechanism? What is your worst-case revenue in your mechanism? Is your mechanism Pareto efficient? Justify your answer.

   (b) What is the optimal (i.e., revenue-maximizing) auction for the setting? What is your expected revenue in that auction? What is your worst-case revenue in that auction? Is that auction Pareto efficient? Justify your answer.

3. (40pts) (War of attrition [from Auction Theory, 2nd edition, by Krishna]) Consider a two-bidder war of attrition in which the bidder with the highest bid wins the object but both bidders pay the losing bid. The bidders' values are independently and identically distributed according to F.

   (a) Derive a symmetric equilibrium bidding strategy.

   (b) Directly show the symmetric equilibrium strategy and the seller's revenue when the bidders' values are uniformly distributed on $[0, 1]$.

   Hint: Use the final equation of the following proof of the revenue equivalence theorem [slightly adapted from Auction Theory, 2nd edition, by Krishna].

Consider any auction where the highest bidder wins, and fix a symmetric equilibrium β of it. Let p(v) be the equilibrium expected payment by a bidder with value v. Suppose β is such that $p(0) = 0$.

Consider the expected payoff of bidder 1 with value v when he bids $\beta(z)$ instead of $\beta(v)$. Bidder 1 wins when his bid $\beta(z)$ exceeds the highest competing bid $\beta(Y_1)$, i.e., when $z > Y_1$. His expected payoff is

$$u(z, v) = G(z)v - p(z)$$

where $G(z) = F(z)^{N-1}$ is the distribution of Y_1.

Maximization results in the first-order condition

$$\frac{\partial u(z, v)}{\partial z} = g(z)v - \frac{d}{dz}p(z) = 0$$

At an equilibrium it is optimal to bid according to $z = v$, so we obtain that for all y,

$$\frac{d}{dy}p(y) = g(y)y$$

Thus,

$$\begin{aligned} p(v) &= p(0) + \int_0^v yg(y)dy \\ &= \int_0^v yg(y)dy \\ &= G(x)\ E[Y_1|Y_1 < x] \end{aligned}$$

Since the right hand side does not depend on the particular auction form, revenue equivalence follows. □