Near Optimal Auctions with Unknown Demand

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(Joint work with Kaustaubh Desmukh, Amos Fiat,

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September 30, 2003

Overview

1. Basic (Unlimited Supply) Auction Problem
2. Motivation: limited supply & overview of extensions.
3. Competitive Analysis.
4. Competitive Auctions.
   (a) Random Sampling: market analysis on the fly.
   (b) Private Value Decision Problem.
   (c) Reduction to decision problem, via random sampling.
   (d) Reduction to decision problem, via consensus estimates.
5. Extensions.
   (a) Cancellable Auction and the Multicast Problem.
   (b) Multi-unit Double Auction.

Optimization with Private Values

Optimization Algorithms:

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Goal: Optimization in the Private Value Model

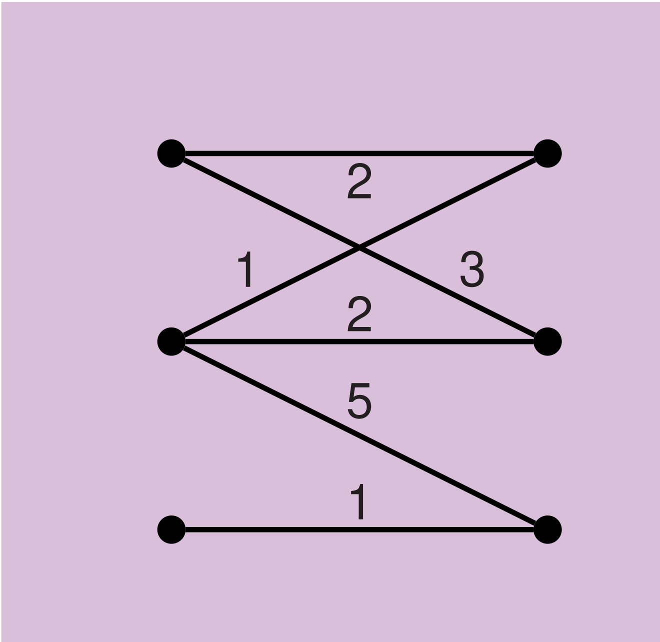
Example Problem

Classical Optimization Problem: Maximum Weighted Matching

Input: Weighted bipartite graph.

Output: Matching that maximize the sum of matched edge weights.

Example:



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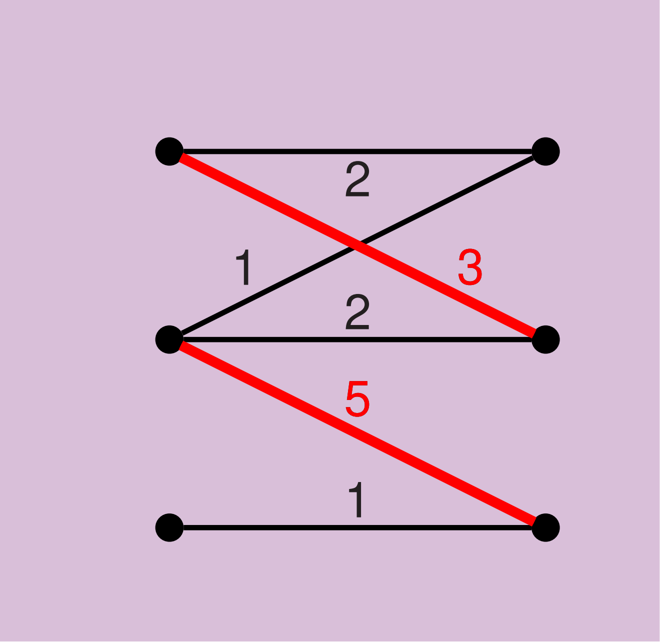

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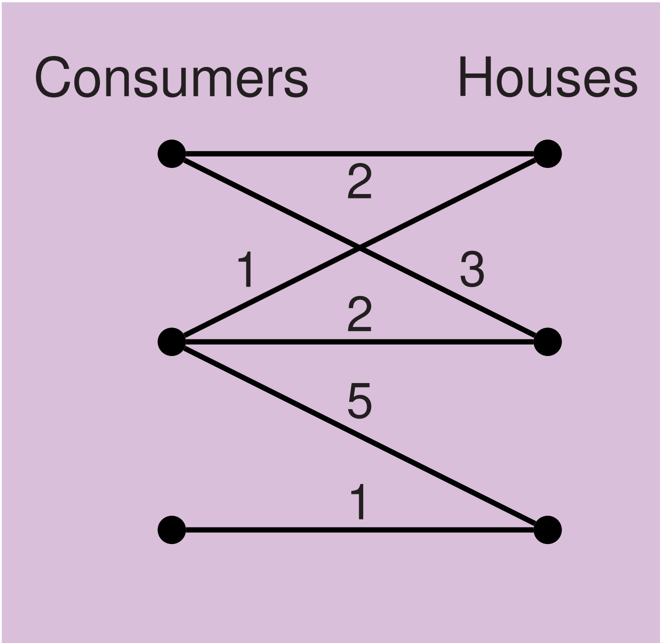
- Real estate agency has houses for sale.
- Consumers are willing to pay different amounts for each house.



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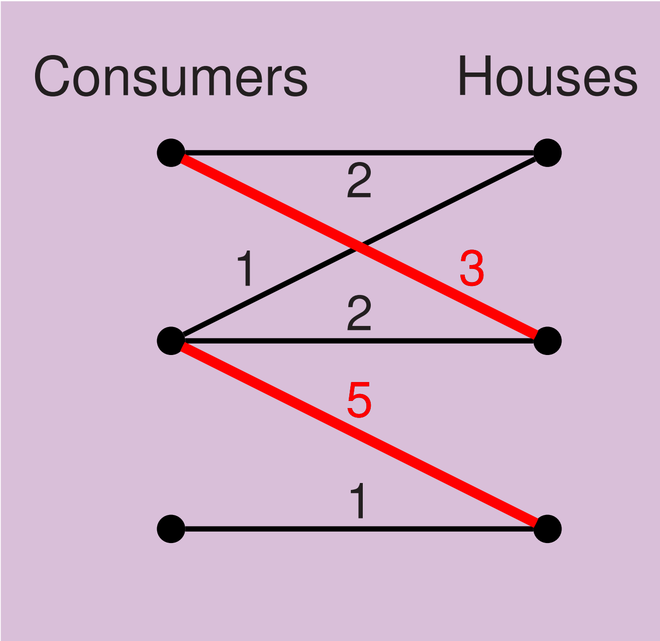
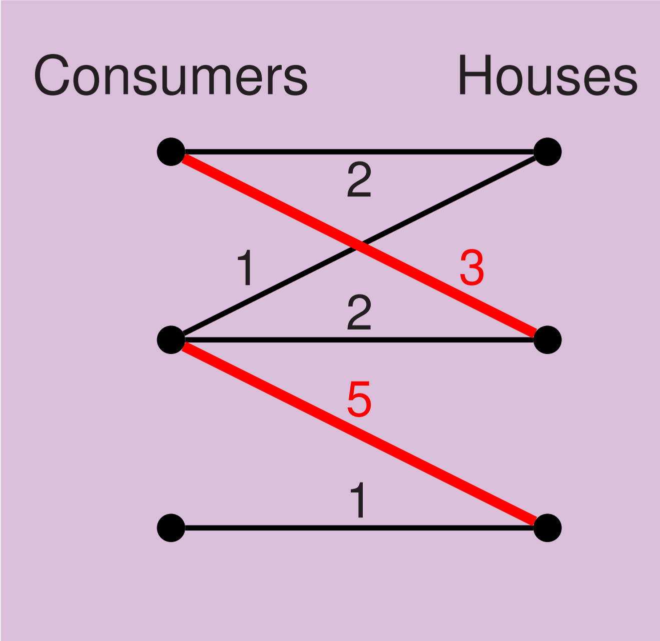


- Optimal Sale = Maximum Weighted Matching.

Private Values

For Private Values:

- Algorithm must solicit values.
- Consumers may lie to get better deal.



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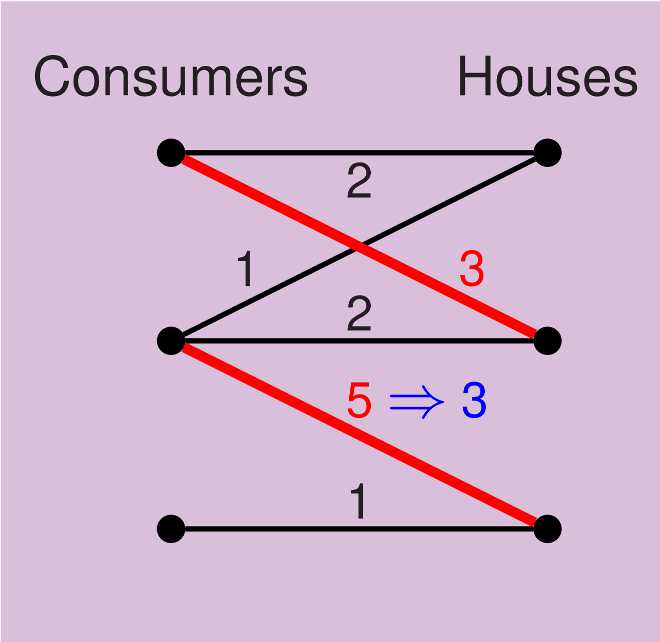

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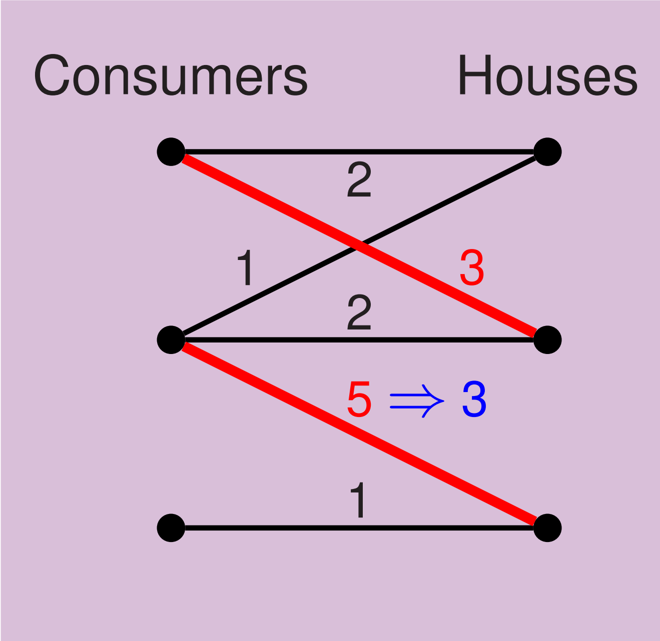

- What if all consumers speculate?

Objective

Research Question: For any optimization problem, what happens if values are private?

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For this talk: worst case profit maximization.

- E.g., for electronic commerce and dynamic pricing.
- Focus on an *auction problem*.

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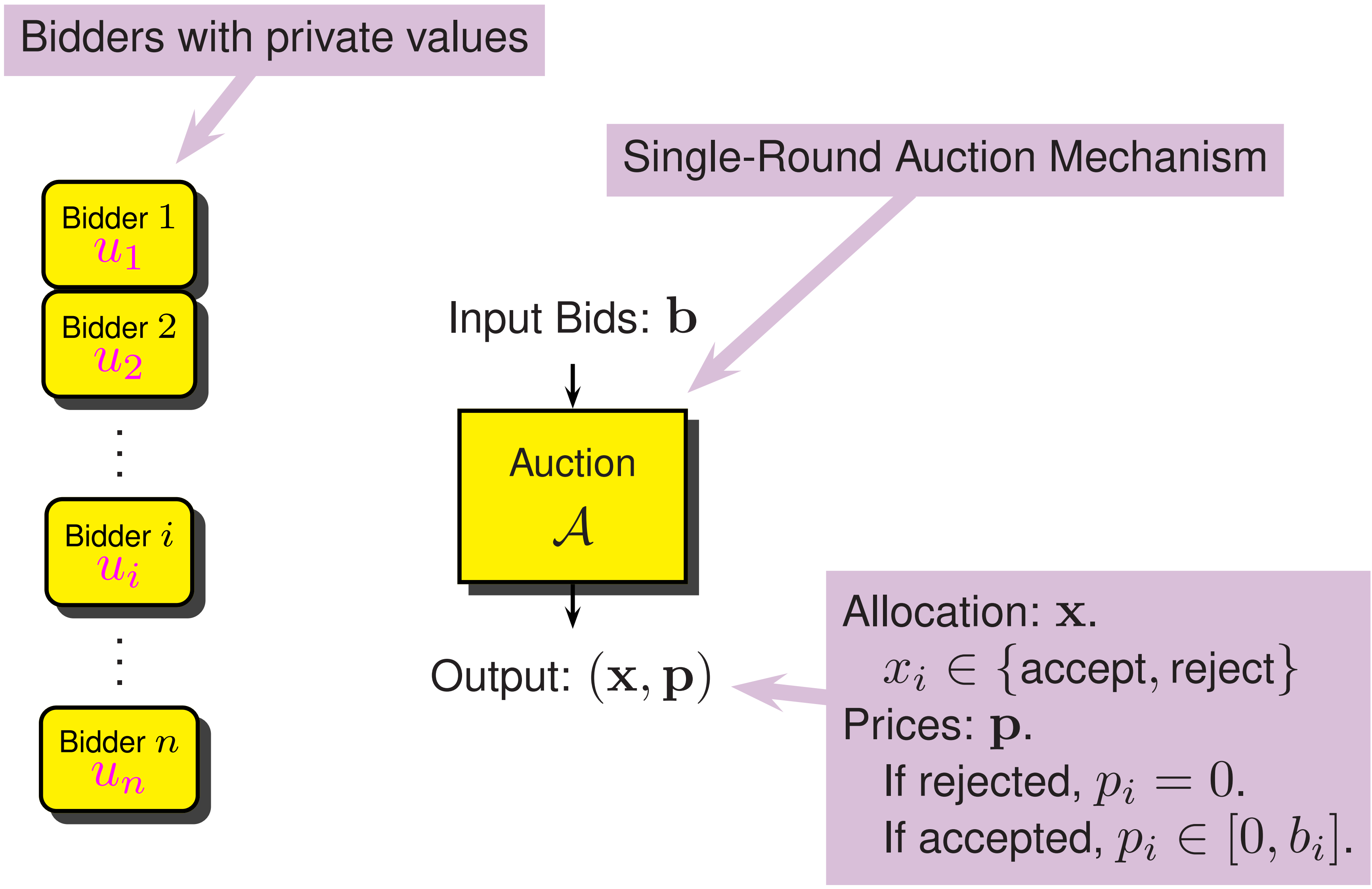
- Special case of many dynamic pricing problems.
- Insight into profit maximization in general.

Single-Round Auction



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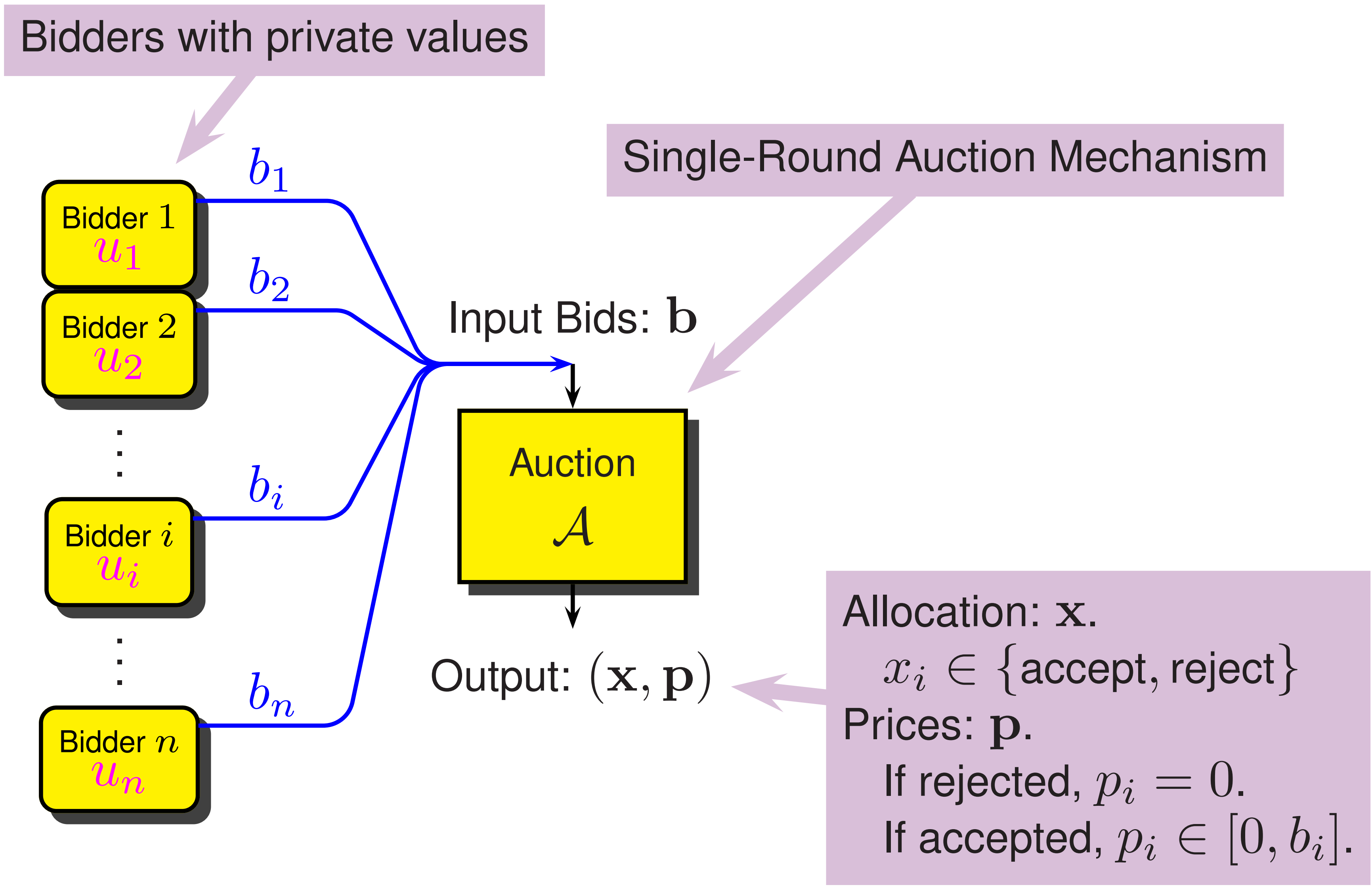

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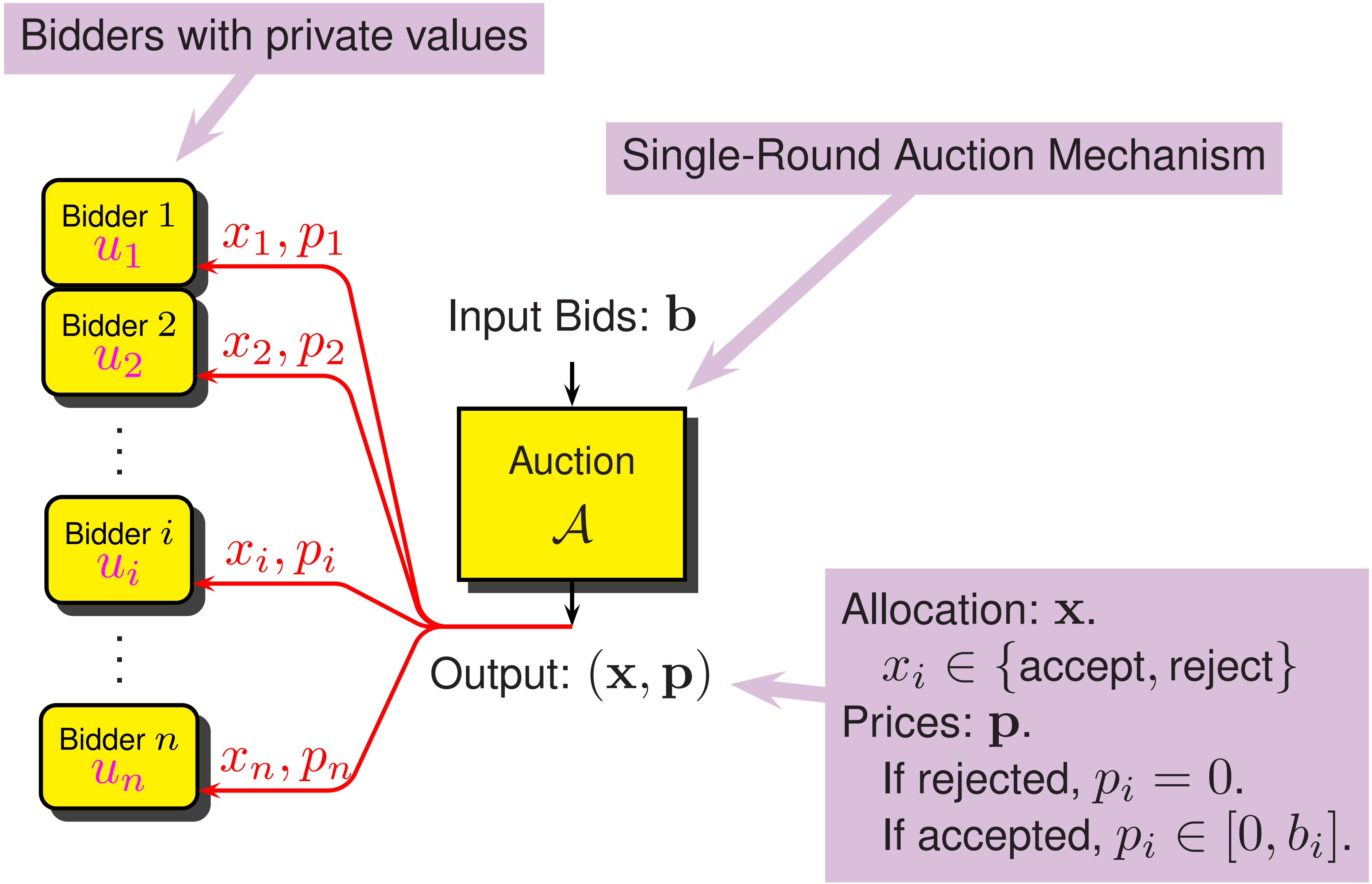

Truthful Mechanism Design

Problem: for auction $\mathcal{A}$,

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New Goal: Design truthful auction to maximize profit.

Standard Pricing Mechanisms

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k-item Vickrey Auction, k-Vickrey

Sell to highest k bidders at $k+1$st highest bid value.

Bid-Independence

Definition: Bids with bidder i removed:

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Bid-Independent Auction: $\mathcal{A}_f$

On input $\mathbf{b}$, for each bidder i:

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Theorem: An auction is truthful iff it is bid-independent.

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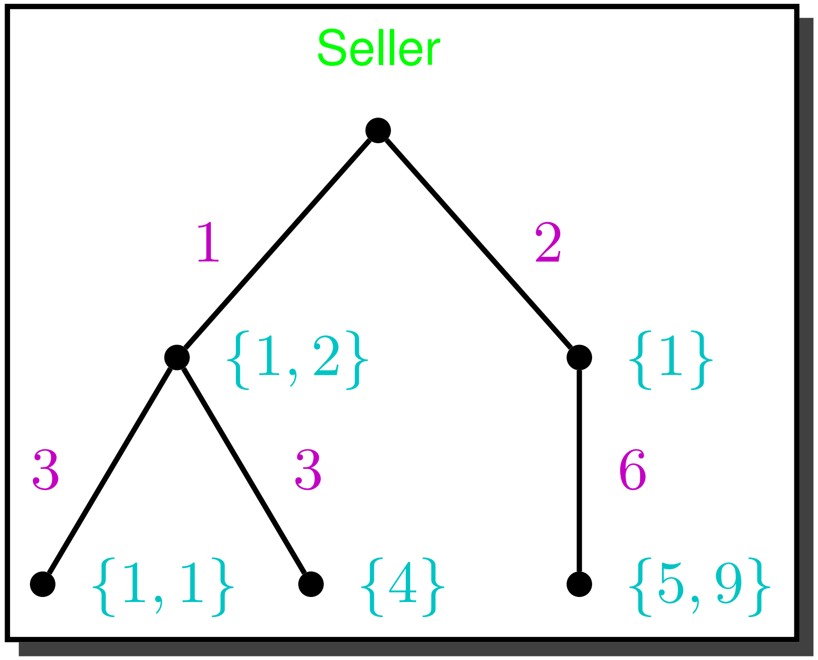
Multicast Tree Problem

Multicast pricing [FPS-00]
Select which consumers receive the multicast (winners).

- seller at root.
- consumers at tree nodes with private values.
- cost for subtree to winners.

profit = revenue − cost



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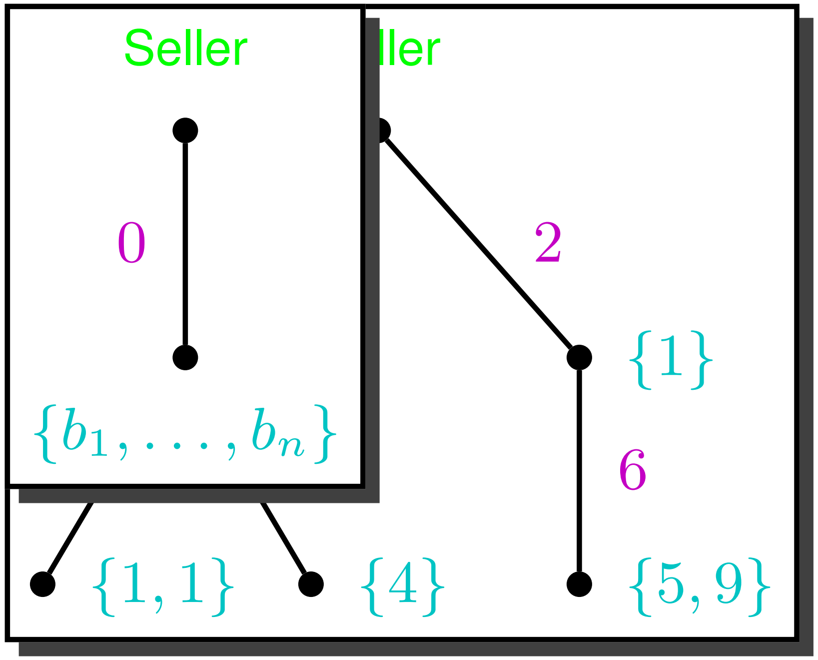

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Multi-unit Double Auction Problem

- n buyers and n sellers want to exchange a commodity.
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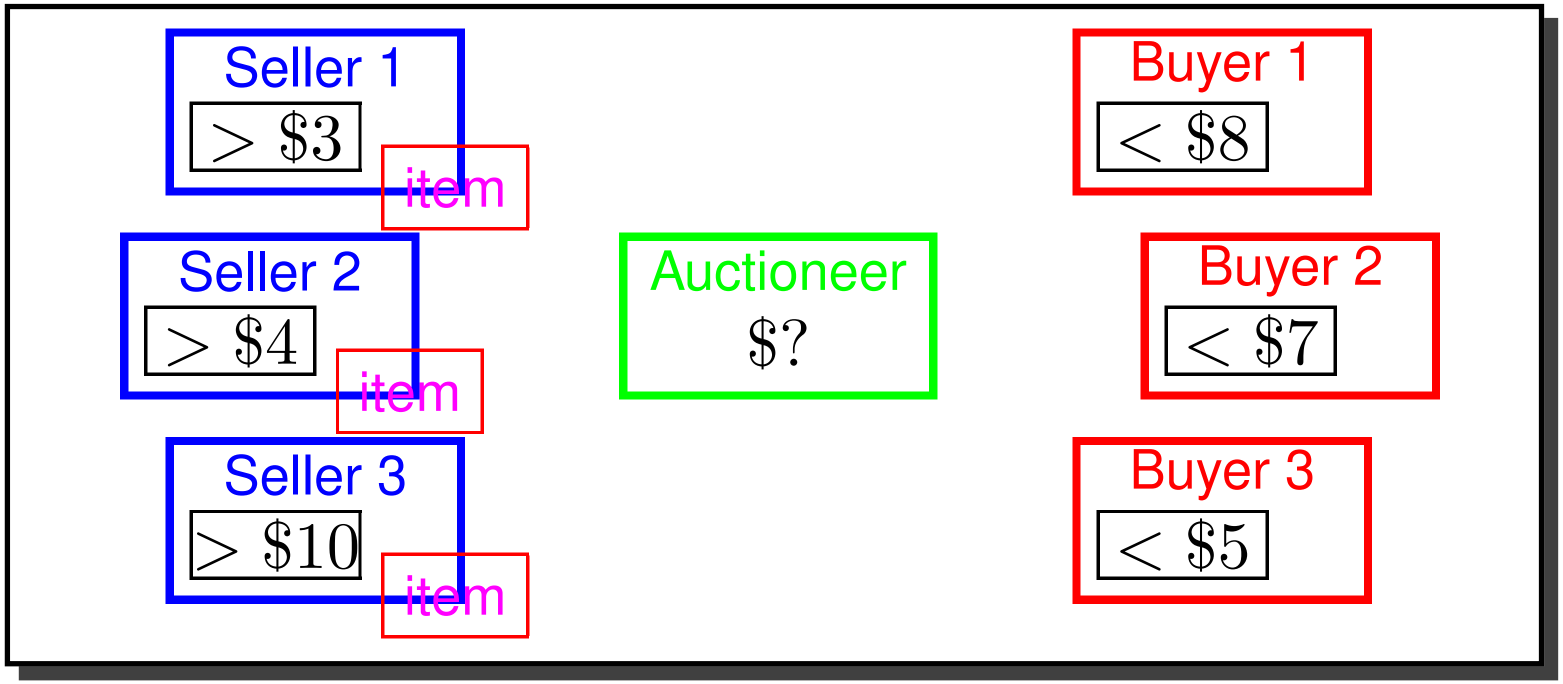
- Third party "auctioneer" mediates the exchange.
  (i.e., reallocates the items, collects, and makes payments)



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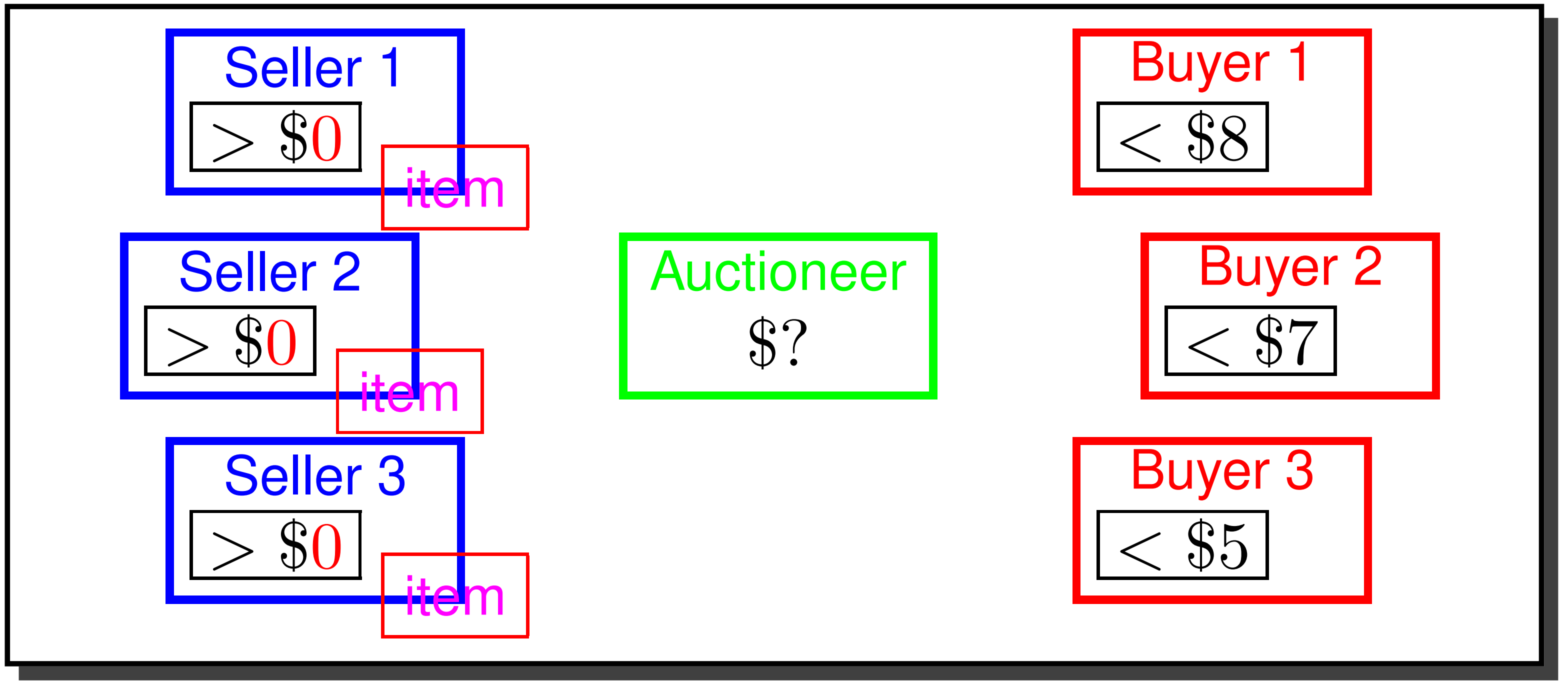

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Main Point: with unlimited supply, focus is on profit maximization.

Limited Supply

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Reduction:

1. Simulate k-Vickrey $\longrightarrow$ k winners at sale price p.
2. Run unlimited supply auction on winners of k-Vickrey.
   (but require price at least p)

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What is optimal public value auction?

Public Value Optimal Multi-price

Optimal Public Value Mechanism: $\mathcal{T}(\mathbf{b})$

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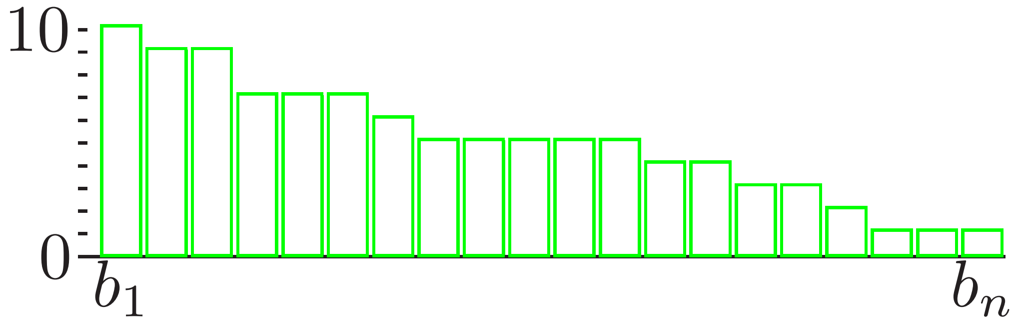

Clearly: $\mathcal{T}(\mathbf{b}) = \sum_i b_i$.

The Price of Private Values

Public Value Optimal Single Price, $\mathcal{F}$

- Choose optimal p for $\mathrm{FixedPricing}_p(\mathbf{b})$.
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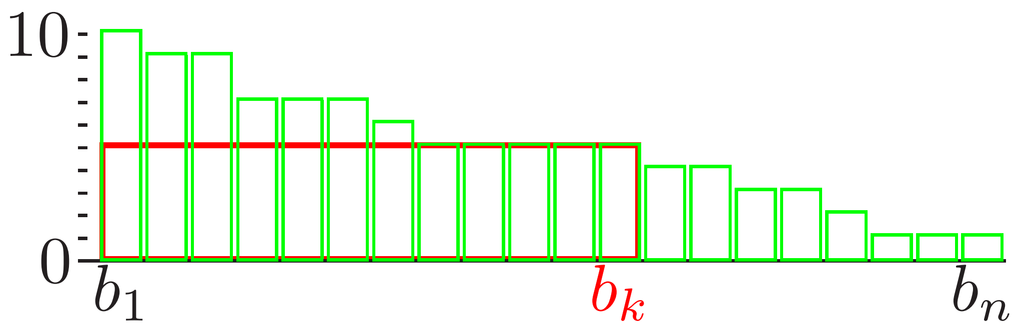

Fact: $\mathcal{F}(\mathbf{b}) = \max_i ib_i$.

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Lemma: No auction gets constant fraction of maximum bid.
E.g., $\mathbf{b} = (1, 1, h, 1, 1, 1)$.

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Worst Case Competitive Framework:

Definition: $\mathcal{F}^{(2)}(\mathbf{b}) = \max_{k \geq 2} kb_k$ – optimal single price-profit with at least two winners.

Definition: Auction $\mathcal{A}$ is β-competitive if $\mathbf{E}[\mathcal{R}] \geq \mathcal{F}^{(2)}(\mathbf{b})/\beta$.

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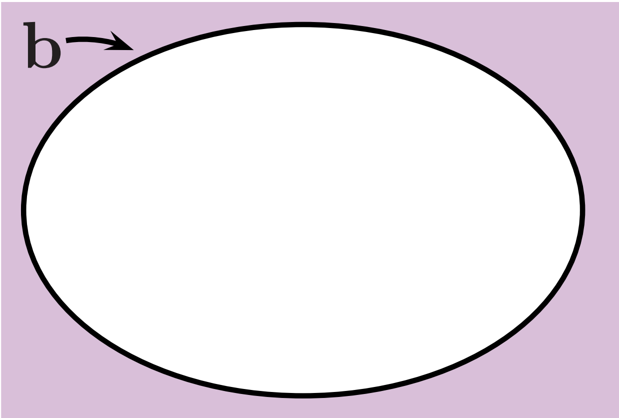
In Worst Case:

- Fixed choice of p or k in $\mathrm{FixedPricing}_p$ and k-Vickrey is not competitive.
- Randomized choice is not better than $(\log n / \log\log n)$-competitive.

Random Sampling

Sampling Optimal Price Auction

1. Randomly partition bids into two sets: $\mathbf{b}'$ and $\mathbf{b}''$.
2. Use $p' = \mathrm{opt}(\mathbf{b}')$ as price for $\mathbf{b}''$.



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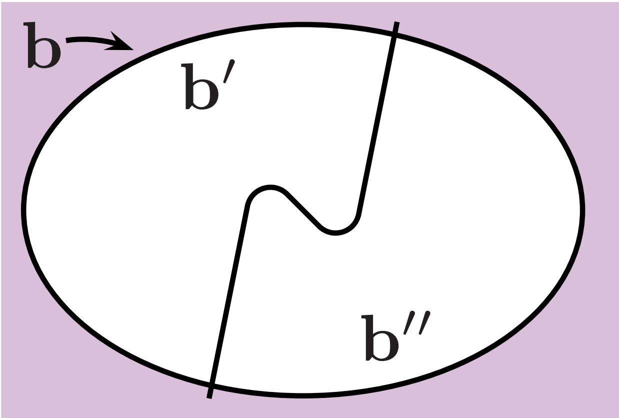

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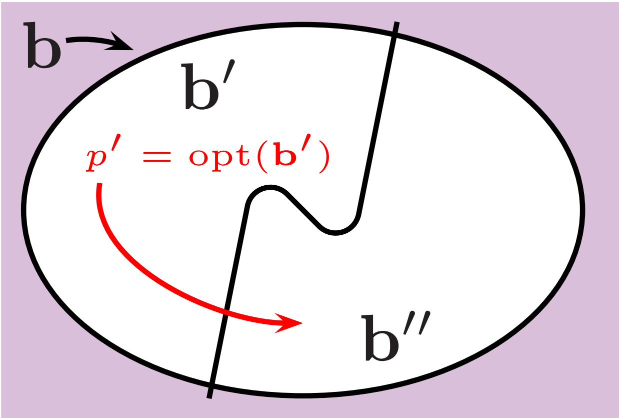

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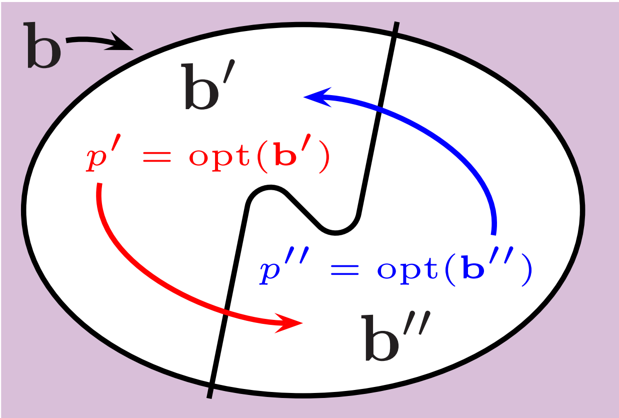

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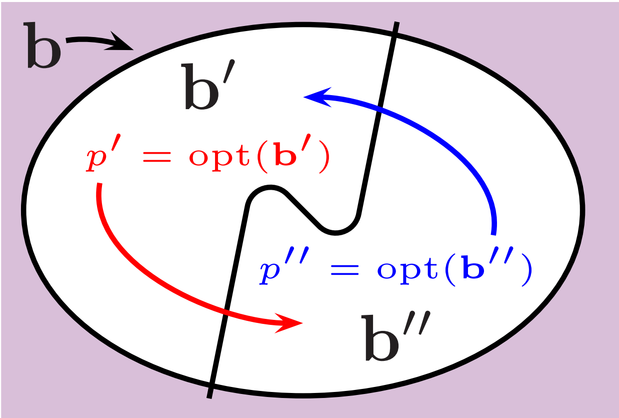

Theorem: [Goldberg, Hartline, Wright 2001] The Sampling Optimal Price Auction is constant competitive.

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Note: This reduction does not work for private value problems. (Simulating several truthful mechanisms and using the outcome of the best one is not truthful)

Basic Auction Decision Problem

The Decision Problem for the Basic Auction:

Given:

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- Target profit R.

Design: auction mechanism that obtains profit R if $R \leq \mathcal{F}$.

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Definition: *Profit extractor* is solution to private value decision problem.

Profit Extraction

Result: Profit extractor for basic auction problem: $\mathrm{ProfitExtract}_R$ [Moulin, Shenker 1996]

$\mathrm{ProfitExtract}_R$

1. Find largest k such that k highest bids satisfy $b_i \geq R/k$.
2. Sell at price R/k.
3. Reject low bidders.

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Compare

Truthful mechanism for 1 bidder:

1. Pick price p,
2. If $b \geq p$, bidder pays p,
3. Otherwise, bidder is rejected.

With $\mathrm{ProfitExtract}_R$, can treat a set of bidders like one bidder:

1. Pick revenue R,
2. If $\mathcal{F} \geq R$, we collect R,
3. Otherwise, no winners.

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Sampling Profit Extration Auction, SPE

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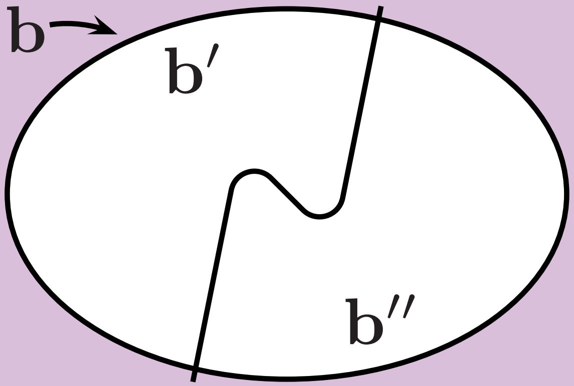

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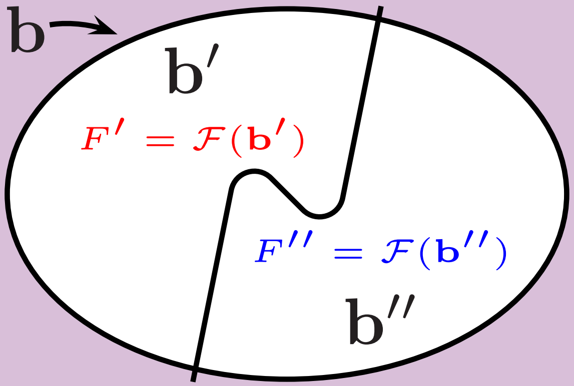

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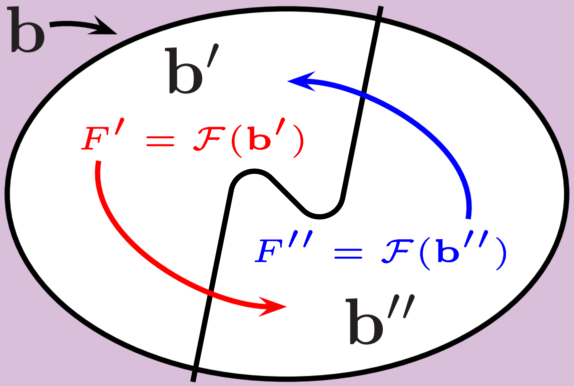

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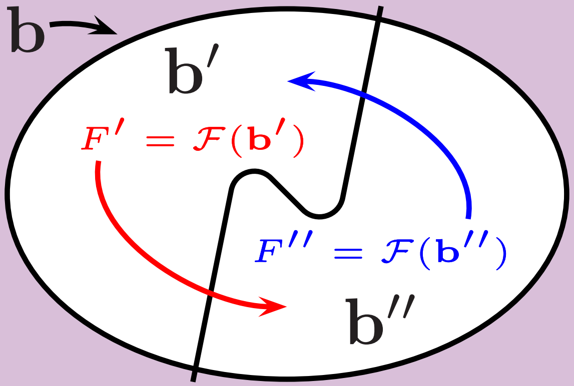

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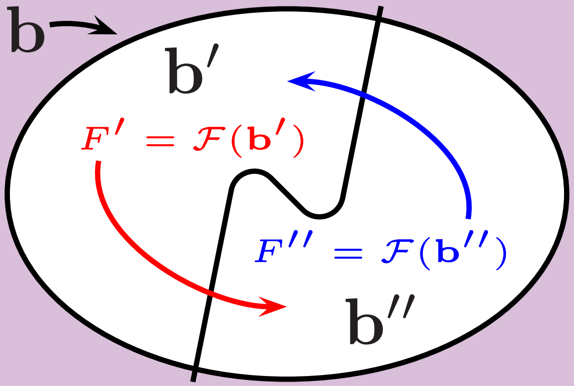

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1. sampling gets two "similar" sets.
2. treat each set like a single bidder and run Vickrey auction.
3. Thus, SPE revenue $= \min(F', F'')$.

SPE 4-Competitive

Theorem: SPE is 4-competitive (and this bound is tight).

Two Steps:

1. $\mathcal{R} = \min(F', F'') = \min(\mathcal{F}(\mathbf{b}'), \mathcal{F}(\mathbf{b}''))$.

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   NEXT!

SPE Proof: Step 2

Proof: (Step 2: $\mathbf{E}[\min(F', F'')] \geq \mathcal{F}^{(2)}(\mathbf{b})/4$)

Let $kv = \mathcal{F}^{(2)}(\mathbf{b})$. ($\mathcal{F}^{(2)}$ sells to top $k \geq 2$ bidders at price v)

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Reduction Idea:

1. Get a bid-independent *consensus estimate* of $\mathcal{F}(\mathbf{b})$.

   Pick function $r(\cdot)$ at random so:

   - *(Consensus)* With high probability, $r(\mathbf{b}_{-i}) = R$ for all i.
   - *(Estimate)* R is close to but less than $\mathcal{F}(\mathbf{b})$.

2. Truthfully extract protit R.

Consensus Discussion

Recall Goal: pick r randomly so

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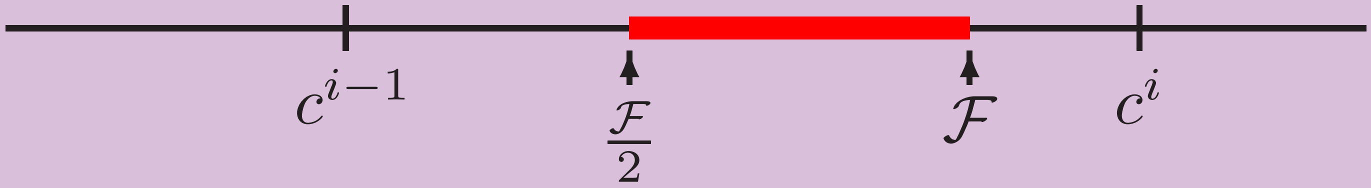

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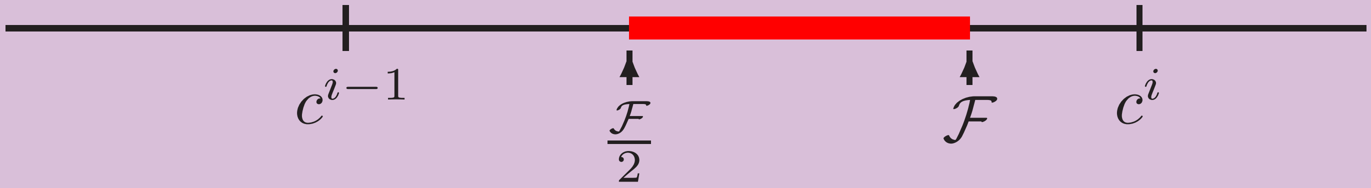

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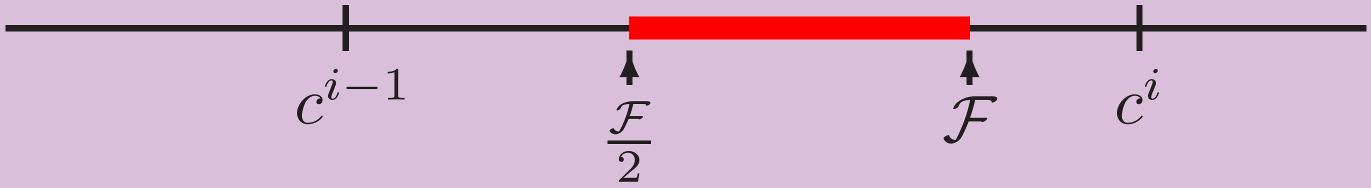

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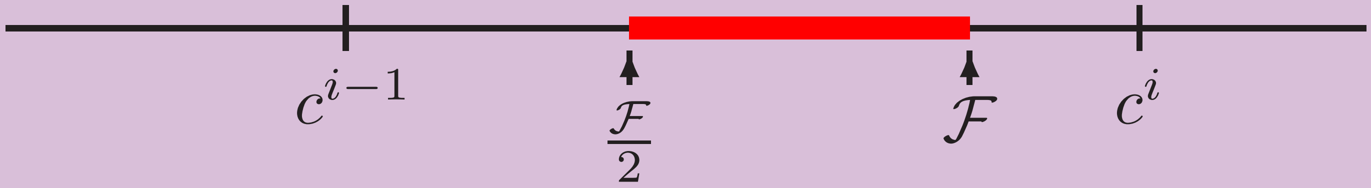

Case 2: No Consensus!

Consensus Estimate Solution (cont)

Recall Goal: $r(\cdot)$ is consensus with high probability.

Solution: [Goldberg, Hartline 2003] For U uniform $[0, 1]$,

$r(\mathbf{b}_{-i}) = \mathcal{F}(\mathbf{b}_{-i})$ rounded down to nearest c^{i+U} (for i integer).

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$r(\mathbf{b}_{-i}) = \mathcal{F}(\mathbf{b}_{-i})$ rounded down to nearest c^{i+U} (for i integer).

Lemma: Probability of Consensus: $1 - \log_c 2$.

Lemma: Expected payoff: $\frac{\mathcal{F}}{\ln c}\left(\frac{1}{2} - \frac{1}{c}\right)$

Private Value Reduction

Main Theorem: [Goldberg, Hartline 2003]

When $\mathcal{F}(\mathbf{b}_{-i})$ is bounded, e.g.

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Can combine

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Corollary: [Goldberg, Hartline 2003] A variant gives 3.39-competitive basic auction.

Competitive Auction Results

What is the optimal competitive ratio?

Sampling Optimal Price:	large constant
Sampling Profit Extraction:	4
Consensus Revenue Estimate:	3.39
Lower bound:	2.42

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Note on worst case bounds:

- Worst case is when small number of items are sold.
- If many items are sold, auctions approach 1-competitive.

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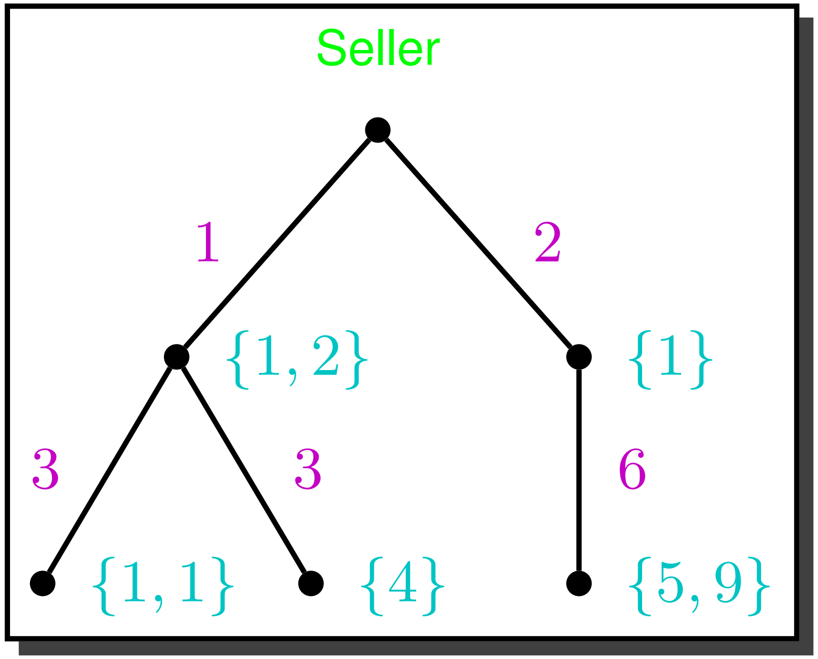
Market Segmented Auction:
Select which consumers/markets receive the good (winners).

- Bidders partitioned into *markets*.
  (E.g., nodes in multicast tree)
- Seller pays cost
  (function of which markets win)
  (E.g., cost of subtree in multicast)

profit = revenue − cost



Goal: profit close to optimal.

General Competitive Framework

Definition: $\mathcal{F}(\mathbf{b}, c(\cdot))$ – optimal public value profit that uses a single price for each market.

Definition: A generalized auction is β-competitive if

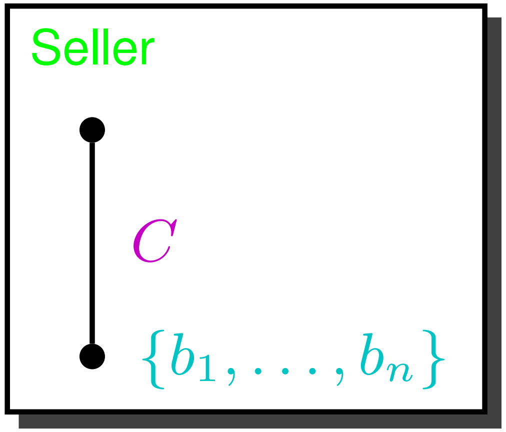
$$\mathbf{E}[\mathcal{R}] \geq \mathcal{F}(\mathbf{b}, c(\cdot))/\beta$$

when profit margin is high and there is competition in markets.

Fixed Cost Auction

The *fixed cost auction* problem is a special case of multicast:

Same as unlimited supply except must pay C if any bidders win.



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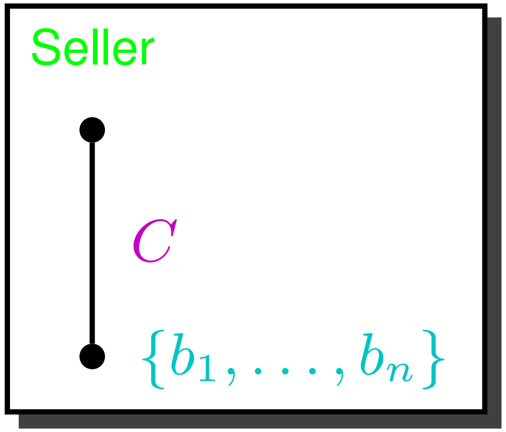

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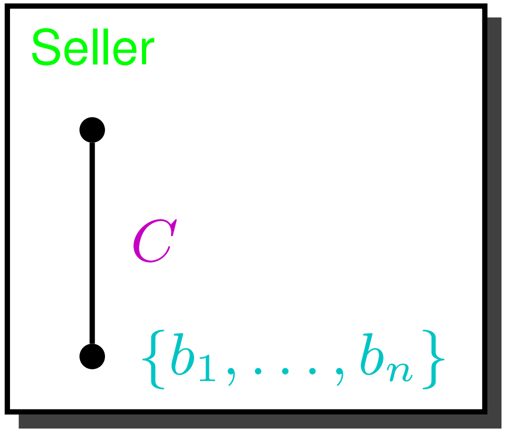

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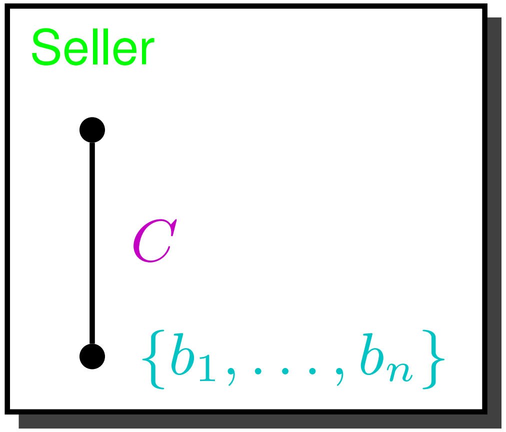

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Definition: An auction is *cancellable* if it is truthful even if cancelled.

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Intuition:

1. Assume $F' > F''$.
2. Winners are in $\mathbf{b}'$ only and $\mathcal{R} = F'' = \mathcal{F}(\mathbf{b}'')$.
3. Winning bids can not change $\mathcal{R}$ without losing.

Local Auction Solution

Local *Cancellable* Auction

1. Run cancellable auction locally on bidders in each *market*.
   (E.g., nodes in tree)

2. Choose optimal market allocation for cost function.
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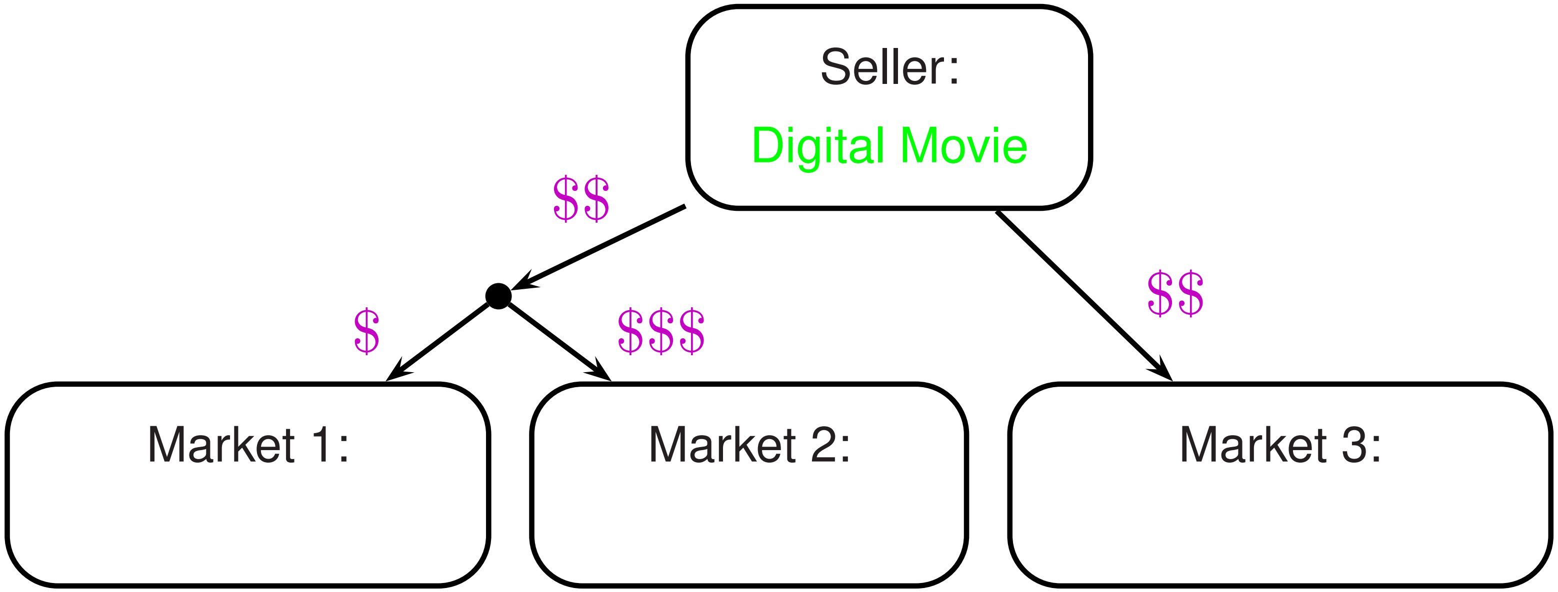

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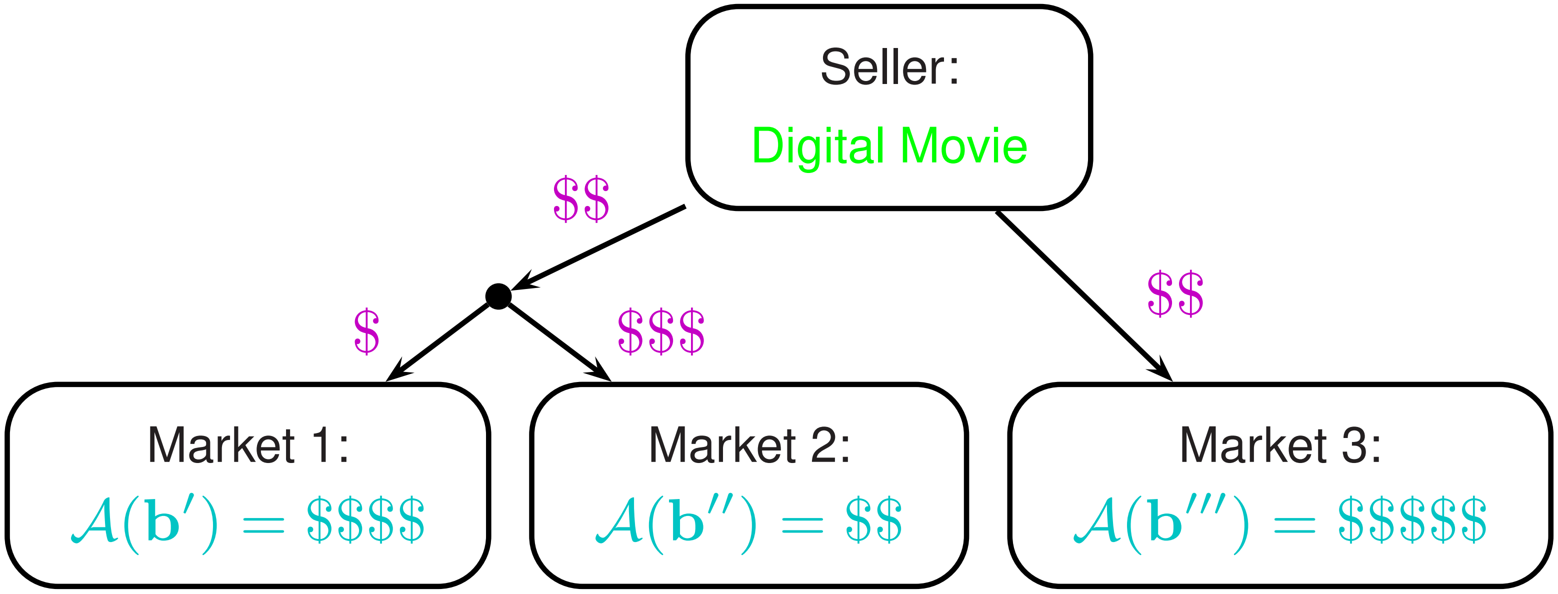

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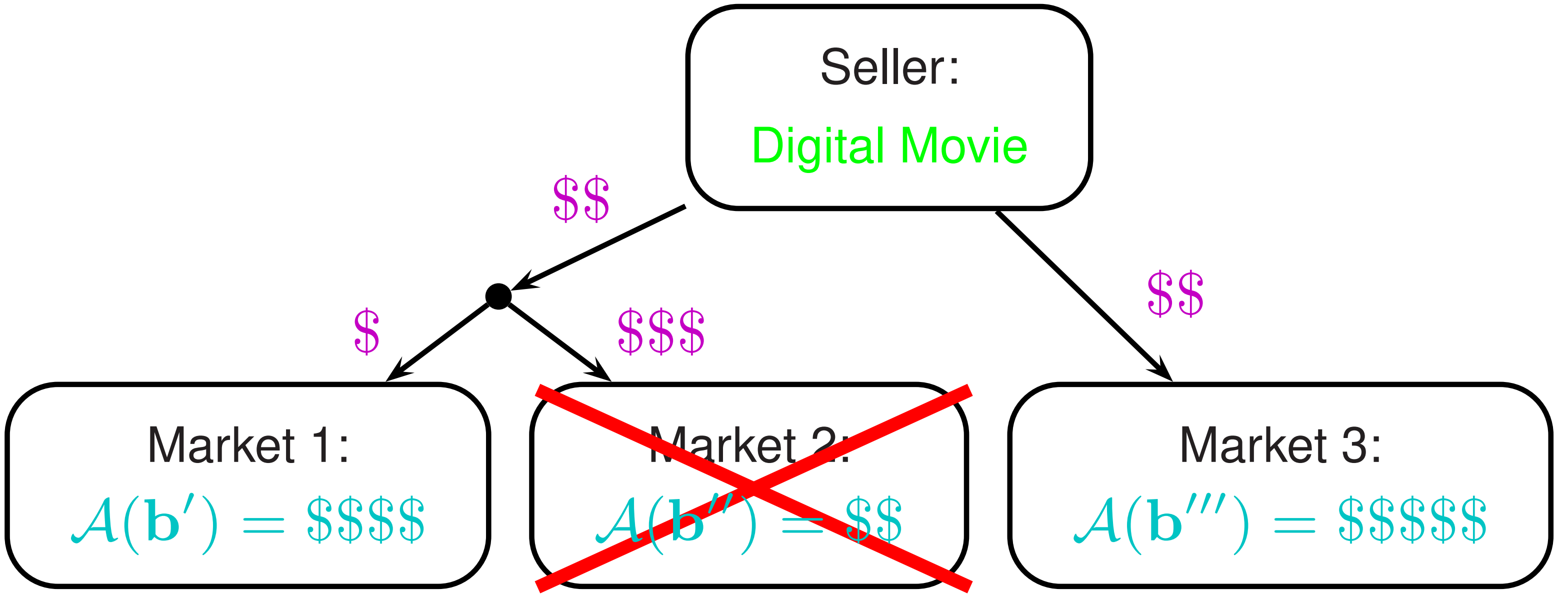

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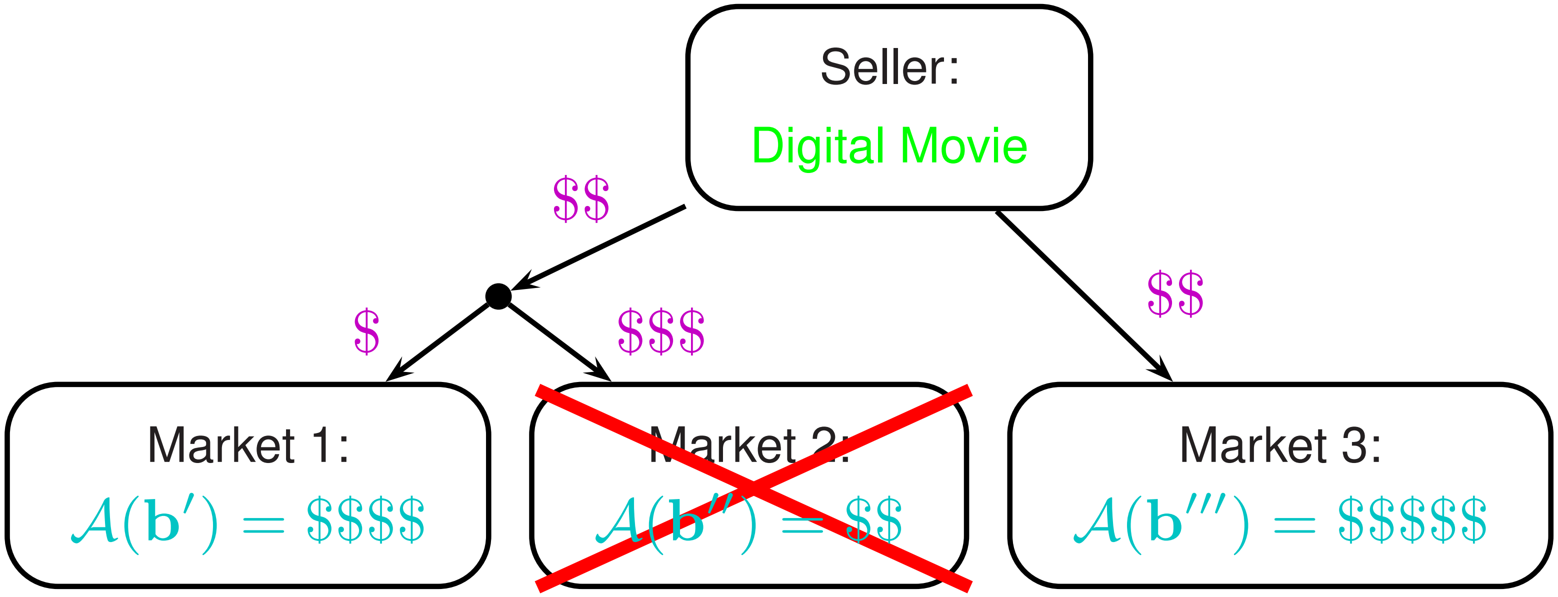

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   E.g. fixed cost auction

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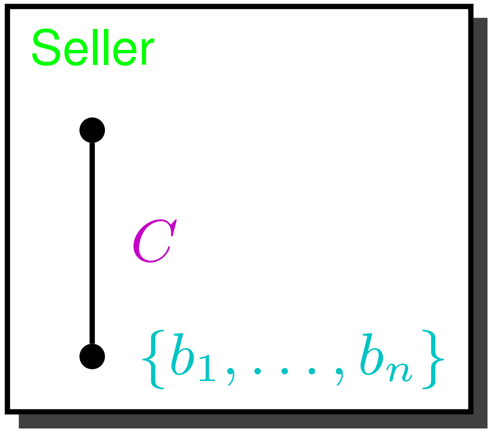
   - If $\mathcal{F}(\mathbf{b}) < 4C$ then $\mathbf{E}[\text{SPE}(\mathbf{b})] \leq C$.



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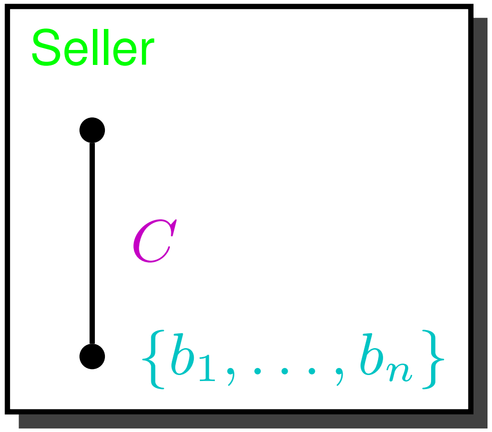
   E.g. fixed cost auction

   - SPE is 4-competitive.
   - If $\mathcal{F}(\mathbf{b}) < 4C$ then **E**[SPE$(\mathbf{b})$] $\leq C$.



2. Requires there is competition in markets.
   (I.e., $\mathcal{F}(\mathbf{b}) = \mathcal{F}^{(2)}(\mathbf{b})$)

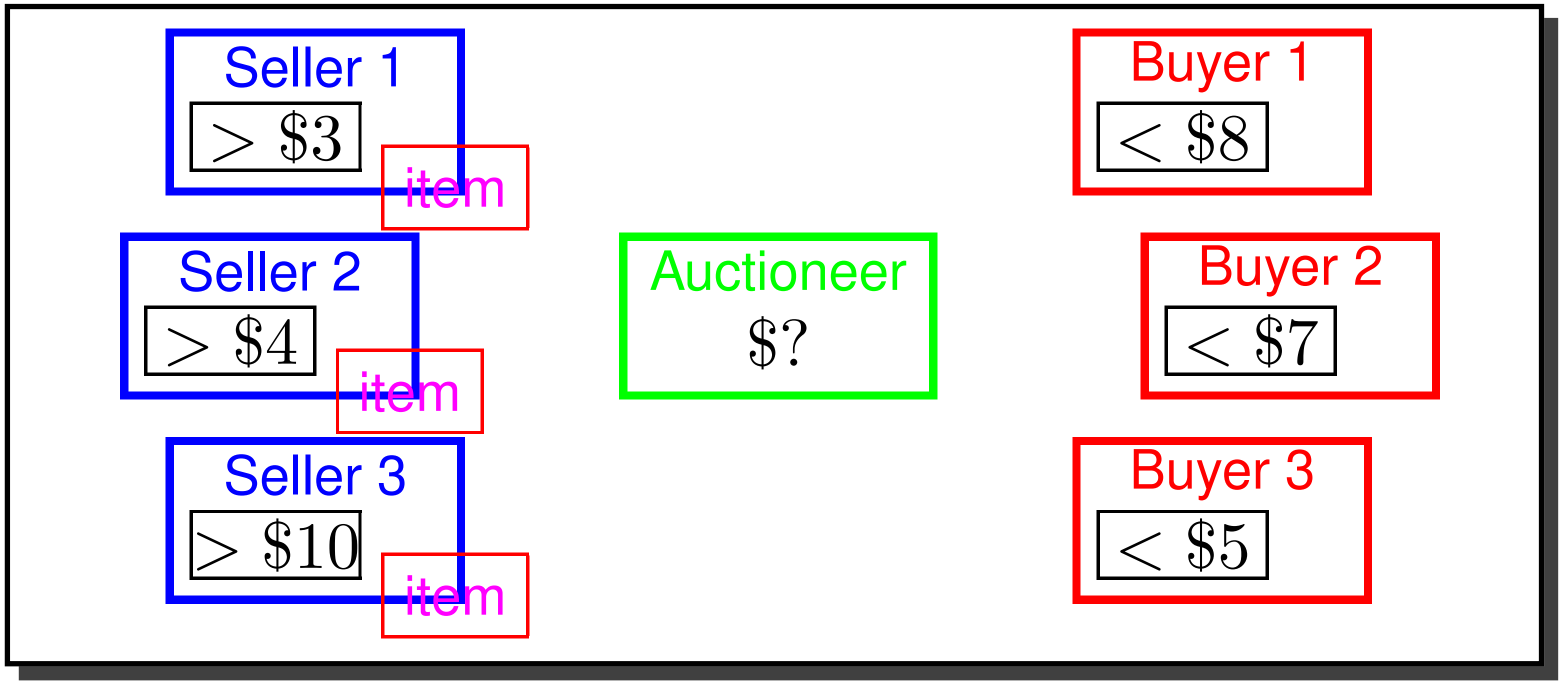
   E.g. if all markets are singleton sets, auction profit is zero.

Overview

1. Basic (Unlimited Supply) Auction Problem
2. Motivation: limited supply & overview of extensions.
3. Competitive Analysis.
4. Competitive Auctions.
   (a) Random Sampling: market analysis on the fly.
   (b) Private Value Decision Problem.
   (c) Reduction to decision problem, via random sampling.
   (d) Reduction to decision problem, via consensus estimates.
5. Extensions.
   (a) Cancellable Auction and the Multicast Problem.
   $\Longrightarrow$ (b) Multi-unit Double Auction.

Multi-unit Double Auction Problem

- n buyers and n sellers want to exchange a commodity.
- Each seller has one unit to sell.
- Each buyer wants to purchase one unit.
- Third party "auctioneer" mediates the exchange.
  (i.e., reallocates the items, collects, and makes payments)



Goal: Maximize profit auctioneer profit!

Double Auctions

The Double Auction Problem:

Buyers and sellers bid to buy or sell one copy of an identical good.

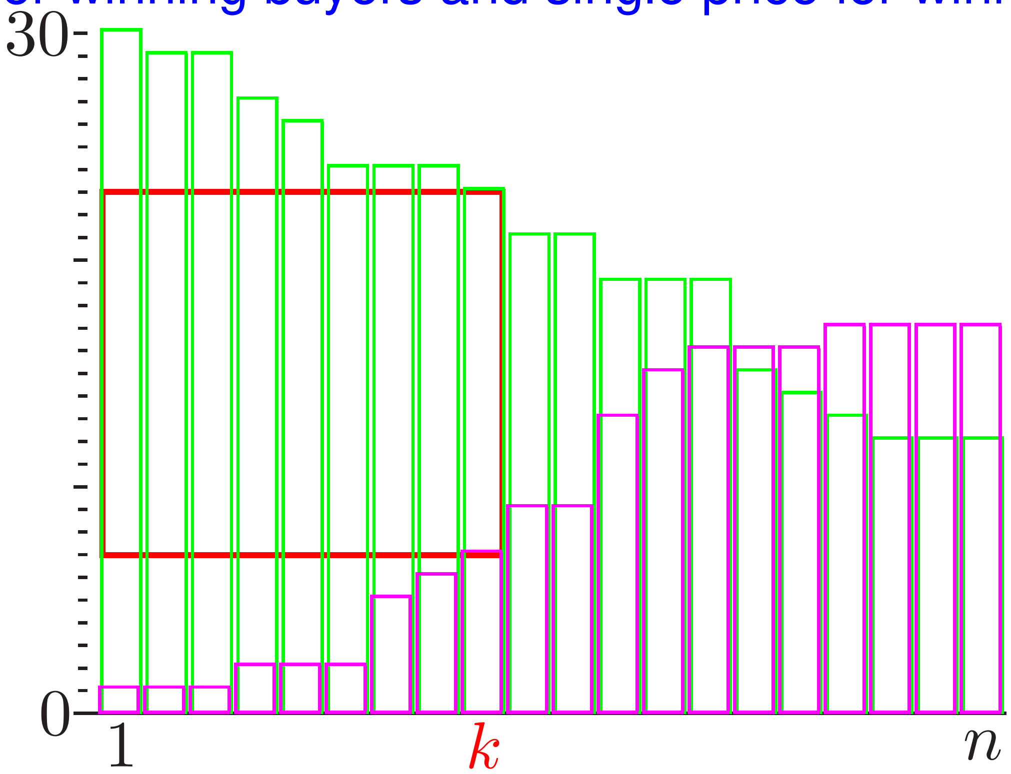
Input: Buyer bids $\mathbf{b} = (b_1, \ldots, b_n)$,
Seller bids $\mathbf{s} = (s_1, \ldots, s_n)$.

Output: Buyer allocation $\mathbf{x}$ and prices $\mathbf{p}$,
Seller allocation $\mathbf{y}$ and prices $\mathbf{q}$.
$(\sum_i x_i = \sum_i y_i)$

Profit: $\sum_i p_i - \sum_i q_i$.

Competitive Framework

Definition: $\mathcal{F}(\mathbf{b}, \mathbf{s}) = \max_k k(b_k - s_k)$ – the optimal profit with single price for winning buyers and single price for winning sellers.



Definition: Double auction is β-competitive if **E**$[\mathcal{R}] \geq \mathcal{F}^{(2)}(\mathbf{b}, \mathbf{s})/\beta$.

$(\mathcal{F}^{(2)}(\mathbf{b}, \mathbf{s}) = \max_{k \geq 2}(b_k - s_k))$

Consensus Framework

Recall Theorem for Basic Auctions:

When $\mathcal{F}(\mathbf{b}_{-i})$ is bounded, e.g.

$$\frac{1}{2}\mathcal{F}(\mathbf{b}) \leq \mathcal{F}(\mathbf{b}_{-i}) \leq \mathcal{F}(\mathbf{b})$$

Can combine

- the bid-independent consensus estimate solution, and
- a truthful profit extractor

to get constant-competitive mechanism.

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For double auction:

- If two items exchanged by $\mathcal{F}(\mathbf{b}, \mathbf{s})$,

$$\frac{1}{2}\mathcal{F}(\mathbf{b}, \mathbf{s}) \leq \mathcal{F}(\mathbf{b}_{-i}, \mathbf{s}), \mathcal{F}(\mathbf{b}, \mathbf{s}_{-i}) \leq \mathcal{F}(\mathbf{b}, \mathbf{s}).$$

- Profit extractor for double auction $\Rightarrow$ competitive double auction.

Double Auction Profit Extraction

Is there a profit extractor for double auctions?

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Double Auction Profit Extractor

- Find largest k such that the $k(b_k - s_k) \geq R$.
- Charge $k - 1$ highest buyers b_k and $k - 1$ lowest sellers s_k.
- Reject other buyers and sellers.

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- Reject other buyers and sellers.

Properties:

- Truthful.
- Has revenue $\geq \frac{k-1}{k} R$ if $R \leq \mathcal{F}(\mathbf{b}, \mathbf{s})$ and 0 otherwise.
  (with k items exchanged by in $\mathcal{F}(\mathbf{b}, \mathbf{s})$)

Result

Theorem: Double Auction Consensus on Revenue Estimates Auction is 3.75-competitive.

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General Reduction to Private Value Decission Problem:

- Given target profit metric $\mathcal{O}$, and (e.g., $\mathcal{F}$)
- Input $\mathbf{I}$ with $\frac{1}{\rho}\mathcal{O}(\mathbf{I}) \leq \mathcal{O}(\mathbf{I}_{-i}) \leq \mathcal{O}(\mathbf{I})$, and
- Profit Extractor for $\mathcal{O}$; (small print here)

$\Longrightarrow$

- The Consensus on Revenue Estimates Mechanism is constant competitive with $\mathcal{O}$.

Conclusions

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