SCHOOL OF Life Sciences
ARIZONA STATE UNIVERSITY

Numerical Methods within the Ant Colony: The Illuminating Case of Multi-Objective Macronutrient Regulation in Eusocial Insects

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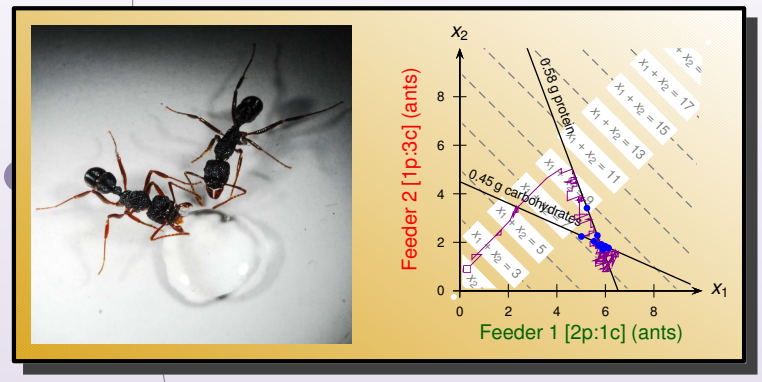
Sunday, October 12, 2014, 3:50 – 4:10



Colony-level Macronutrient Regulation
(Dussutour and Simpson 2009)

A. Dussutour and S. J. Simpson. Communal nutrition in ants. *Curr. Biol.*, 19(9): 740–744, May 12, 2009. doi:10.1016/j.cub.2009.03.015

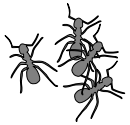
S. C. Cook and S. T. Behmer. Macronutrient regulation in the tropical terrestrial ant *Ectatomma ruidum* (Formicidae): a field study in Costa Rica. *Biotropica*, 42(2):135–139, March 2010. doi:10.1111/j.1744-7429.2009.00616.x

S. C. Cook, M. D. Eubanks, R. E. Gold, and S. T. Behmer. Colony-level macronutrient regulation in ants: mechanisms, hoarding, and associated costs. *Anim. Behav.*, 79(2):429–437, February 2010. doi:10.1016/j.anbehav.2009.11.022

S. C. Cook, R. A. Wynalda, R. E. Gold, and S. T. Behmer. Macronutrient regulation in the Rasberry crazy ant (*Nylanderia* sp. nr. *pubens*). *Insectes Sociaux*, 59(1):93–100, February 2012. doi:10.1007/s00040-011-0193-7

Colony-level Macronutrient Regulation
(Dussutour and Simpson 2009)

(250 ants)



1p:3c

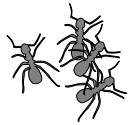
Feeder 2
(high carbohydrate)

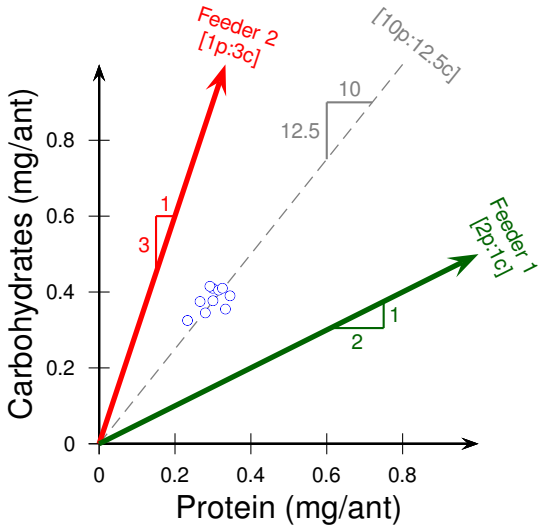
2p:1c

Feeder 1
(high protein)

Colony-level Macronutrient Regulation
(Dussutour and Simpson 2009)

(250 ants)





1p:3c — Feeder 2 (high carbohydrate)

2p:1c — Feeder 1 (high protein)

Colony-level Macronutrient Regulation
(Dussutour and Simpson 2009)

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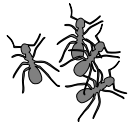
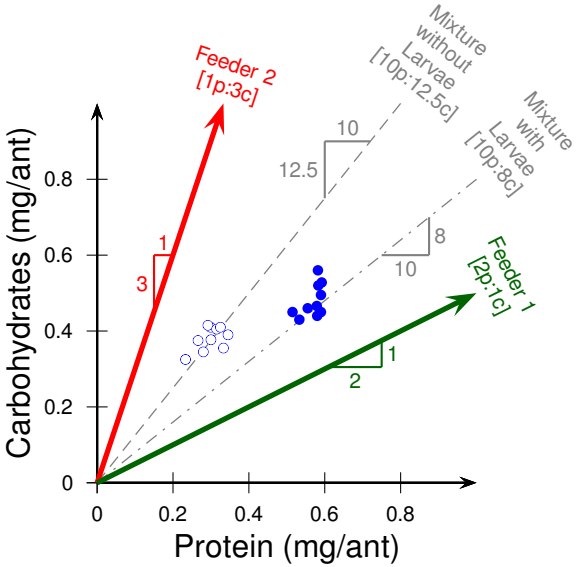


(100 larvae)



1p:3c — Feeder 2 (high carbohydrate)

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Colony-level Macronutrient Regulation
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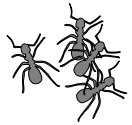

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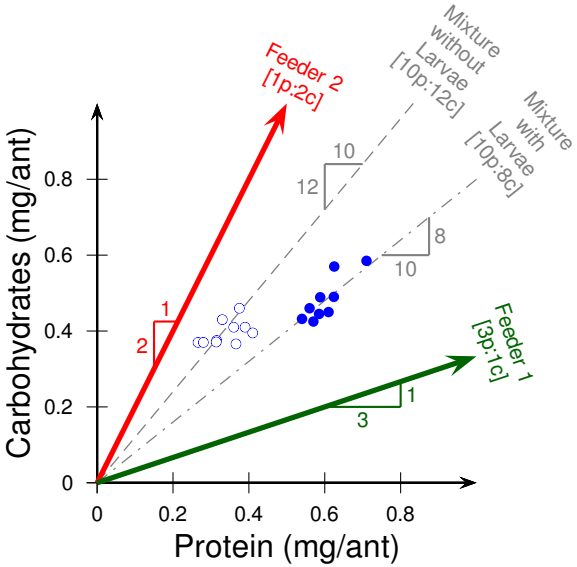

1p:2c — Feeder 2 (high carbohydrate)

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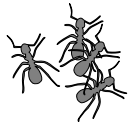
Colony-level Macronutrient Regulation
(Dussutour and Simpson 2009)

(250 ants)



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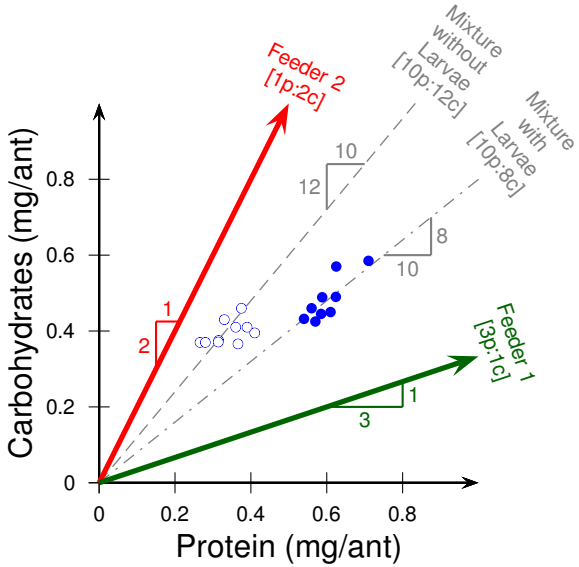


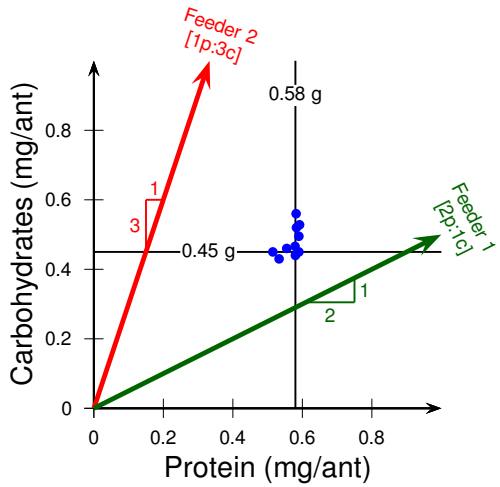
1p:2c — Feeder 2 (high carbohydrate)

3p:1c — Feeder 1 (high protein)

Ultimate cause is clear, but
what is the mechanism/implementation?

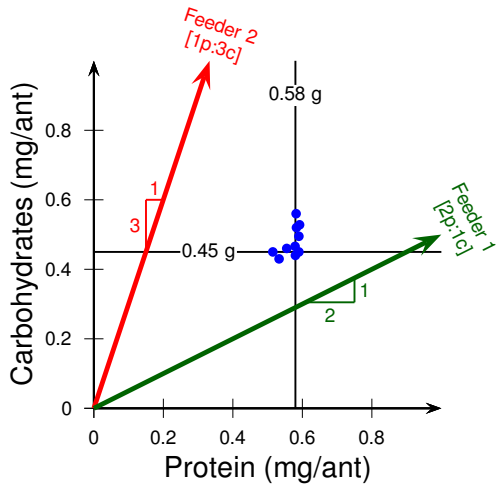
Colony-level Macronutrient Regulation

Mathematics of Allocation

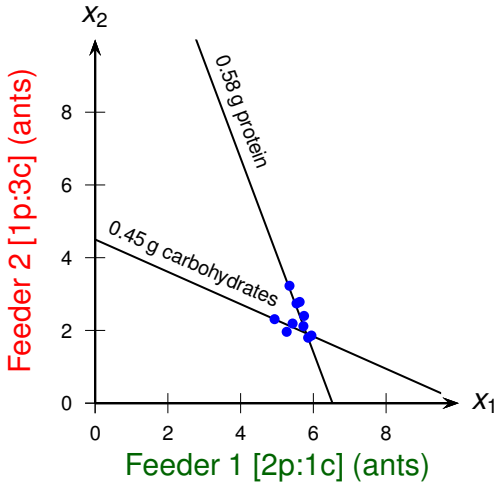


Colony-level Macronutrient Regulation

Mathematics of Allocation

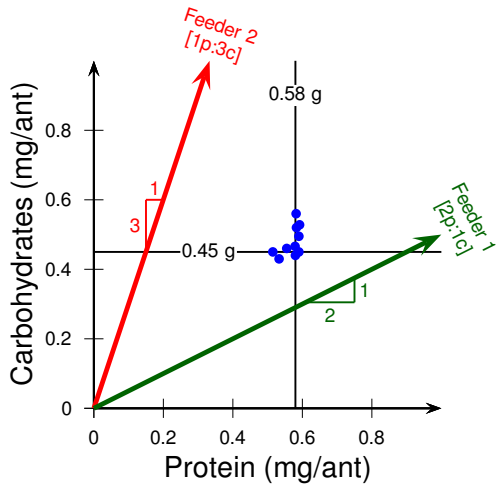


$\Longleftrightarrow$



Colony-level Macronutrient Regulation

Mathematics of Allocation



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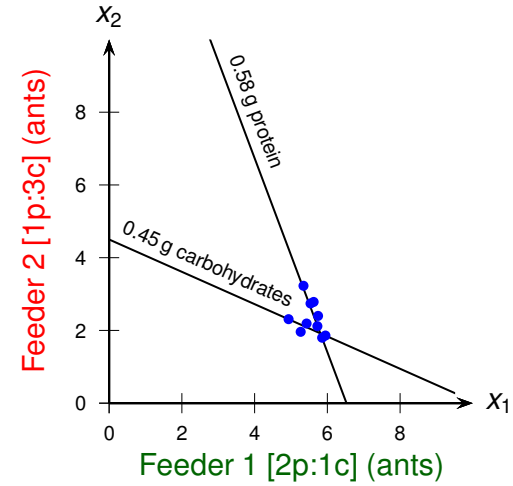
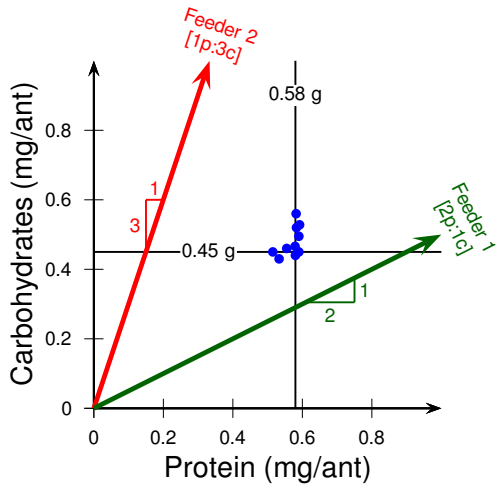


$$\begin{cases} a_{p1}x_1^* + a_{p2}x_2^* = c_p \\ a_{c1}x_1^* + a_{c2}x_2^* = c_c \end{cases}$$

Colony-level Macronutrient Regulation

Mathematics of Allocation



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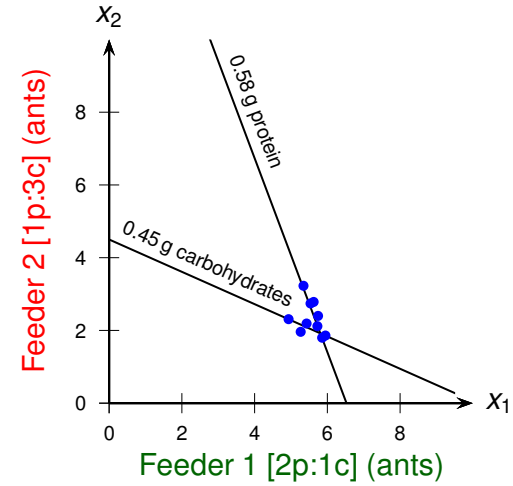
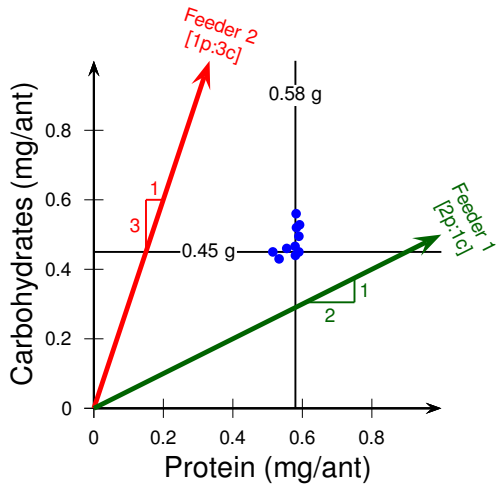


$$\begin{cases} a_{p1}x_1^* + a_{p2}x_2^* = c_p \\ a_{c1}x_1^* + a_{c2}x_2^* = c_c \end{cases} \quad \Longleftrightarrow \quad A\vec{x}^* = \vec{c}$$



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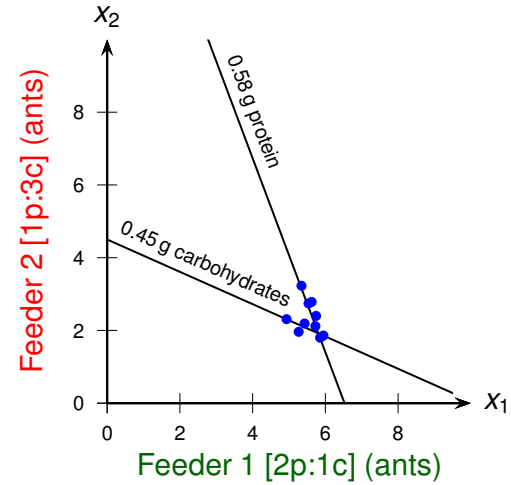

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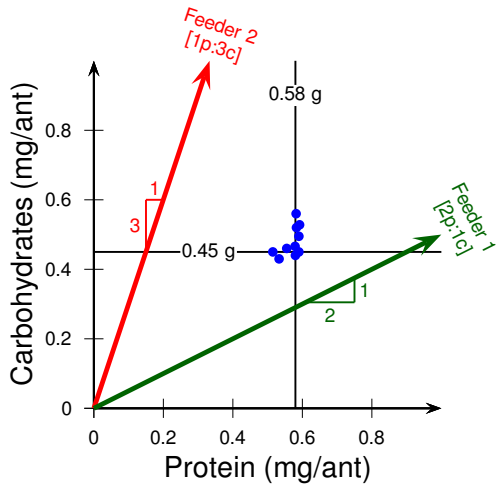
$\vec{x}^* = A^{-1}\vec{c}$ – Ants are doing *decentralized* matrix inversion?

Colony-level Macronutrient Regulation

Mathematics of Allocation



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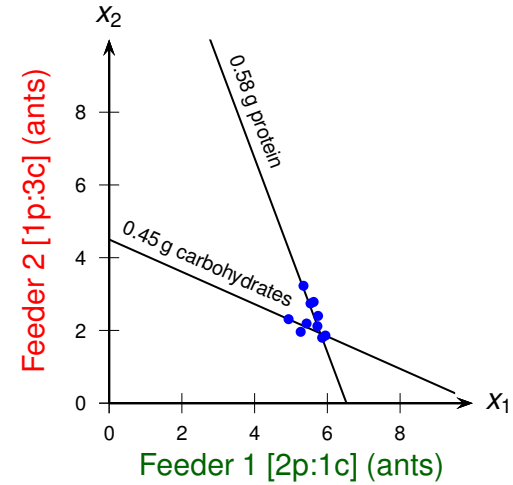
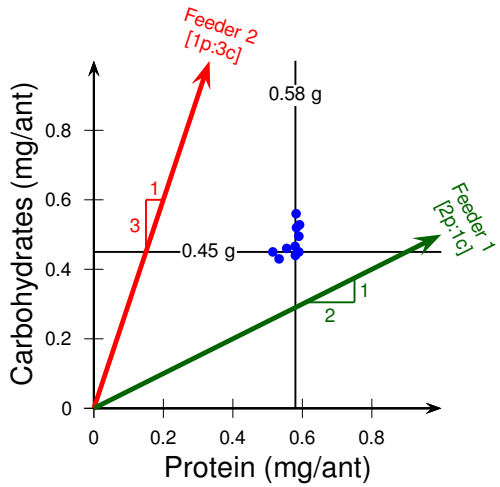


$$\begin{cases} a_{11}x_1^* + \cdots + a_{1n}x_n^* = c_1 \\ \vdots \\ a_{m1}x_1^* + \cdots + a_{mn}x_n^* = c_m \end{cases} \Longleftrightarrow \quad A_{m\times n}\vec{x}^* = \vec{c}_m$$

In natural settings, there are more feeders than vital nutrients.



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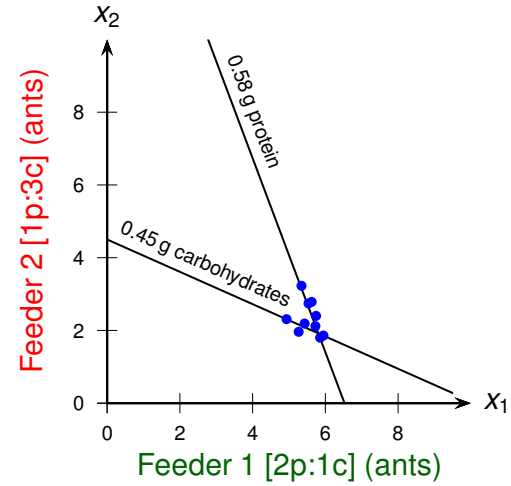

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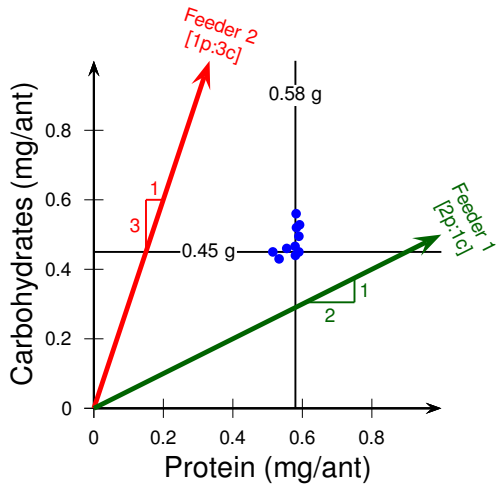
Solutions may not exist.



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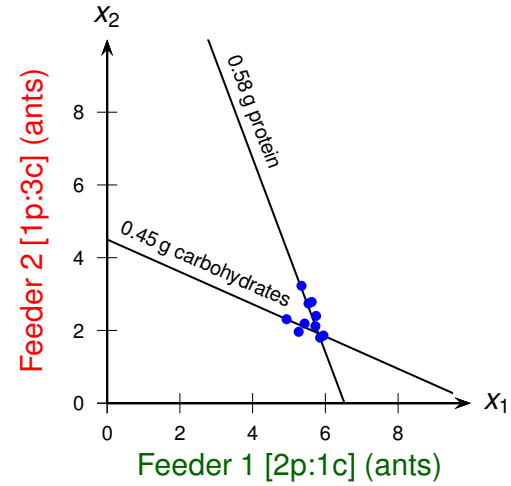

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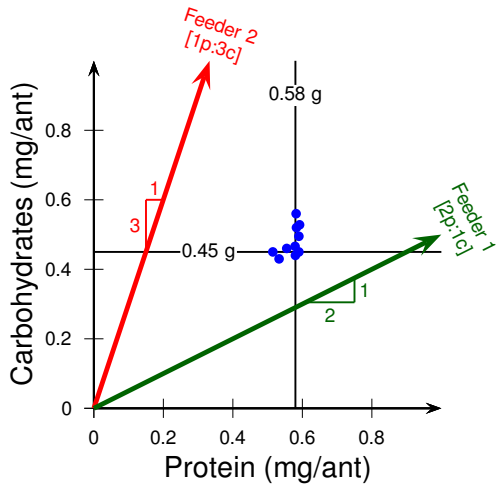
Continuum of solutions! Which solution is best?
Do ants have a pseudoinverse?



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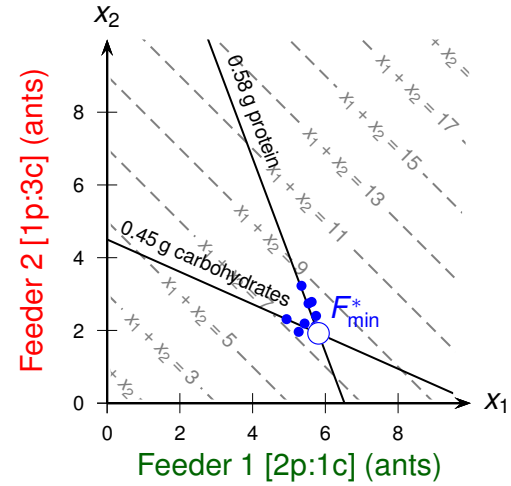


minimize $F(\vec{x})$

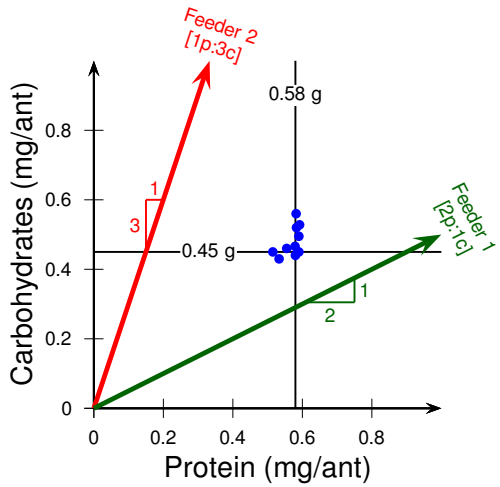
subject to $A_{m\times n}\vec{x} \geq \vec{c}_m$

minimal effort example:

$F(\vec{x}) \triangleq x_1 + \cdots + x_n$

Alternative model: ***Optimization under constraints***

Solutions *exist* and are *unique*!



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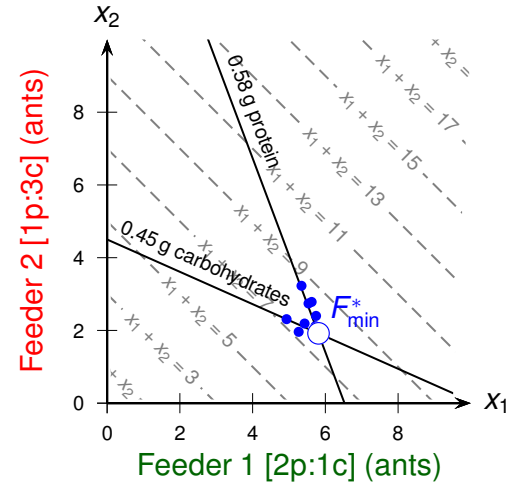

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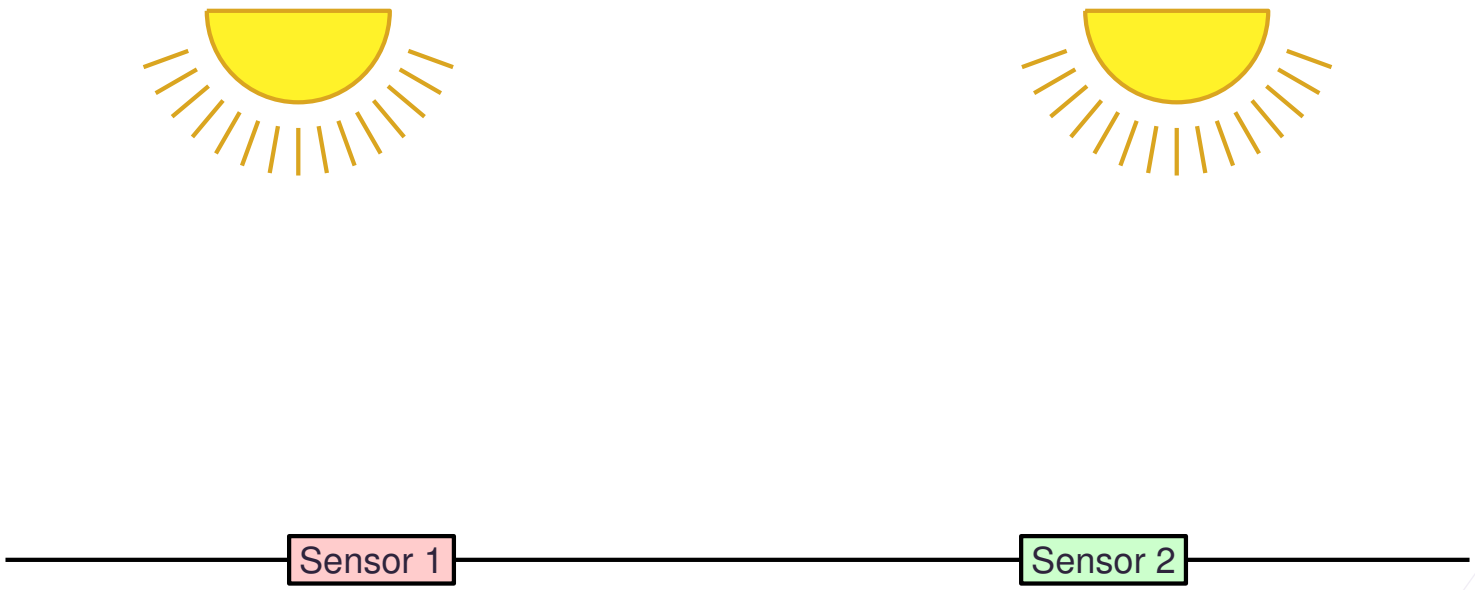
Alternative model: ***Optimization under constraints***

Decentralized solver implementable on ants?



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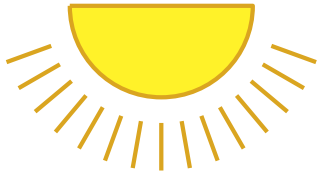
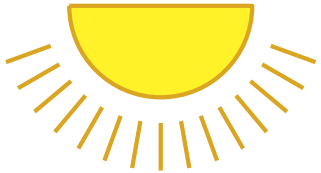
Feeder 1
(high protein)



Feeder 2
(high carbohydrate)



Sensor 1

0.58 g protein
requirement

Sensor 2

0.45 g carbohydrate
requirement

minimize $F(\vec{x})$

subject to $A_{m\times n}\vec{x} \geq \vec{c}_m$

"minimal effort:"

$$F(\vec{x}) \triangleq x_1 + \cdots + x_n$$

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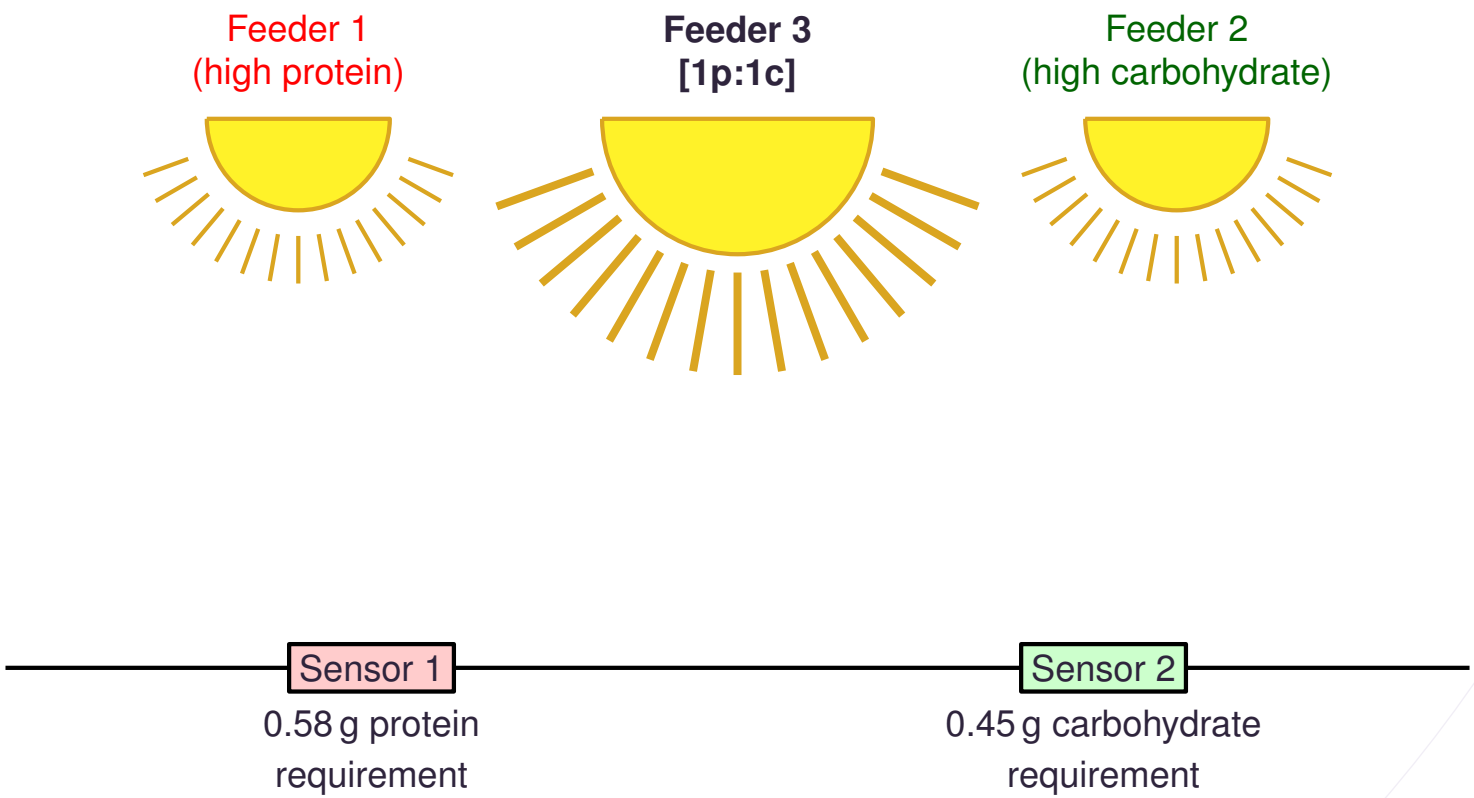

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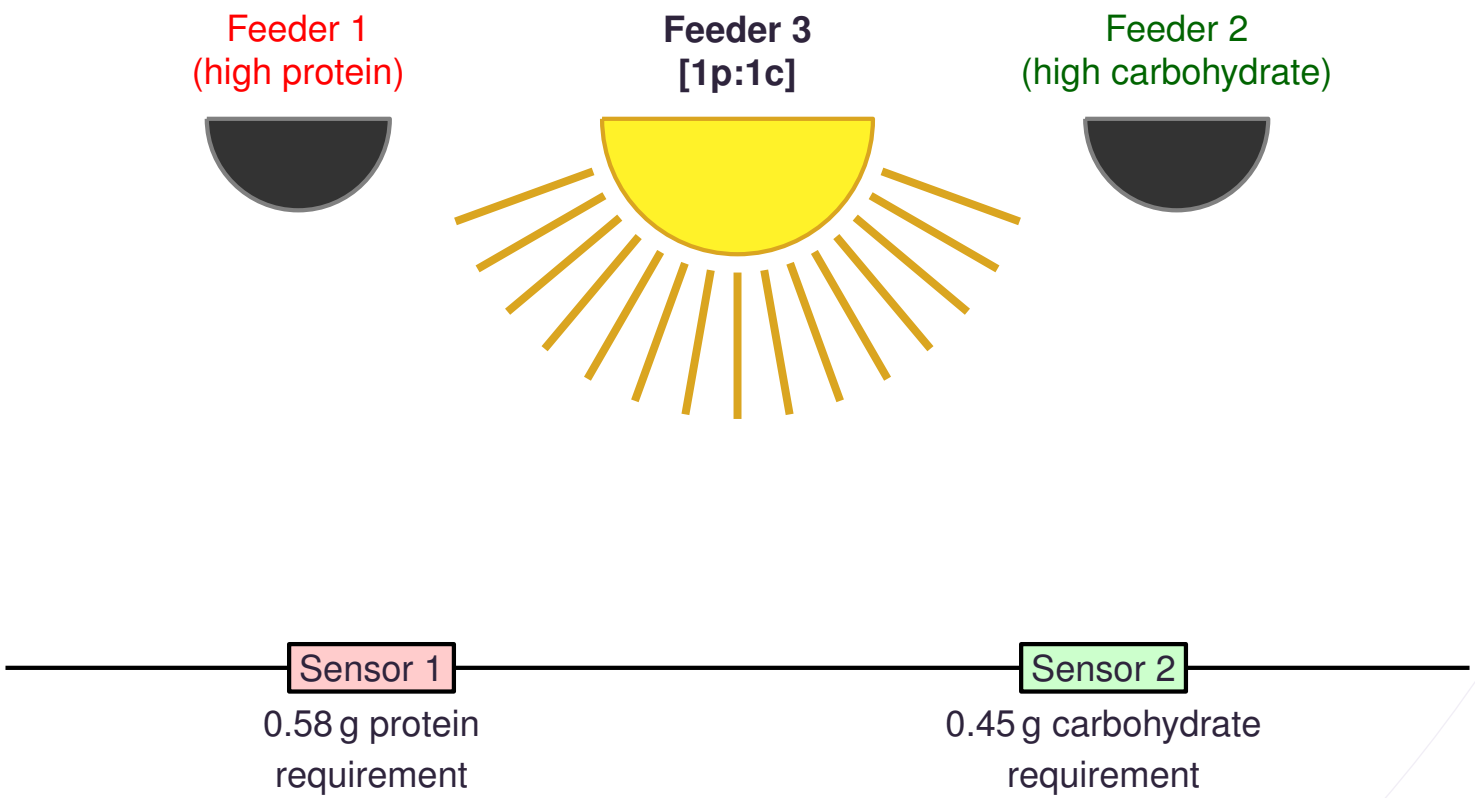

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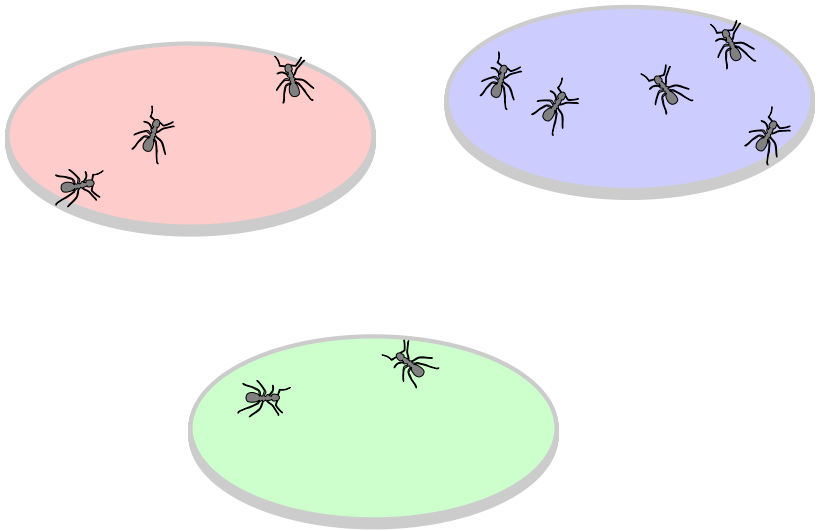
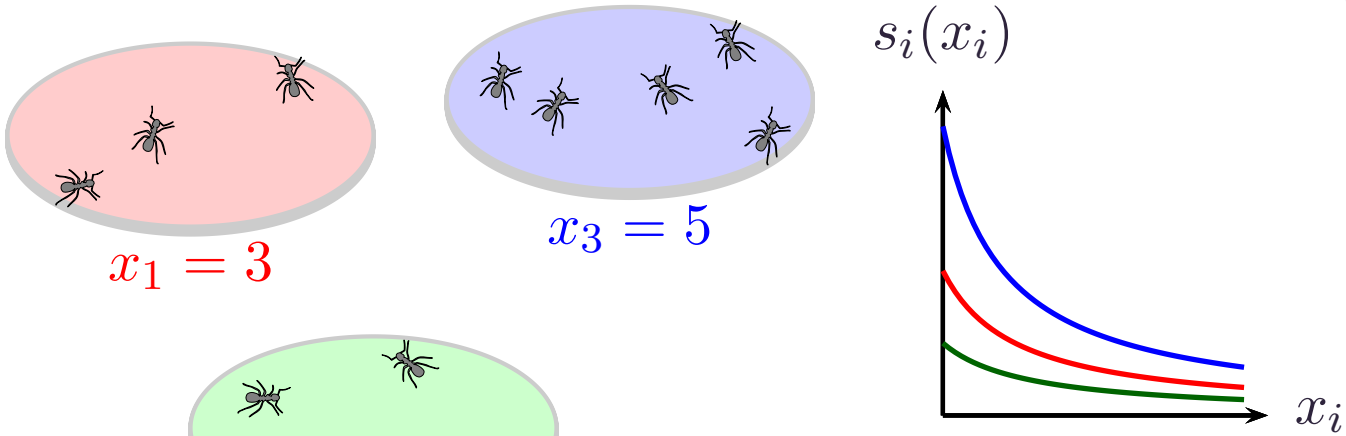
Algorithmic Insights

Social Foraging – Ideal Free Distribution (Fretwell and Lucas 1969; Fretwell 1972)



- Existing IFD-inspired dynamic resource allocation strategies in engineering
  - AAV cooperative control (Finke and Passino 2007; Moore et al. 2009)
  - Water distribution (Ramirez-Llanos and Quijano 2010)
  - Temperature control (Pantoja et al. 2011)

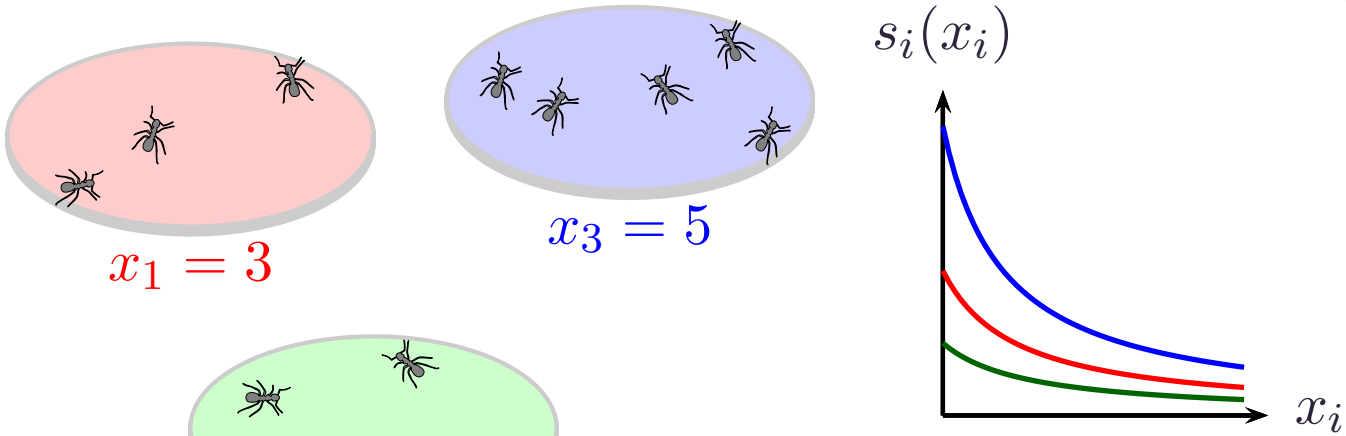
Social Foraging – Ideal Free Distribution (Fretwell and Lucas 1969; Fretwell 1972)



$$x_1 + x_2 + x_3 = N$$
$$s_1(x_1^*) \approx s_2(x_2^*) \approx s_3(x_3^*)$$

$$\begin{aligned} \text{minimize} \quad & \max\{s_i(x_i)\} \\ \text{subject to} \quad & x_1 + \cdots + x_n = N \end{aligned}$$

Social Foraging – Ideal Free Distribution (Fretwell and Lucas 1969; Fretwell 1972)



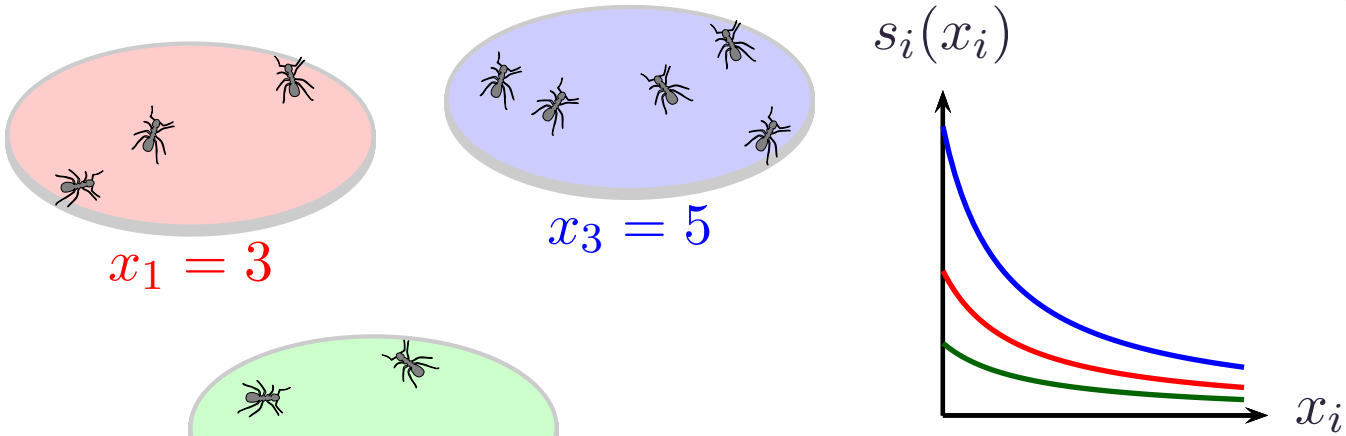
$$x_1 + x_2 + x_3 = N$$

$$s_1(x_1^*) \approx s_2(x_2^*) \approx s_3(x_3^*)$$

$$\text{minimize} \quad \sum \int_0^{x_i} \frac{1}{s_i(\tau)} \mathrm{d}\tau$$

$$\text{subject to} \quad x_1 + \cdots + x_n \geq N$$

Social Foraging – Ideal Free Distribution (Fretwell and Lucas 1969; Fretwell 1972)



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$$s_1(x_1^*) \approx s_2(x_2^*) \approx s_3(x_3^*)$$

$$\begin{aligned} \text{minimize} \quad & F(\vec{x}) \\ \text{subject to} \quad & a_1 x_1 + \cdots + a_n x_n \geq c \end{aligned}$$

"MultiIFD" Asynchronous Distributed Solver

- *MultiIFD* discrete-time realization with speed–accuracy tradeoff parameter δ:
  - A violation of constraint $j \in \{1, 2, \ldots, m\}$ induces **marginal** IFD:

$$\vec{x}^{\text{next}} - \vec{x}^{\text{prev}} \propto \left[\frac{a_{j1}}{\nabla_1 F(\vec{x})}, \quad \frac{a_{j2}}{\nabla_2 F(\vec{x})}, \quad \ldots, \quad \frac{a_{jn}}{\nabla_n F(\vec{x})} \right]^\top .$$

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  - For each patch $i \in \{1, 2, \ldots, n\}$, animals regularly deallocate:

$$x_i^{\text{next}} - x_i^{\text{prev}} = -\delta.$$

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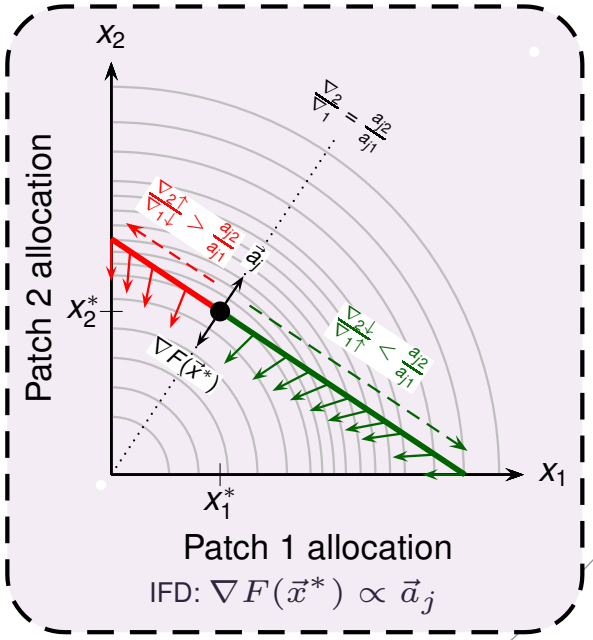

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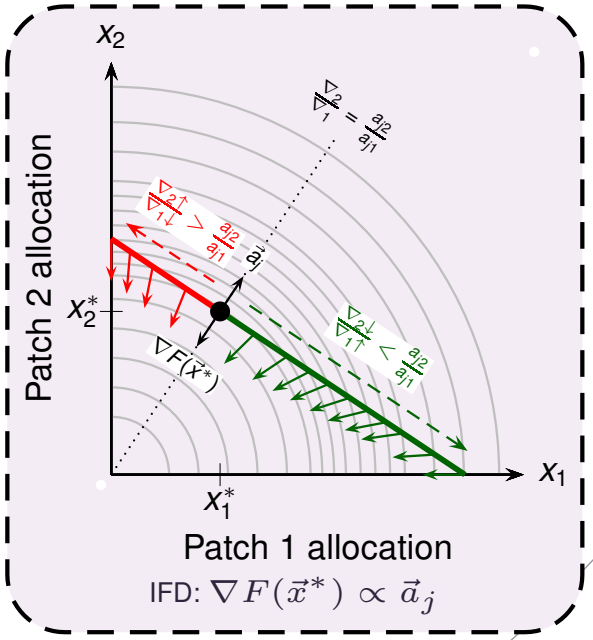
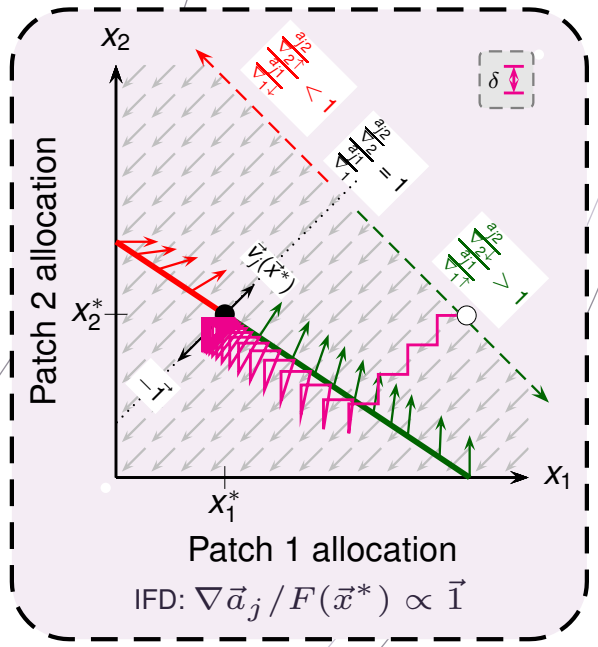
- *MultiIFD* discrete-time realization with speed–accuracy tradeoff parameter δ:
  - A violation of constraint $j \in \{1, 2, \ldots, m\}$ induces **marginal** IFD:

$$\vec{x}^{\text{next}} - \vec{x}^{\text{prev}} \propto \left[\frac{a_{j1}}{\nabla_1 F(\vec{x})}, \quad \frac{a_{j2}}{\nabla_2 F(\vec{x})}, \quad \ldots, \quad \frac{a_{jn}}{\nabla_n F(\vec{x})} \right]^\top.$$

  - For each patch $i \in \{1, 2, \ldots, n\}$, animals regularly deallocate:

$$x_i^{\text{next}} - x_i^{\text{prev}} = -\delta.$$



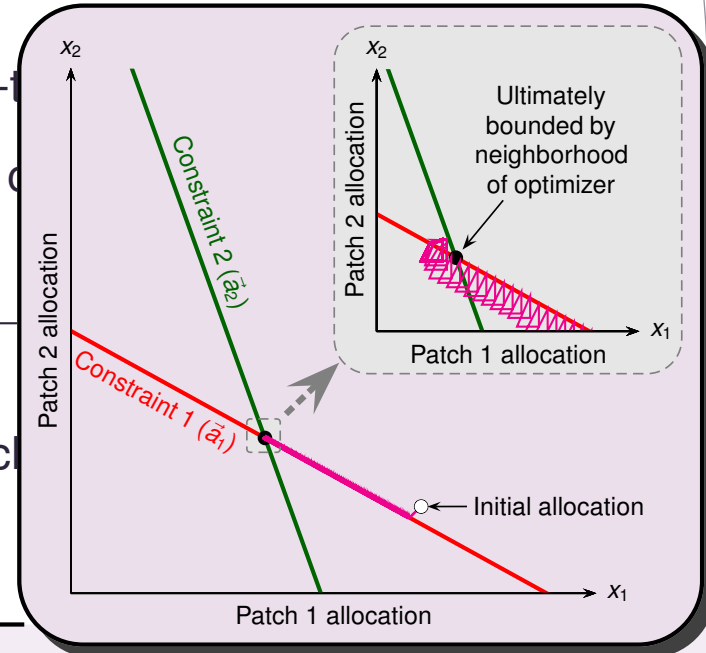


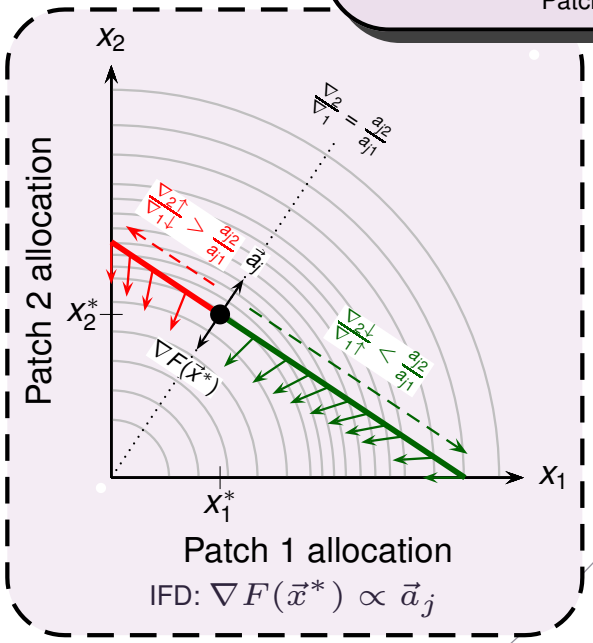
“MultiIFD” Asynchronous Distributed Solver

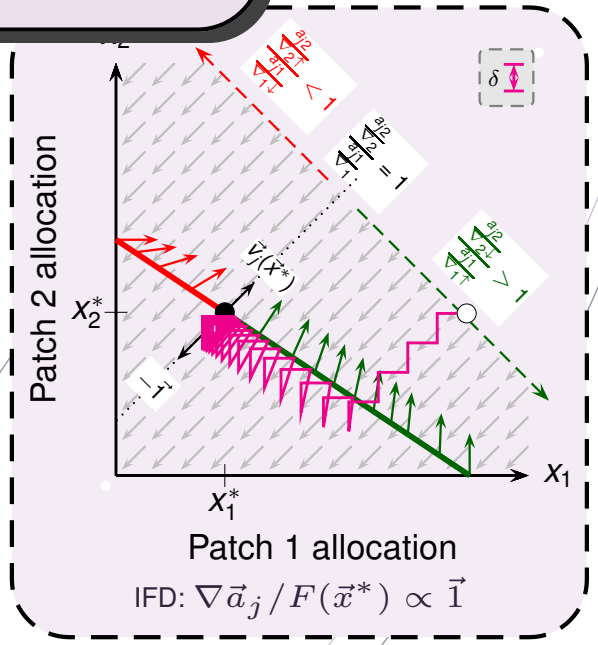
- *MultiIFD* discrete-t… …cy tradeoff parameter δ:
  - A violation of c… …ces **marginal** IFD:

$$\vec{x}^{\text{next}} \ldots \quad , \quad \frac{a_{jn}}{\nabla_n F(\vec{x})}\Big]^{\top} .$$

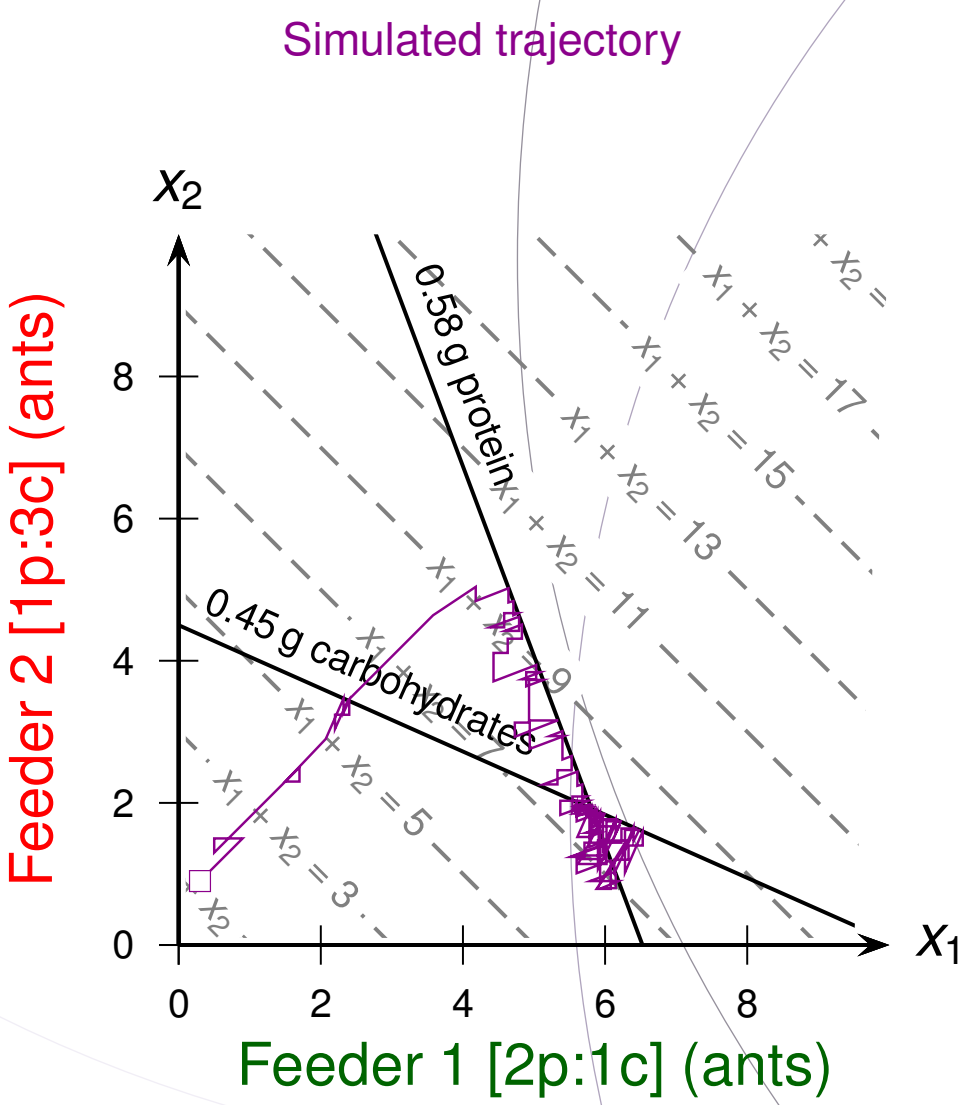
  - For each patc… …arly deallocate:







Validation with Animal Models



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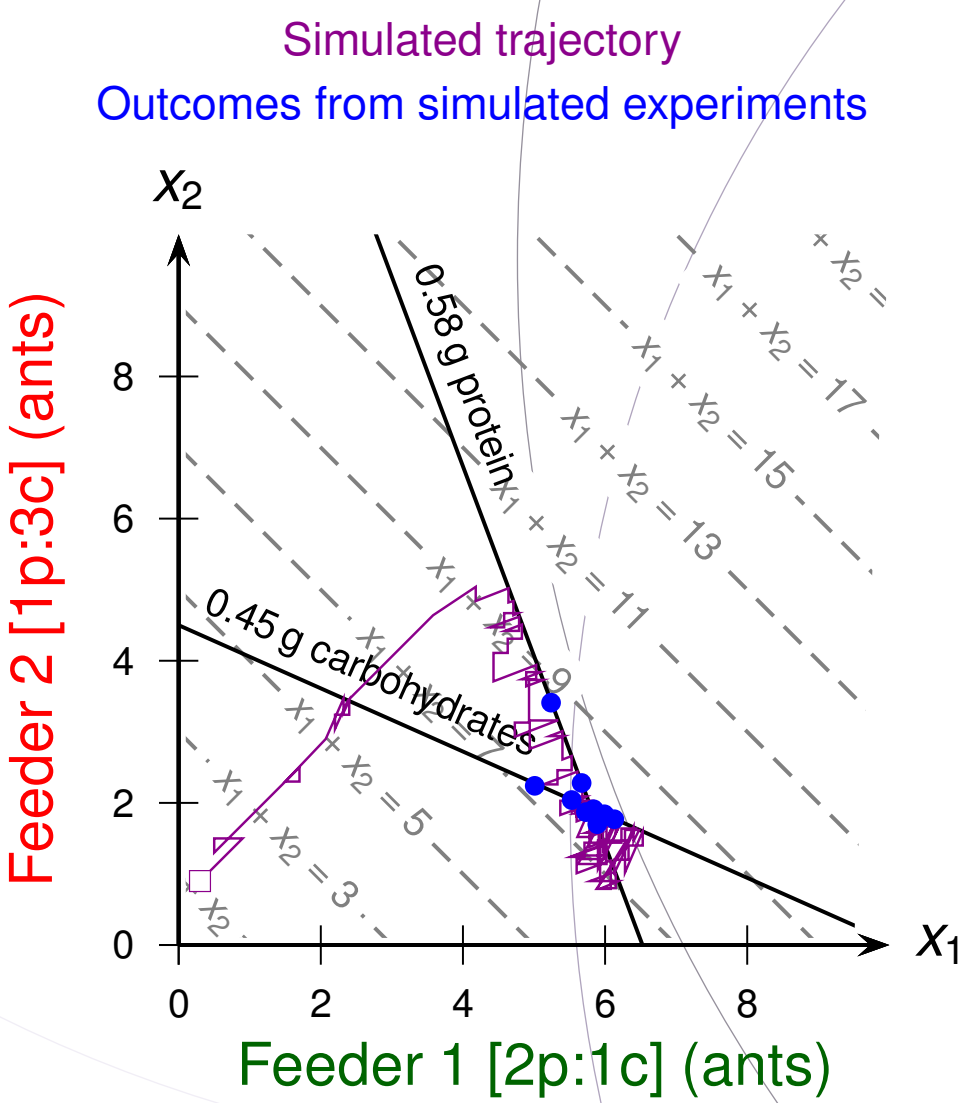
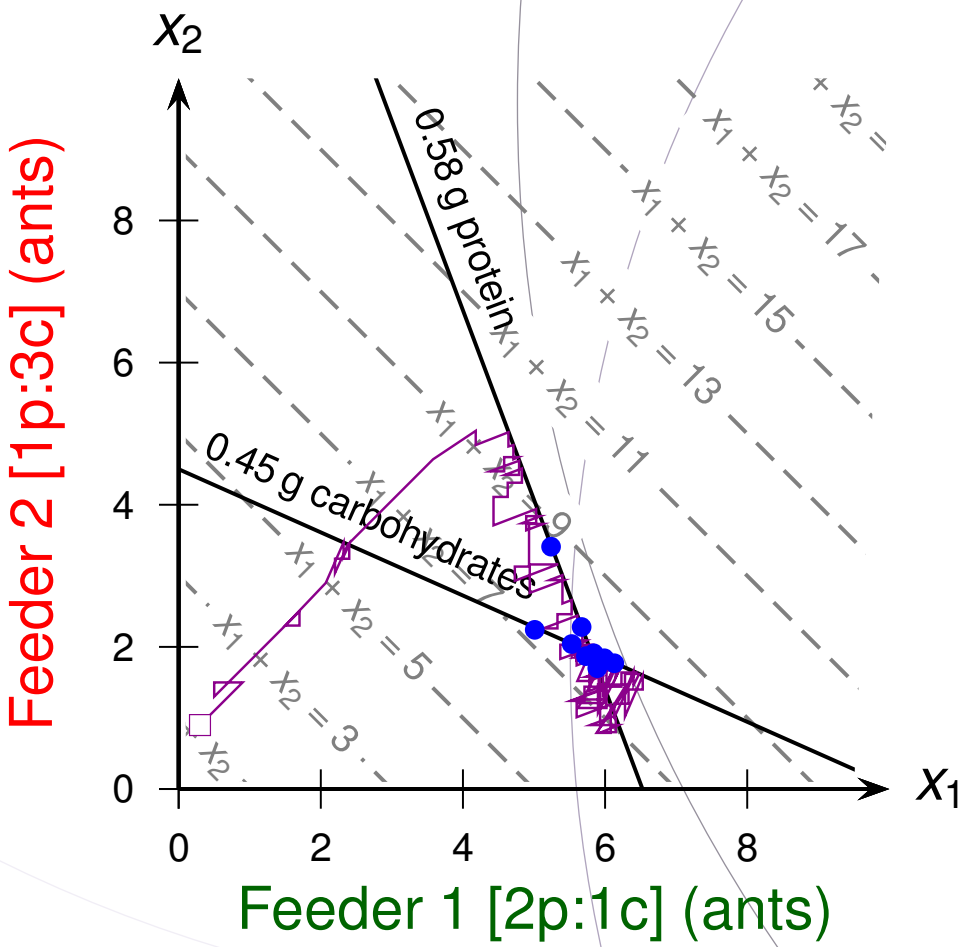


Validation with Animal Models

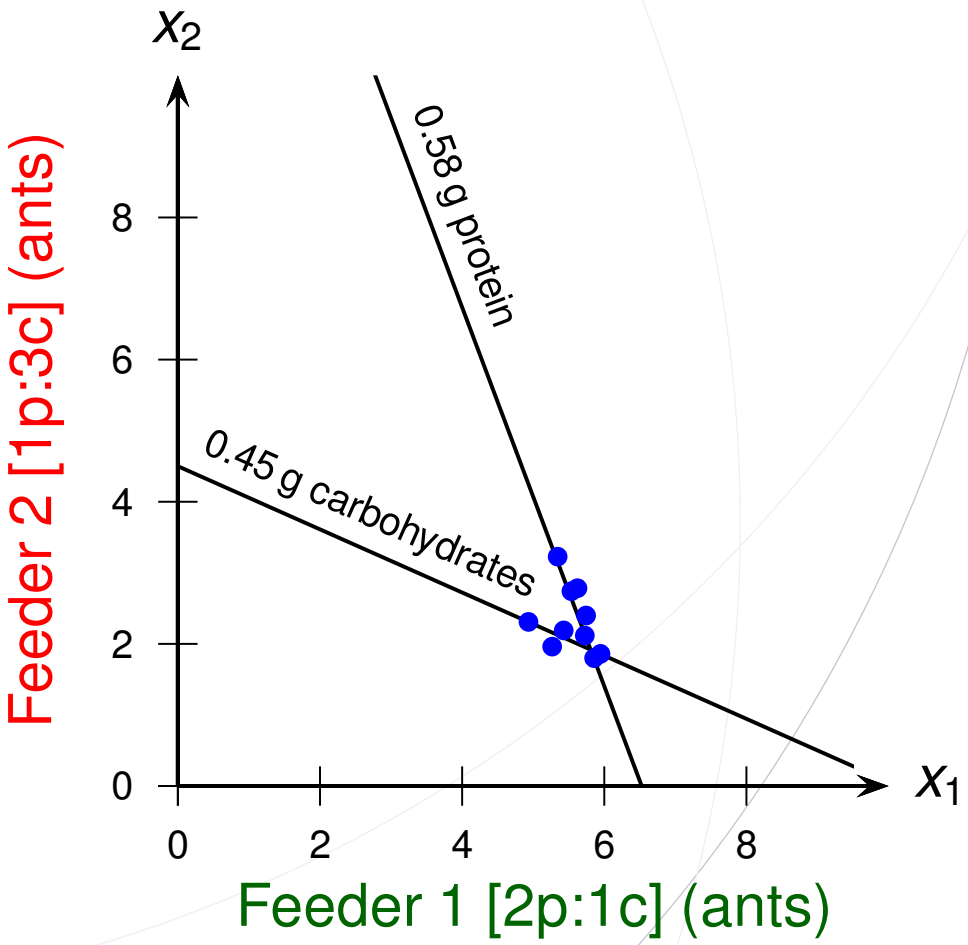
Simulated trajectory

Outcomes from simulated experiments



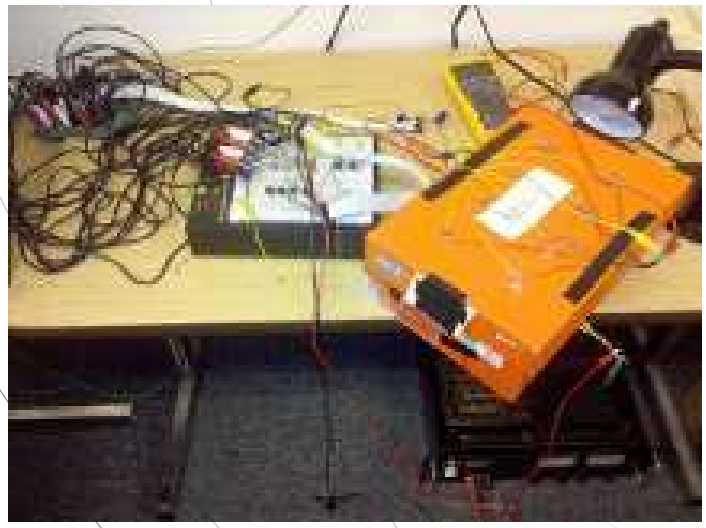
Outcomes from

Dussutour and Simpson (2009)



Validation on Small-scale Lighting Testbed

Real-time Shoebox

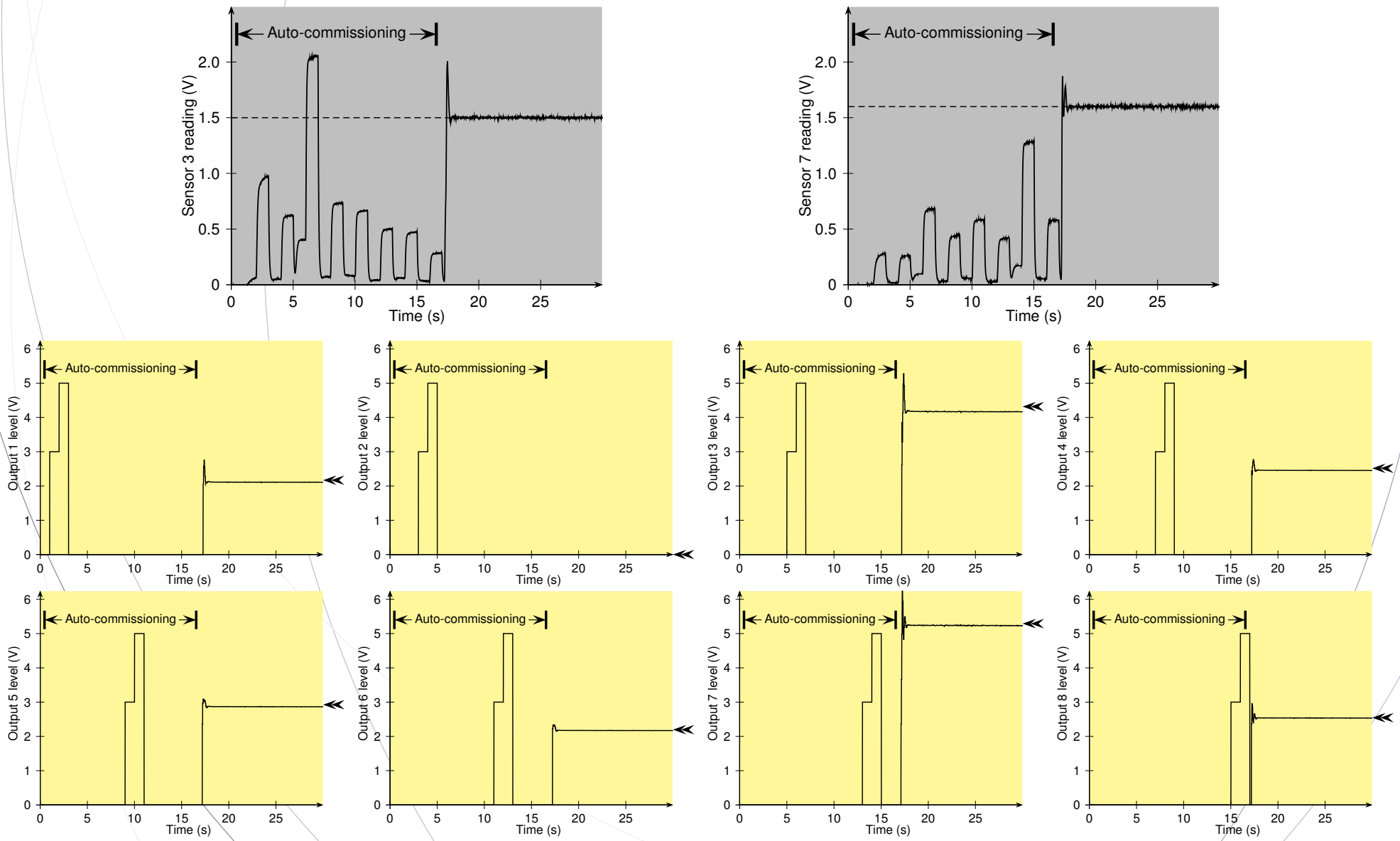


- Real-time control hardware
- Asynchronous events
- Auto-commissioning period

nb: CDMA and VLC (Linnartz et al. 2008)

Validation on Small-scale Lighting Testbed

Centralized Solver Results



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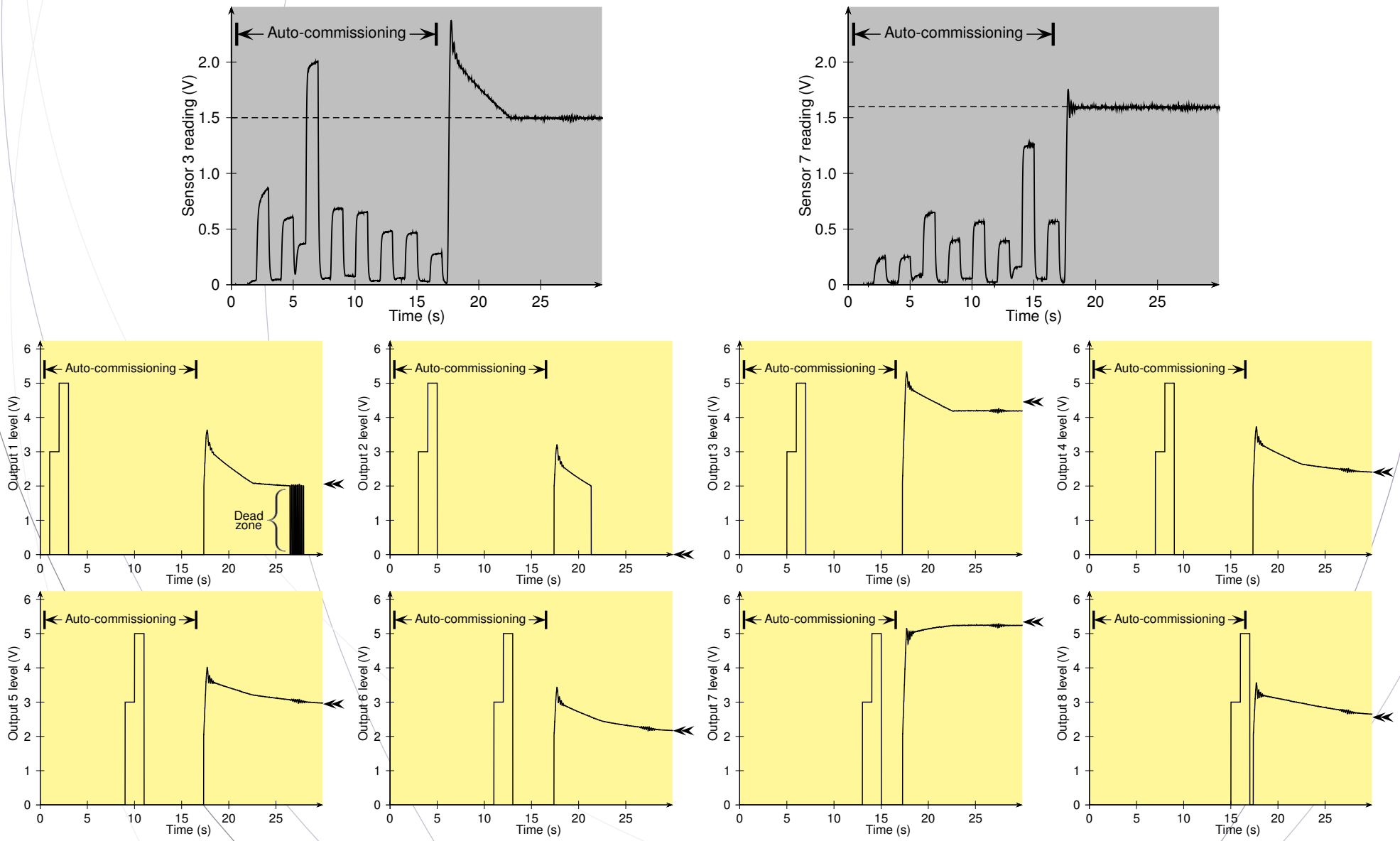
Validation on Small-scale Lighting Testbed

MultiIFD Results – Slide Along Constraint



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Summary and Conclusions

- Summary:
  - Social-insect colonies regulate macronutrient intake
  - Colonies *somehow* solve non-separable allocation problem
  - Optimization under multiple constraints is useful conceptual tool
  - Lighting analogy suggests new experiments
- MultiIFD principles:
  - *Stigmergic* coordination – colony nutrients are a *shared memory*
  - Decentralized implementation is robust and adaptive
- Ongoing work: *Temnothorax* as model system
  - Very high resolution possible
  - Measurement of small quantities of ingested food is challenging
- Future work: Stoichiometry; *Camponotus*; *Solenopsis*; *Paratrechina*

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NSF EECS-0931669 (CPS)
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BDA 2014

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