Division of Labor in Ant Colonies

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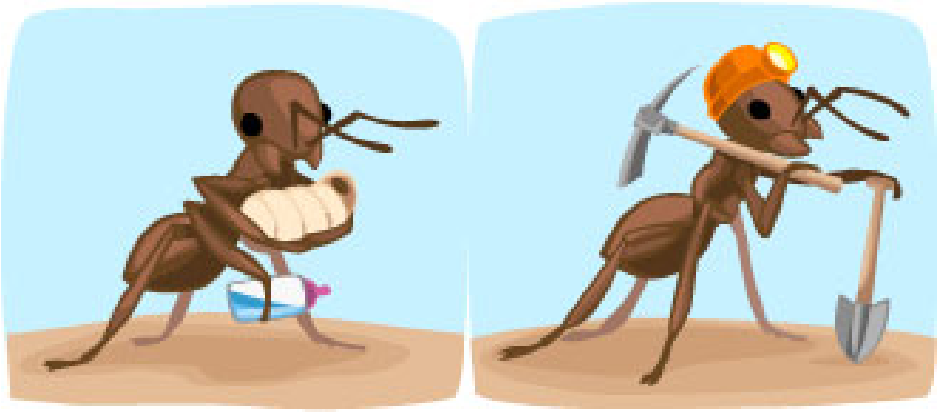

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Contributions

- A very general mathematical formulation for the phenomenon of division of labor in ant colonies.

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- A distributed algorithm that quickly reaches a near optimal task allocation and imposing only minimal assumptions on the capabilities of individual ants.

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- For task $\tau \in T$, ant $a \in A$ and time $t \in \mathbb{R}_{\geq 0}$:
  - $d(\tau, t)$ is the total energy demand for task τ at time t.
  - $e(\tau, a, t)$ is the energy that can be supplied by ant a if engaged at task τ at time t.

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- Observe that by definition $A = Y(\tau, t) \cup I(t)$.

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- An *optimal* task assignment is one that minimizes $\sum_{\tau \in T} q(\tau, t)^2$

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- We consider synchronous model where time proceeds in lock-step rounds $1, 2, \ldots$, and for the duration of each round $i \in \mathbb{N}$ each ant $a \in A$ works at task $y(a, i)$.

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- For each task $\tau \in T$ ants can sense only a binary feedback function $f(\tau, i)$ where:

$$f(\tau, i) = \begin{cases} +1 & q(\tau, i) \geq 0, \\ -1 & q(\tau, i) < 0. \end{cases}$$

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- Idea 1: Offload the burden of memory from the individuals to the colony.
- Idea 2: Let ants use a response-threshold strategy to pick tasks, and allow them to specialize.

Algorithm Details

- Each ant stores a task $currentTask \in T \cup \{\perp\}$ and a potential table $\varrho[\tau]$.

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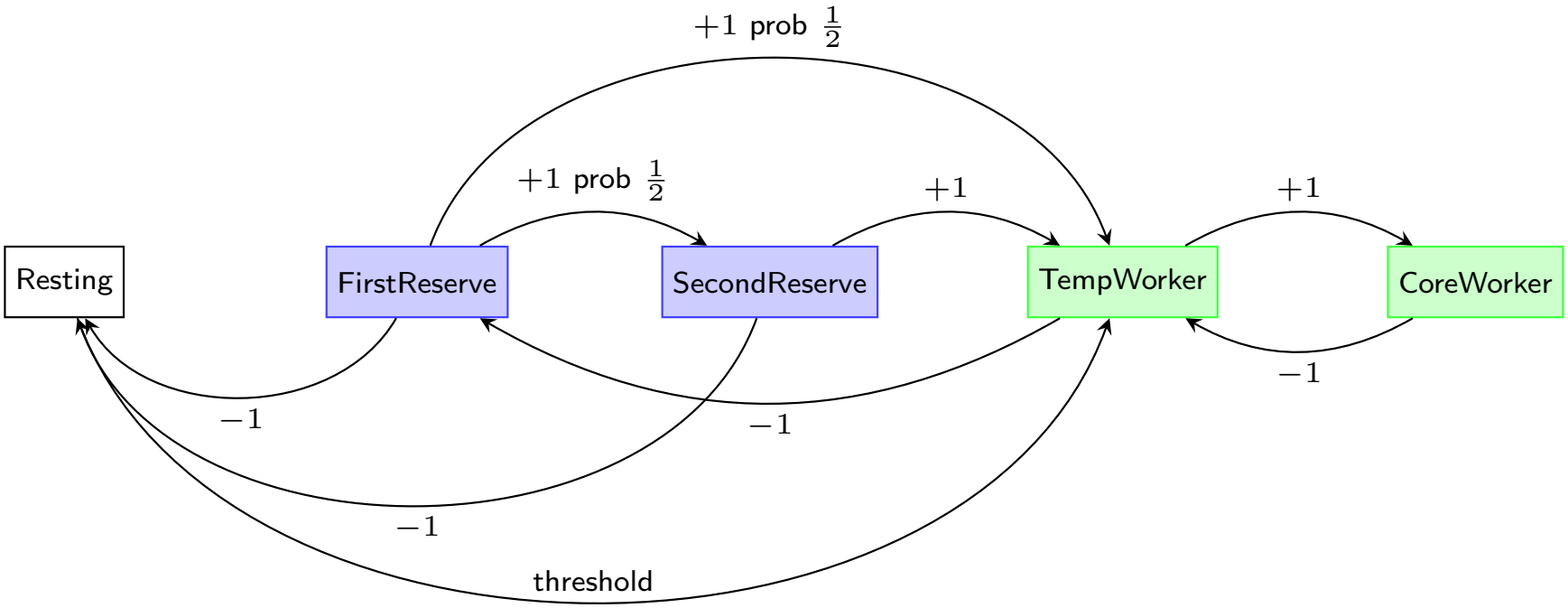
- Initially ants start in the $\textsc{Resting}$ state with $currentTask = \perp$ and a potential of zero for every task $\forall \tau \in T, \varrho[\tau] = 0$.
- Ants can be in one of the five states $\textsc{Resting}$, $\textsc{FirstReserve}$, $\textsc{SecondReserve}$, $\textsc{TempWorker}$ and $\textsc{CoreWorker}$.

Simplified State Machine



Resting State

$$\forall \tau \in T, \varrho[\tau] \leftarrow \begin{cases} 0 & \text{if } f(\tau, i) < 0 \\ \min(\varrho[\tau] + 1, 3) & \text{if } f(\tau, i) > 0 \end{cases}$$

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candidate $\leftarrow \{\tau \in T \mid \varrho[\tau] = 3\}$

if candidate $\neq \varnothing$ **then**

  with probability $\frac{1}{2}$ **do**

    $\forall \tau \in T, \varrho[\tau] \leftarrow 0$

    $currentTask \leftarrow$ random candidate task

    state $\leftarrow$ TempWorker

  end with

end if

Working States

case state $= \textsc{TempWorker}$

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  end if

Working States

case state $=$ $\textsc{TempWorker}$
  if $f(currentTask, i) < 0$ **then**
    state $\leftarrow$ $\textsc{FirstReserve}$
  else
    state $\leftarrow$ $\textsc{CoreWorker}$
  end if
case state $=$ $\textsc{CoreWorker}$

Working States

```
case state = TempWorker
    if f(currentTask, i) < 0 then
        state ← FirstReserve
    else
        state ← CoreWorker
    end if
case state = CoreWorker
    if f(currentTask, i) < 0 then
        state ← TempWorker
    end if
end case
```

Reserve States

$$\textbf{case}\ \mathsf{state} = \textsc{FirstReserve}$$

Reserve States

case state $=$ $\textsc{FirstReserve}$
  if $f(currentTask, i) < 0$ **then**
    state $\leftarrow$ $\textsc{Resting}$
  else

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```
case state = FirstReserve
    if f(currentTask, i) < 0 then
        state ← Resting
    else
        with probability 1/2 do
            state ← TempWorker
        otherwise
            state ← SecondReserve
        end with
    end if
```

Reserve States

```
case state = FirstReserve
    if f(currentTask, i) < 0 then
        state ← Resting
    else
        with probability 1/2 do
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case state = SecondReserve
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case state = SecondReserve
    if f(currentTask, i) < 0 then
        state ← Resting
    else
        state ← TempWorker
    end if
end case
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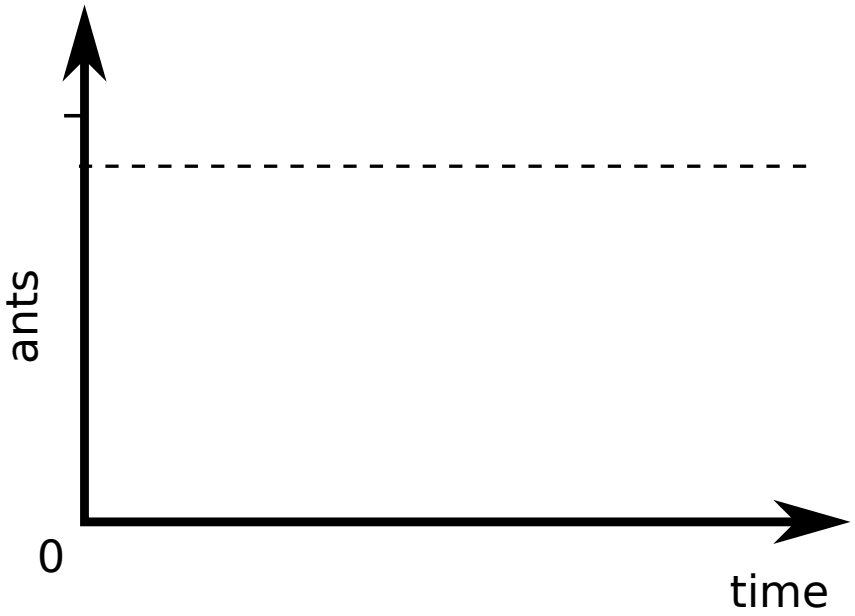

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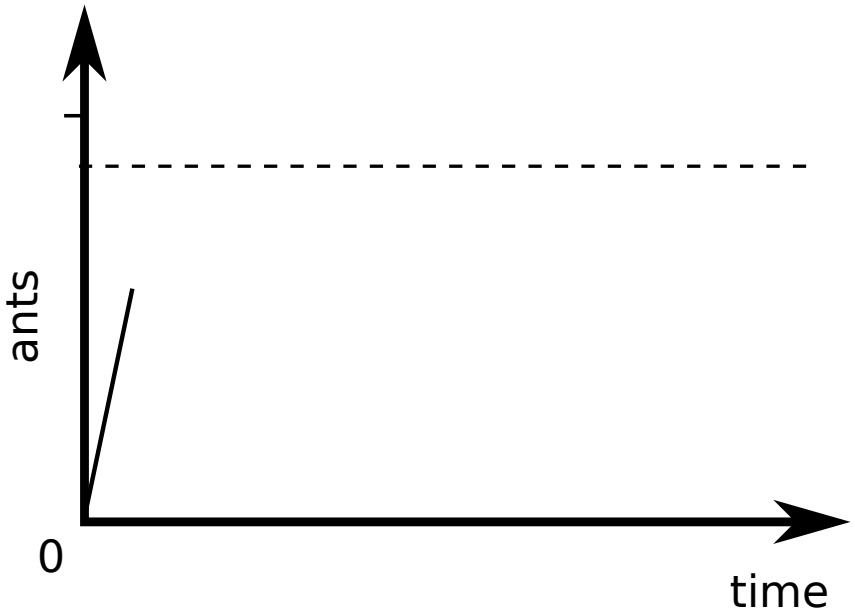

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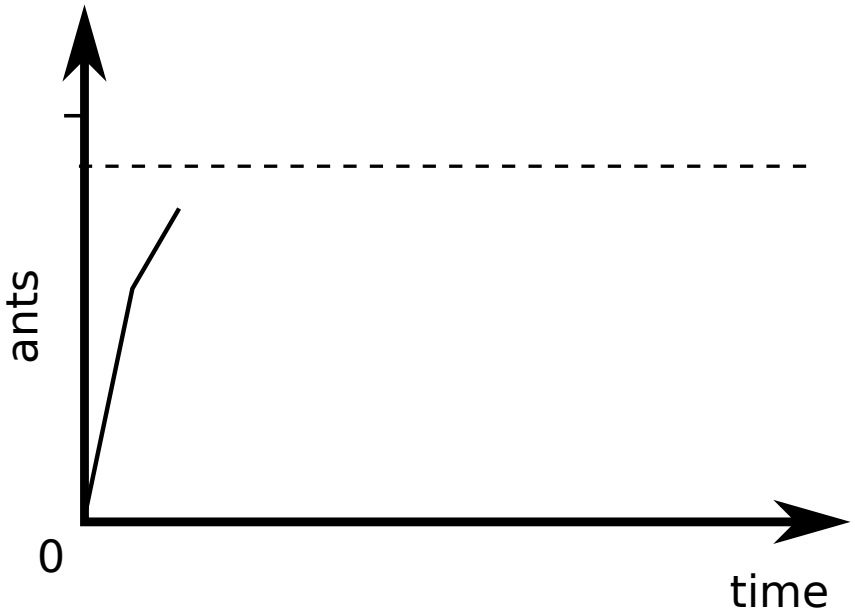

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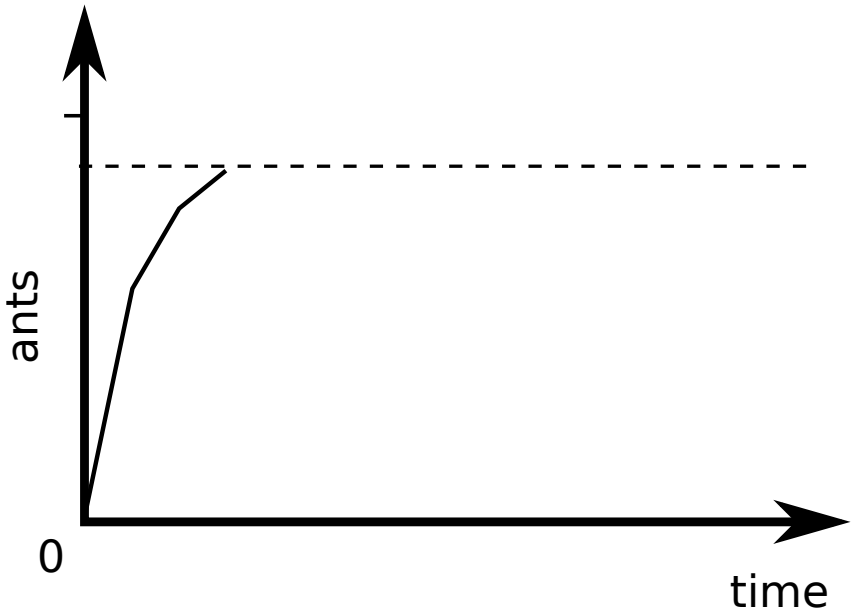

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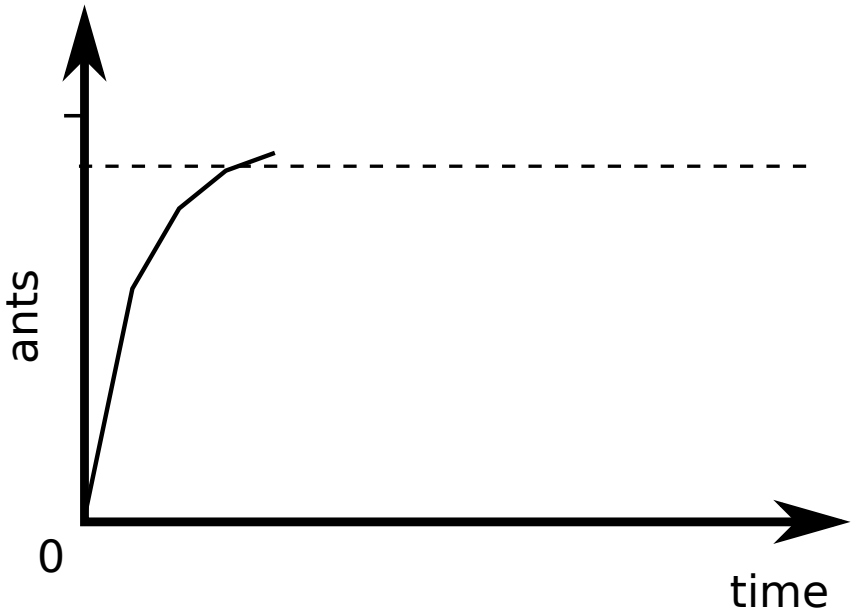

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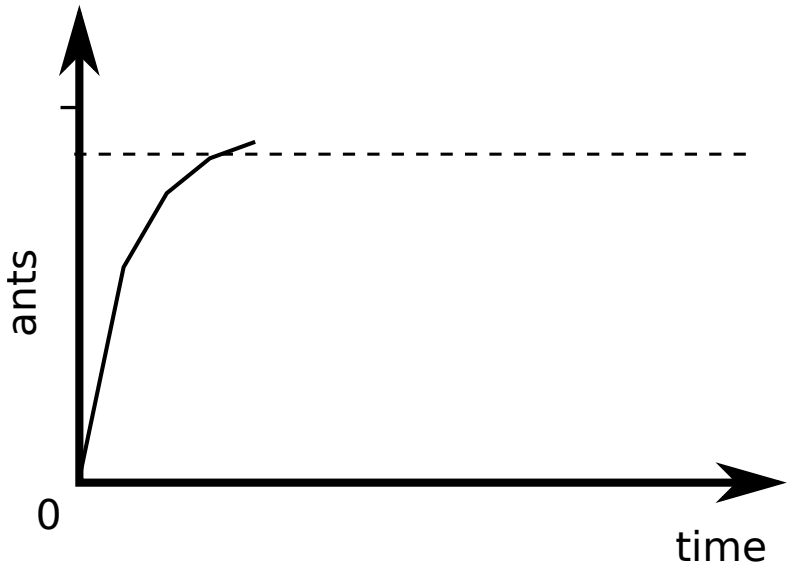

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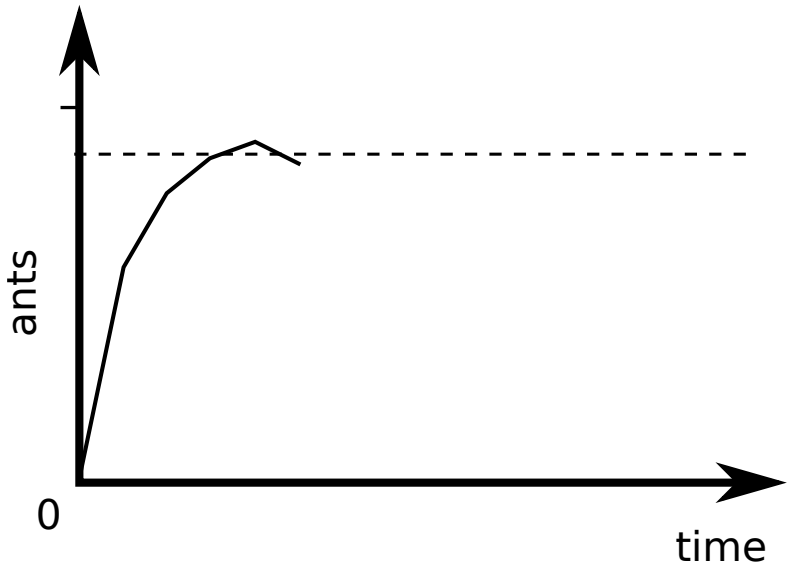

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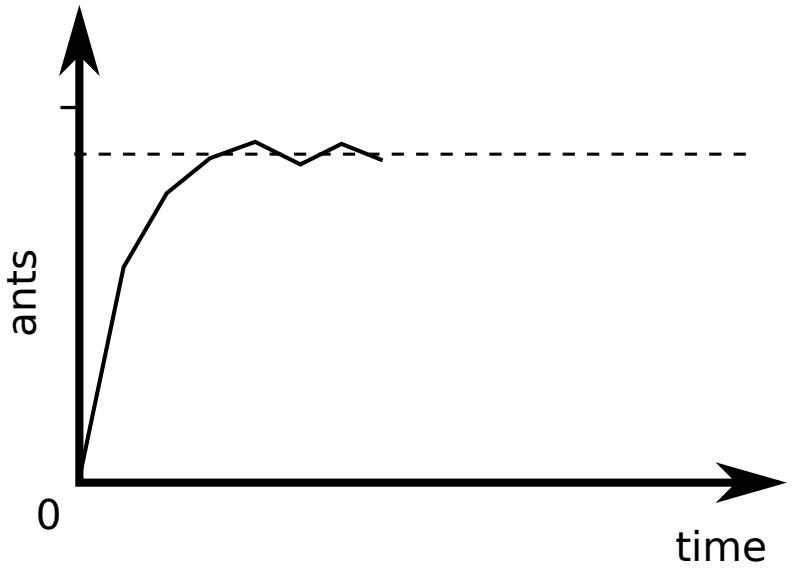

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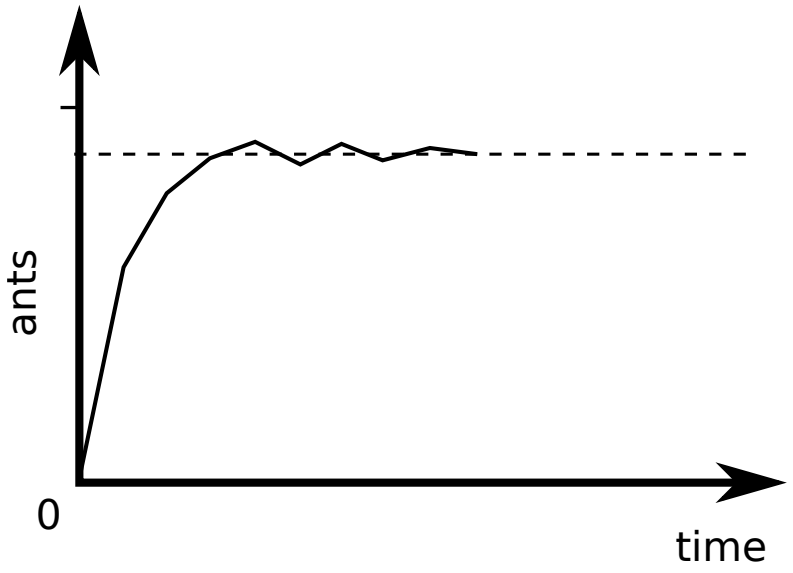

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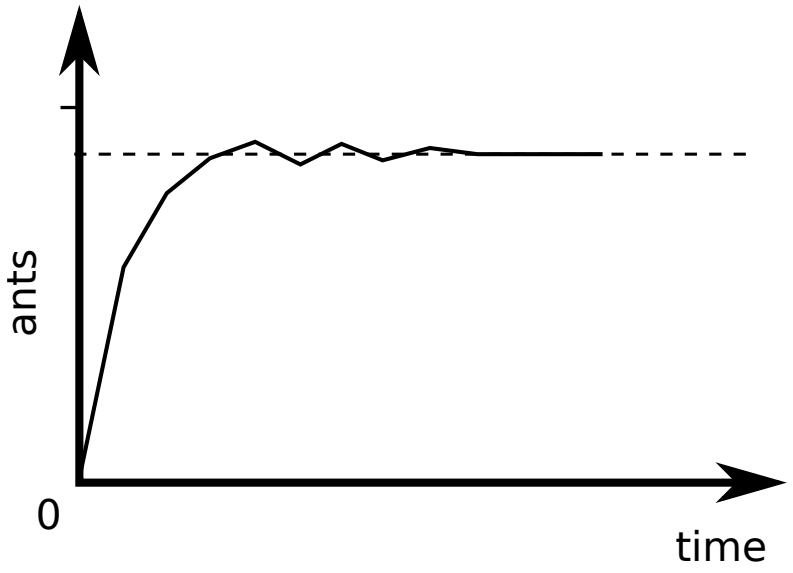

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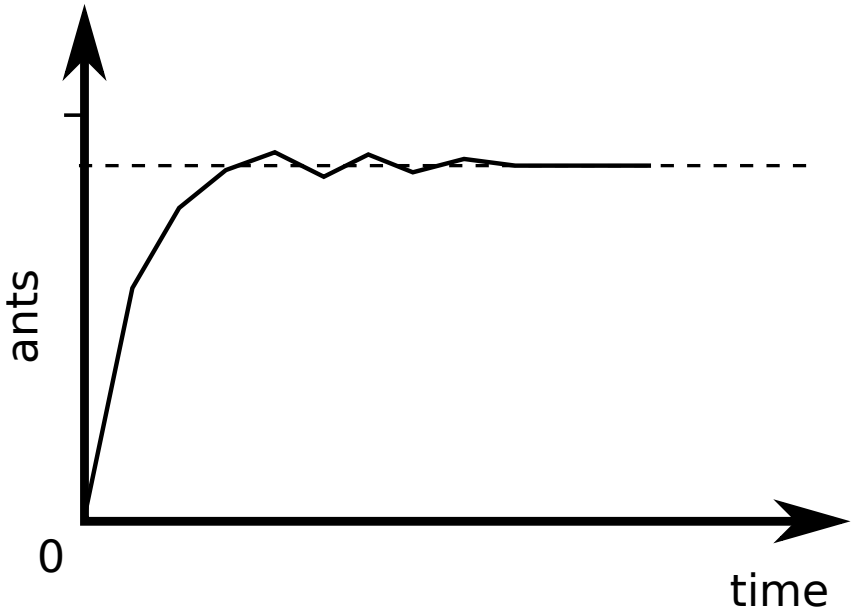
- **Theorem:** After $O(\log n)$ rounds, with high probability, ants reach an optimal task allocation.



Questions?