A (Logical) Framework for Collaboration

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Frank Pfenning 60th Celebration
August, 2022

Coming to CMU

Frank and I have been colleagues since 1988!

Many projects, many students, many interactions over many years.

A central theme: logical frameworks.

Frank has long taken the lead on developing and extending these ideas.

- Substructurality: linear and concurrent LF's.
- Implementation: unification, argument synthesis, theorem proving.
- Meta-theory: normalization, decidability.

Frank's work is exemplary in integrating theory and practice, the ethos of the Principles of Programming Group at CMU.

Twelf: An Implementation of LF

A major theme of Frank's work has been the design and development of Twelf.

- What's in a name? First there was Elf, then there was ~~Zwölf~~ Twelf.
- See `https://twelf.org` for code, examples, and ask The Twelf Elf.

Many components, variations, experiments:

- Type checking: higher-order abstract syntax, judgments-as-types.
- Unification-based argument synthesis, definition mechanism.
- Logic programming interpretation: backchaining (cf λ-Prolog)
- Coverage and totality checking: a prover for $\forall\exists$ formulae.

The tool of choice for language definition, both in-the-small and in-the-large.

Language Definition in Twelf

Higher-order Abstract Syntax:

- Syntactic categories: $\texttt{tp} : \texttt{Type}$, $\texttt{tm} : \texttt{Type}$.
- Hierarchical structure: $\texttt{arr} : \texttt{tp} \to \texttt{tp} \to \texttt{tp}$.
- Binding and scope: $\texttt{lam} : (\texttt{tm} \to \texttt{tm}) \to \texttt{tm}$.

Statics via Judgments-as-Types:

- Typing: $\texttt{of} : \texttt{tm} \to \texttt{tp} \to \texttt{Type}$.
- Example: $\texttt{arr-I} : (x : \texttt{tm} \to \texttt{of}\ x\ T_1 \to \texttt{of}\ (F\ x)\ T_2) \to \texttt{of}\ (\texttt{lam}\ F)\ (\texttt{arr}\ T_1\ T_2)$.

Dynamics via Plotkin's SOS:

- Transition: $\texttt{step} : \texttt{tm} \to \texttt{tm} \to \texttt{Type}$.
- Example: $\texttt{beta} : \texttt{step}\ (\texttt{ap}\ (\texttt{lam}\ F)\ M)\ (F\ M)$.

Language Definition in Twelf (jww Crary)

Type preservation:

- Judgment: $\texttt{pres} : \texttt{of}\ e\ T \to \texttt{step}\ e\ e' \to \texttt{of}\ e'\ T \to \text{Type}$.
- Proofs relate derivations to derivations:

$$\texttt{beta-pres} : \texttt{pres}\ (\texttt{arr-E}\ (\texttt{arr-I}\ DF)\ DA)\ (\texttt{beta})\ (DF\ \_\ DA).$$

Preservation Thm: $\forall\, Dst : \texttt{step}\ e\ e' \ \forall\, Dof : \texttt{of}\ e\ T \ \exists\, Dof : \texttt{of}\ e'\ T$.

- Worlds specification: $\%\texttt{worlds}\ ()\ (\texttt{pres}\ \_\ \_\ \_)$.
- Mode specification: $\%\texttt{mode}\ \texttt{pres}\ {+}Dst\ {+}Dof\ {-}Dof$.
- Totality check: $\%\texttt{total}\ D\ (\texttt{pres}\ D\ \_\ \_)$.

The proof is entirely readable and maintainable! (cf ML programming)

Scaling Up

Remarkably, Twelf scales to handle all of Standard ML!

- Elaboration: human-oriented external language to well-behaved internal language.
- Statics of IL: standard typing rules plus some tricks for state.
- Dynamics of IL: SOS rules with state.
- Progress and Preservation: well-elaborated programs are well-behaved.

Key points:

- Pair-programmed weekly over about six months.
- Proofs are readable: enumeration of cases + coverage and totality.
- Size: about 30,000 lines of Twelf in total.

But is it the same language? (Long discussion ensues.)

Meta-Theory of LF Itself

On Equivalence and Canonical Forms in the LF Type Theory is memorable.

- Reduction-free algorithm for equality checking with canonical "witness."
- Soundness and completeness by type-directed logical relations.
- (Later extended by Crary to families.)

Referees objected to there being no proof of "strong normalization" (of what?).

Mechanized independently by Urban in Nominal Isabelle.

- Mistake in the treatment of canonical families.
- We suggested two fixes, both worked.
- I chaired presentation at conference.

Above and Beyond

I have been very fortunate to be included as an honorary member of Frank's family.

- Nancy, his wife, and her parents, brothers, and sister.
- Their children, Andreas, Marina, and Nils.
- Their nieces, nephews, in-laws, and friends.
- Many family occasions and events, both joyous and otherwise.

I am enormously grateful for the friendship and support that Frank and Nancy have given me from (literally) the day I arrived in Pgh through many years of ups and downs, both personal and professional.

Happy Birthday, Frank!

Thanks for many years of collaboration and friendship!