(1)	$1 \vdash x_o$	Hyp
(2)	$1 \vdash x_o \equiv \top$	RuleP: 1
(3)	$1 \vdash x_o = \top$	Ext=: 2
(4)	$\vdash x_o \supset x = \top$	Deduct: 3
(5)	$5 \vdash x_o = \top$	Hyp
(6)	$5 \vdash \top$	RuleP
(7)	$5 \vdash x_o$	Sub=: 6 5
(8)	$\vdash x_o = \top \supset x$	Deduct: 7
(9)	$\vdash [x_o \supset x = \top] \wedge . x = \top \supset x$	Conj: 4 8
(10)	$\vdash x_o \equiv x = \top$	ImpEquiv: 9
(11)	$\vdash x_o = . x = \top$	Ext=: 10
(12)	$\vdash \forall x_o . x = . x = \top$	UGen: x_o 11
(13)	$13 \vdash \sim x_o$	Hyp
(14)	$13 \vdash x_o \equiv \bot$	RuleP: 13
(15)	$13 \vdash x_o = \bot$	Ext=: 14
(16)	$\vdash \sim x_o \supset x = \bot$	Deduct: 15
(17)	$17 \vdash x_o = \bot$	Hyp
(18)	$17 \vdash \sim \bot$	RuleP
(19)	$17 \vdash \sim x_o$	Sub=: 18 17
(20)	$\vdash x_o = \bot \supset \sim x$	Deduct: 19
(21)	$\vdash [\sim x_o \supset x = \bot] \wedge . x = \bot \supset \sim x$	Conj: 16 20
(22)	$\vdash \sim x_o \equiv x = \bot$	ImpEquiv: 21
(23)	$\vdash \sim x_o = . x = \bot$	Ext=: 22
(24)	$\vdash \forall x_o . \sim x = . x = \bot$	UGen: x_o 23
(25)	$\vdash a_\alpha = z_\alpha \equiv a = z$	RuleP
(26)	$\vdash [\lambda q_\alpha . a_\alpha = q] z_\alpha \equiv a = z$	Lambda: 25
(27)	$\vdash \forall z_\alpha . [\lambda q_\alpha . a_\alpha = q] z \equiv a = z$	UGen: z_α 26
(28)	$\vdash \forall z_\alpha [[\lambda q_\alpha . a_\alpha = q] z \equiv a = z] \supset \iota [\lambda q . a = q] = a$	Assert T5310
(29)	$\vdash \iota [\lambda q_\alpha . a_\alpha = q] = a$	MP: 27 28
(30)	$\vdash \forall a_\alpha . \iota [\lambda q_\alpha . a = q] = a$	UGen: a_α 29
(31)	$\vdash \iota [\lambda q_\alpha . a_\alpha = q] = a$	UI: a_α 30
(32)	$\vdash \top \wedge a_\alpha = q_\alpha \vee \sim \top \wedge b_\alpha = q \equiv a = q$	RuleP
(33)	$\vdash [\top \wedge a_\alpha = q_\alpha \vee \sim \top \wedge b_\alpha = q] = . a = q$	Ext=: 32
(34)	$\vdash \iota [\lambda q_\alpha . \top \wedge a_\alpha = q \vee \sim \top \wedge b_\alpha = q] = a$	Sub=: 31 33
(35)	$\vdash \mathrm{THAT}\ q_\alpha [\top \wedge a_\alpha = q \vee \sim \top \wedge b_\alpha = q] = a$	Defn: 34
(36)	$\vdash \mathbf{COND}\, a_\alpha\, b_\alpha\, \top = a$	Defn: 35
(37)	$\vdash \forall b_\alpha . \mathbf{COND}\, a_\alpha\, b\, \top = a$	UGen: b_α 36
(38)	$\vdash \forall a_\alpha \forall b_\alpha . \mathbf{COND}\, a\, b\, \top = a$	UGen: a_α 37
(39)	$\vdash \forall a_o \forall b_o . \mathbf{COND}\, a\, b\, \top = a$	Type Subst: $\circ_\alpha$ 38
(40)	$\vdash \iota [\lambda q_\alpha . b_\alpha = q] = b$	UI: b_α 30
(41)	$\vdash \bot \wedge a_\alpha = q_\alpha \vee \sim \bot \wedge b_\alpha = q \equiv b = q$	RuleP
(42)	$\vdash [\bot \wedge a_\alpha = q_\alpha \vee \sim \bot \wedge b_\alpha = q] = . b = q$	Ext=: 41
(43)	$\vdash \iota [\lambda q_\alpha . \bot \wedge a_\alpha = q \vee \sim \bot \wedge b_\alpha = q] = b$	Sub=: 40 42
(44)	$\vdash \mathrm{THAT}\ q_\alpha [\bot \wedge a_\alpha = q \vee \sim \bot \wedge b_\alpha = q] = b$	Defn: 43
(45)	$\vdash \mathbf{COND}\, a_\alpha\, b_\alpha\, \bot = b$	Defn: 44
(46)	$\vdash \forall b_\alpha . \mathbf{COND}\, a_\alpha\, b\, \bot = b$	UGen: b_α 45
(47)	$\vdash \forall a_\alpha \forall b_\alpha . \mathbf{COND}\, a\, b\, \bot = b$	UGen: a_α 46
(48)	$\vdash \forall a_o \forall b_o . \mathbf{COND}\, a\, b\, \bot = b$	Type Subst: $\circ_\alpha$ 47
(49)	$\vdash \lambda z_{o\alpha} \exists zp_\alpha [z = [= zp]] = \lambda z \exists zp . z = [= zp]$	Refl=
(50)	$\vdash \exists sing_{o(o\alpha)} . sing = \lambda z_{o\alpha} \exists zp_\alpha . z = [= zp]$	EGen: $sing_{o(o\alpha)}$ 49
(51)	$51 \vdash sing_{o(o\alpha)} = \lambda z_{o\alpha} \exists zp_\alpha . z = [= zp]$	Choose: $sing_{o(o\alpha)}$
(52)	$51 \vdash [= a_\alpha] = [= a]$	Assert REFL=
(53)	$51 \vdash \exists zp_\alpha . [= a_\alpha] = [= zp]$	EGen: a_α 52

(54) $51 \vdash [\lambda\, z_{o\alpha}\ \exists\, zp_{\alpha}.\ z = [= zp]].\ = a_{\alpha}$ Lambda: 53
(55) $51 \vdash sing_{o(o\alpha)}.\ = a_{\alpha}$ Sub=: 54 51
(56) $51 \vdash \forall\, a_{\alpha}\, sing_{o(o\alpha)}.\ = a$ UGen: a_{α} 55
(57) $\vdash sing_{o(o\alpha)}[= a_{\alpha}] = .\, sing[= a] = \top$ UI: $[sing_{o(o\alpha)}.\ = a_{\alpha}]$ 12
(58) $51 \vdash sing_{o(o\alpha)}.\ = a_{\alpha}$ UI: a_{α} 56
(59) $51 \vdash sing_{o(o\alpha)}[= a_{\alpha}] = \top$ Sub=: 58 57
(60) $51 \vdash \forall\, a_{\alpha}.\, sing_{o(o\alpha)}[= a] = \top$ UGen: a_{α} 59
(61) $51,61 \vdash \sim sing_{o(o\alpha)}\, B_{o\alpha}$ Hyp
(62) $\vdash \sim sing_{o(o\alpha)}\, B_{o\alpha} = .\, sing\, B = \bot$ UI: $[sing_{o(o\alpha)}\, B_{o\alpha}]$ 24
(63) $51,61 \vdash sing_{o(o\alpha)}\, B_{o\alpha} = \bot$ Sub=: 61 62
(64) $51,61 \vdash sing_{o(o\alpha)}\, A_{o\alpha} \vee \sim sing\, A$ RuleP
(65) $51,61,65 \vdash sing_{o(o\alpha)}\, A_{o\alpha}$ Case 1: 64
(66) $\vdash sing_{o(o\alpha)}\, A_{o\alpha} = .\, sing\, A = \top$ UI: $[sing_{o(o\alpha)}\, A_{o\alpha}]$ 12
(67) $51,61,65 \vdash sing_{o(o\alpha)}\, A_{o\alpha} = \top$ Sub=: 65 66
(68) $\vdash \forall\, b_o.\mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, A_{o\alpha}].\iota\, B_{o\alpha}]\ \bot\ \bot]\, b\ \top = \mathbf{COND}[lt[\iota\, A].\iota\, B]\ \bot\ \bot$
UI: $[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, A_{o\alpha}].\iota\, B_{o\alpha}]\bot\bot]$ 39
(69) $\vdash \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, A_{o\alpha}].\iota\, B_{o\alpha}]\ \bot\ \bot][\mathbf{COND}\ \top\ \bot\ \bot]\ \top = \mathbf{COND}[lt[\iota\, A].\iota\, B]\ \bot\ \bot$
UI: $[\mathbf{COND}\top\bot\bot]$ 68
(70) $\vdash \forall\, b_o.\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, A_{o\alpha}].\iota\, B_{o\alpha}]\, b\ \bot = b$ UI: $[lt_{o\alpha\alpha}[\iota\, A_{o\alpha}].\iota\, B_{o\alpha}]$ 48
(71) $\vdash \mathbf{COND}[lt_{o\alpha\alpha}[\iota\, A_{o\alpha}].\iota\, B_{o\alpha}]\ \bot\ \bot = \bot$ UI: $\bot$ 70
(72) $51,61,65 \vdash \bot = \bot$ Assert REFL=
(73) $51,61,65 \vdash \mathbf{COND}[lt_{o\alpha\alpha}[\iota\, A_{o\alpha}].\iota\, B_{o\alpha}]\ \bot\ \bot = \bot$ Sub=: 72 71
(74) $51,61,65 \vdash \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, A_{o\alpha}].\iota\, B_{o\alpha}]\ \bot\ \bot][\mathbf{COND}\ \top\ \bot\ \bot]\ \top = \bot$ Sub=: 73 69
(75) $51,61,65 \vdash \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, A_{o\alpha}].\iota\, B_{o\alpha}]\ \bot\ \bot][\mathbf{COND}\ \top\ \bot\ \bot][sing_{o(o\alpha)}\, A] = \bot$
Sub=: 74 67
(76) $51,61,76 \vdash \sim sing_{o(o\alpha)}\, A_{o\alpha}$ Case 2: 64
(77) $\vdash \sim sing_{o(o\alpha)}\, A_{o\alpha} = .\, sing\, A = \bot$ UI: $[sing_{o(o\alpha)}\, A_{o\alpha}]$ 24
(78) $51,61,76 \vdash sing_{o(o\alpha)}\, A_{o\alpha} = \bot$ Sub=: 76 77
(79) $\vdash \forall\, b_o.\mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, A_{o\alpha}].\iota\, B_{o\alpha}]\ \bot\ \bot]\, b\ \bot = b$
UI: $[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, A_{o\alpha}].\iota\, B_{o\alpha}]\bot\bot]$ 48
(80) $\vdash \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, A_{o\alpha}].\iota\, B_{o\alpha}]\ \bot\ \bot][\mathbf{COND}\ \top\ \bot\ \bot]\ \bot = \mathbf{COND}\ \top\ \bot\ \bot$
UI: $[\mathbf{COND}\top\bot\bot]$ 79
(81) $\vdash \forall\, b_o.\mathbf{COND}\ \top\, b\ \bot = b$ UI: $\top$ 48
(82) $51,61,76 \vdash \mathbf{COND}\ \top\ \bot\ \bot = \bot$ UI: $\bot$ 81
(83) $51,61,76 \vdash \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, A_{o\alpha}].\iota\, B_{o\alpha}]\ \bot\ \bot][\mathbf{COND}\ \top\ \bot\ \bot]\ \bot = \bot$ Sub=: 82 80
(84) $51,61,76 \vdash \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, A_{o\alpha}].\iota\, B_{o\alpha}]\ \bot\ \bot][\mathbf{COND}\ \top\ \bot\ \bot][sing_{o(o\alpha)}\, A] = \bot$
Sub=: 83 78
(85) $51,61 \vdash \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, A_{o\alpha}].\iota\, B_{o\alpha}]\ \bot\ \bot][\mathbf{COND}\ \top\ \bot\ \bot][sing_{o(o\alpha)}\, A] = \bot$
Cases: 64 75 84
(86) $51,61 \vdash \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, A_{o\alpha}].\iota\, B_{o\alpha}]\ \bot.\, sing_{o(o\alpha)}\, B][\mathbf{COND}\ \top\ \bot.\, sing\, B][sing\, A] = \bot$
Sub=: 85 63
(87) $51 \vdash \sim sing_{o(o\alpha)}\, B_{o\alpha} \supset \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, A_{o\alpha}].\iota\, B]\ \bot.\, sing\, B][\mathbf{COND}\ \top\ \bot.\, sing\, B][sing\, A] = \bot$
Deduct: 86
(88) $51 \vdash \forall\, B_{o\alpha}.\ \sim sing_{o(o\alpha)}\, B \supset \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, A_{o\alpha}].\iota\, B]\ \bot.\, sing\, B][\mathbf{COND}\ \top\ \bot.\, sing\, B][sing\, A] = \bot$
UGen: $B_{o\alpha}$ 87
(89) $51 \vdash \forall\, A_{o\alpha}\, \forall\, B_{o\alpha}.\ \sim sing_{o(o\alpha)}\, B \supset \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, A].\iota\, B]\ \bot.\, sing\, B][\mathbf{COND}\ \top\ \bot.\, sing\, B][sing\, A] = \bot$
UGen: $A_{o\alpha}$ 88
(90) $51,90 \vdash \sim sing_{o(o\alpha)}\, A_{o\alpha}$ Hyp
(91) $51,90,91 \vdash sing_{o(o\alpha)}\, B_{o\alpha}$ Hyp

(92) $\vdash \sim sing_{o(o\alpha)}\, A_{o\alpha} = .\, sing\, A = \bot$ UI: $[sing_{o(o\alpha)}\, A_{o\alpha}]$ 24

(93) 51,90 $\vdash sing_{o(o\alpha)}\, A_{o\alpha} = \bot$ Sub=: 90 92

(94) $\vdash sing_{o(o\alpha)}\, B_{o\alpha} = .\, sing\, B = \top$ UI: $[sing_{o(o\alpha)}\, B_{o\alpha}]$ 12

(95) 51,90,91 $\vdash sing_{o(o\alpha)}\, B_{o\alpha} = \top$ Sub=: 91 94

(96) $\vdash \forall b_o.\mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, A_{o\alpha}].\iota\, B_{o\alpha}]\, \bot\, \top]\, b\, \bot = b$
UI: $[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, A_{o\alpha}].\iota\, B_{o\alpha}]\bot\top]$ 48

(97) $\vdash \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, A_{o\alpha}].\iota\, B_{o\alpha}]\, \bot\, \top][\mathbf{COND}\, \top\, \bot\, \top]\, \bot = \mathbf{COND}\, \top\, \bot\, \top$
UI: $[\mathbf{COND}\top\bot\top]$ 96

(98) $\vdash \forall b_o.\mathbf{COND}\, \top\, b\, \top = \top$ UI: $\top$ 39

(99) $\vdash \mathbf{COND}\, \top\, \bot\, \top = \top$ UI: $\bot$ 98

(100) 51,90,91 $\vdash \top = \top$ Assert REFL=

(101) 51,90,91 $\vdash \mathbf{COND}\, \top\, \bot\, \top = \top$ Sub=: 100 99

(102) 51,90,91 $\vdash \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, A_{o\alpha}].\iota\, B_{o\alpha}]\, \bot\, \top][\mathbf{COND}\, \top\, \bot\, \top]\, \bot = \top$
Sub=: 101 97

(103) 51,90,91 $\vdash \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, A_{o\alpha}].\iota\, B_{o\alpha}]\, \bot.\, sing_{o(o\alpha)}\, B][\mathbf{COND}\, \top\, \bot.\, sing\, B]\, \bot = \top$
Sub=: 102 95

(104) 51,90,91 $\vdash \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, A_{o\alpha}].\iota\, B_{o\alpha}]\, \bot.\, sing_{o(o\alpha)}\, B][\mathbf{COND}\, \top\, \bot.\, sing\, B][sing\, A] = \top$
Sub=: 103 93

(105) 51,90 $\vdash sing_{o(o\alpha)}\, B_{o\alpha} \supset \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, A_{o\alpha}].\iota\, B]\, \bot.\, sing\, B][\mathbf{COND}\, \top\, \bot.\, sing\, B][sing\, A] = \top$
Deduct: 104

(106) 51 $\vdash \sim sing_{o(o\alpha)}\, A_{o\alpha} \supset .\, sing\, B_{o\alpha} \supset \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, A].\iota\, B]\, \bot.\, sing\, B][\mathbf{COND}\, \top\, \bot.\, sing\, B][sing\, A] = \top$
Deduct: 105

(107) 51 $\vdash \forall B_{o\alpha}.\ \sim sing_{o(o\alpha)}\, A_{o\alpha} \supset .\, sing\, B \supset \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, A].\iota\, B]\, \bot.\, sing\, B][\mathbf{COND}\, \top\, \bot.\, sing\, B][sing\, A] = \top$
UGen: $B_{o\alpha}$ 106

(108) 51 $\vdash \forall A_{o\alpha}\, \forall B_{o\alpha}.\ \sim sing_{o(o\alpha)}\, A \supset .\, sing\, B \supset \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, A].\iota\, B]\, \bot.\, sing\, B][\mathbf{COND}\, \top\, \bot.\, sing\, B][sing\, A] = \top$
UGen: $A_{o\alpha}$ 107

(109) 51,109 $\vdash sing_{o(o\alpha)}\, A_{o\alpha}$ Hyp

(110) 51,109,110 $\vdash sing_{o(o\alpha)}\, B_{o\alpha}$ Hyp

(111) $\vdash sing_{o(o\alpha)}\, A_{o\alpha} = .\, sing\, A = \top$ UI: $[sing_{o(o\alpha)}\, A_{o\alpha}]$ 12

(112) $\vdash sing_{o(o\alpha)}\, B_{o\alpha} = .\, sing\, B = \top$ UI: $[sing_{o(o\alpha)}\, B_{o\alpha}]$ 12

(113) 51,109 $\vdash sing_{o(o\alpha)}\, A_{o\alpha} = \top$ Sub=: 109 111

(114) 51,109,110 $\vdash sing_{o(o\alpha)}\, B_{o\alpha} = \top$ Sub=: 110 112

(115) $\vdash \forall b_o.\mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, A_{o\alpha}].\iota\, B_{o\alpha}]\, \bot\, \top]\, b\, \top = \mathbf{COND}[lt[\iota\, A].\iota\, B]\, \bot\, \top$
UI: $[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, A_{o\alpha}].\iota\, B_{o\alpha}]\bot\top]$ 39

(116) $\vdash \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, A_{o\alpha}].\iota\, B_{o\alpha}]\, \bot\, \top][\mathbf{COND}\, \top\, \bot\, \top]\, \top = \mathbf{COND}[lt[\iota\, A].\iota\, B]\, \bot\, \top$
UI: $[\mathbf{COND}\top\bot\top]$ 115

(117) $\vdash \forall b_o.\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, A_{o\alpha}].\iota\, B_{o\alpha}]\, b\, \top = lt[\iota\, A].\iota\, B$ UI: $[lt_{o\alpha\alpha}[\iota\, A_{o\alpha}].\iota\, B_{o\alpha}]$ 39

(118) $\vdash \mathbf{COND}[lt_{o\alpha\alpha}[\iota\, A_{o\alpha}].\iota\, B_{o\alpha}]\, \bot\, \top = lt[\iota\, A].\iota\, B$ UI: $\bot$ 117

(119) 51,109,110 $\vdash lt_{o\alpha\alpha}[\iota\, A_{o\alpha}][\iota\, B_{o\alpha}] = lt[\iota\, A].\iota\, B$ Assert REFL=

(120) 51,109,110 $\vdash \mathbf{COND}[lt_{o\alpha\alpha}[\iota\, A_{o\alpha}].\iota\, B_{o\alpha}]\, \bot\, \top = lt[\iota\, A].\iota\, B$ Sub=: 119 118

(121) 51,109,110 $\vdash \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, A_{o\alpha}].\iota\, B_{o\alpha}]\, \bot\, \top][\mathbf{COND}\, \top\, \bot\, \top]\, \top = lt[\iota\, A].\iota\, B$
Sub=: 120 116

(122) 51,109,110 $\vdash \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, A_{o\alpha}].\iota\, B_{o\alpha}]\, \bot.\, sing_{o(o\alpha)}\, B][\mathbf{COND}\, \top\, \bot.\, sing\, B]\, \top = lt[\iota\, A].\iota\, B$
Sub=: 121 114

(123) 51,109,110 $\vdash \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, A_{o\alpha}].\iota\, B_{o\alpha}]\, \bot.\, sing_{o(o\alpha)}\, B][\mathbf{COND}\, \top\, \bot.\, sing\, B][sing\, A] = lt[\iota\, A].\iota\, B$
Sub=: 122 113

(124) $51{,}109 \vdash sing_{o(o\alpha)}\, B_{o\alpha} \supset \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, A_{o\alpha}].\iota\, B]\, \bot.\, sing\, B][\mathbf{COND}\, \top\, \bot.\, sing\, B][sing\, A] = lt[\iota\, A].\iota\, B$

Deduct: 123

(125) $51 \vdash sing_{o(o\alpha)}\, A_{o\alpha} \supset .\, sing\, B_{o\alpha} \supset \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, A].\iota\, B]\, \bot.\, sing\, B][\mathbf{COND}\, \top\, \bot.\, sing\, B][sing\, A] = lt[\iota\, A].\iota\, B$

Deduct: 124

(126) $51 \vdash \forall\, B_{o\alpha}.\, sing_{o(o\alpha)}\, A_{o\alpha} \supset .\, sing\, B \supset \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, A].\iota\, B]\, \bot.\, sing\, B][\mathbf{COND}\, \top\, \bot.\, sing\, B][sing\, A] = lt[\iota\, A].\iota\, B$

UGen: $B_{o\alpha}$ 125

(127) $51 \vdash \forall\, A_{o\alpha}\ \forall\, B_{o\alpha}.\, sing_{o(o\alpha)}\, A \supset .\, sing\, B \supset \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, A].\iota\, B]\, \bot.\, sing\, B][\mathbf{COND}\, \top\, \bot.\, sing\, B][sing\, A] = lt[\iota\, A].\iota\, B$

UGen: $A_{o\alpha}$ 126

(128) $51 \vdash \forall\, B_{o\alpha}.\, \sim sing_{o(o\alpha)}\, B \supset \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, a_{o\alpha}].\iota\, B]\, \bot.\, sing\, B][\mathbf{COND}\, \top\, \bot.\, sing\, B][sing\, a] = \bot$

UI: $a_{o\alpha}$ 89

(129) $51 \vdash \sim sing_{o(o\alpha)}\, b_{o\alpha} \supset \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, a_{o\alpha}].\iota\, b]\, \bot.\, sing\, b][\mathbf{COND}\, \top\, \bot.\, sing\, b][sing\, a] = \bot$

UI: $b_{o\alpha}$ 128

(130) $51{,}130 \vdash \exists\, r_{o\alpha\alpha}\ \forall\, x_\alpha\ \forall\, y_\alpha\ \forall\, z_\alpha.\ \exists\, w_\alpha\, r\, x\, w \wedge \sim r\, x\, x \wedge .\, r\, x\, y \supset .\, r\, y\, z \supset r\, x\, z$ Hyp

(131) $51{,}130{,}131 \vdash \forall\, x_\alpha\ \forall\, y_\alpha\ \forall\, z_\alpha.\ \exists\, w_\alpha\, lt_{o\alpha\alpha}\, x\, w \wedge \sim lt\, x\, x \wedge .\, lt\, x\, y \supset .\, lt\, y\, z \supset lt\, x\, z$

Choose: $lt_{o\alpha\alpha}$ 130

(132) $51{,}130{,}131 \vdash \forall\, y_\alpha\ \forall\, z_\alpha.\ \exists\, w_\alpha\, lt_{o\alpha\alpha}\, x_\alpha\, w \wedge \sim lt\, x\, x \wedge .\, lt\, x\, y \supset .\, lt\, y\, z \supset lt\, x\, z$ UI: x_α 131

(133) $51{,}130{,}131 \vdash \forall\, z_\alpha.\ \exists\, w_\alpha\, lt_{o\alpha\alpha}\, x_\alpha\, w \wedge \sim lt\, x\, x \wedge .\, lt\, x\, y_\alpha \supset .\, lt\, y\, z \supset lt\, x\, z$ UI: y_α 132

(134) $51{,}130{,}131 \vdash \exists\, w_\alpha\, lt_{o\alpha\alpha}\, x_\alpha\, w \wedge \sim lt\, x\, x \wedge .\, lt\, x\, y_\alpha \supset .\, lt\, y\, z_\alpha \supset lt\, x\, z$ UI: z_α 133

(135) $51{,}130{,}131 \vdash \exists\, w_\alpha\, lt_{o\alpha\alpha}\, x_\alpha\, w \wedge \sim lt\, x\, x$ Conj: 134

(136) $51{,}130{,}131 \vdash lt_{o\alpha\alpha}\, x_\alpha\, y_\alpha \supset .\, lt\, y\, z_\alpha \supset lt\, x\, z$ Conj: 134

(137) $51{,}130{,}131 \vdash \exists\, w_\alpha\, lt_{o\alpha\alpha}\, x_\alpha\, w$ Conj: 135

(138) $51{,}130{,}131 \vdash \sim lt_{o\alpha\alpha}\, x_\alpha\, x$ Conj: 135

(139) $51{,}130{,}131 \vdash \forall\, x_\alpha\ \exists\, w_\alpha\, lt_{o\alpha\alpha}\, x\, w$ UGen: x_α 137

(140) $51{,}130{,}131 \vdash \forall\, x_\alpha.\ \sim lt_{o\alpha\alpha}\, x\, x$ UGen: x_α 138

(141) $51{,}130{,}131 \vdash \forall\, z_\alpha.\, lt_{o\alpha\alpha}\, x_\alpha\, y_\alpha \supset .\, lt\, y\, z \supset lt\, x\, z$ UGen: z_α 136

(142) $51{,}130{,}131 \vdash \forall\, y_\alpha\ \forall\, z_\alpha.\, lt_{o\alpha\alpha}\, x_\alpha\, y \supset .\, lt\, y\, z \supset lt\, x\, z$ UGen: y_α 141

(143) $51{,}130{,}131 \vdash \forall\, x_\alpha\ \forall\, y_\alpha\ \forall\, z_\alpha.\, lt_{o\alpha\alpha}\, x\, y \supset .\, lt\, y\, z \supset lt\, x\, z$ UGen: x_α 142

(144) $51{,}130{,}131 \vdash sing_{o(o\alpha)}\, X_{o\alpha} \vee \sim sing\, X$ RuleP

(145) $51{,}130{,}131{,}145 \vdash sing_{o(o\alpha)}\, X_{o\alpha}$ Case 1: 144

(146) $130{,}131{,}145{,}51 \vdash [\lambda\, z_{o\alpha}\ \exists\, zp_\alpha.\, z = [=\, zp]]\, X_{o\alpha}$ Sub=: 145 51

(147) $130{,}131{,}145{,}51 \vdash \exists\, zp_\alpha.\, X_{o\alpha} = [=\, zp]$ Lambda: 146

(148) $51{,}130{,}131{,}145{,}148 \vdash X_{o\alpha} = [=\, xp_\alpha]$ Choose: xp_α 147

(149) $51{,}130{,}131 \vdash \exists\, w_\alpha\, lt_{o\alpha\alpha}\, xp_\alpha\, w$ UI: xp_α 139

(150) $51{,}130{,}131{,}145{,}148{,}150 \vdash lt_{o\alpha\alpha}\, xp_\alpha\, Sxp_\alpha$ Choose: Sxp_α 149

(151) $51 \vdash sing_{o(o\alpha)}.\ = Sxp_\alpha$ UI: Sxp_α 56

(152) $51 \vdash \forall\, B_{o\alpha}.\, sing_{o(o\alpha)}\, X_{o\alpha} \supset .\, sing\, B \supset \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, X].\iota\, B]\, \bot.\, sing\, B][\mathbf{COND}\, \top\, \bot.\, sing\, B][sing\, X] = lt[\iota\, X].\iota\, B$

UI: $X_{o\alpha}$ 127

(153) $51 \vdash sing_{o(o\alpha)}\, X_{o\alpha} \supset .\, sing[=\, Sxp_\alpha] \supset \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, X].\iota.\ = Sxp]\, \bot.\, sing.\ = Sxp][\mathbf{COND}\, \top\, \bot.\, sing.\ = Sxp][sing\, X] = lt[\iota\, X].\iota.\ = Sxp$

UI: $[=\, Sxp_\alpha]$ 152

(154) $130{,}131{,}145{,}51 \vdash sing_{o(o\alpha)}[=\, Sxp_\alpha] \supset \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, X_{o\alpha}].\iota.\ = Sxp]\, \bot.\, sing.\ = Sxp][\mathbf{COND}\, \top\, \bot.\, sing.\ = Sxp][sing\, X] = lt[\iota\, X].\iota.\ = Sxp$

MP: 145 153

(155) $130{,}131{,}145{,}51 \vdash \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, X_{o\alpha}].\iota.\ = Sxp_\alpha]\, \bot.\, sing_{o(o\alpha)}.\ = Sxp][\mathbf{COND}\, \top\, \bot.\, sing.\ = Sxp][sing\, X] = lt[\iota\, X].\iota.\ = Sxp$

MP: 151 154

(156) 51,130,131,145,148,150 $\vdash lt_{o\alpha\alpha}\, xp_\alpha\, Sxp_\alpha$ Same as: 150
(157) $\vdash \iota[= xp_\alpha] = xp$ Assert DESCR
(158) 51,130,131,145,148,150 $\vdash lt_{o\alpha\alpha}[\iota. = xp_\alpha]\, Sxp_\alpha$ Sub=: 156 157
(159) 51,130,131,145,148,150 $\vdash lt_{o\alpha\alpha}[\iota\, X_{o\alpha}]\, Sxp_\alpha$ Sub=: 158 148
(160) $\vdash \iota[= Sxp_\alpha] = Sxp$ Assert DESCR
(161) 51,130,131,145,148,150 $\vdash lt_{o\alpha\alpha}[\iota\, X_{o\alpha}].\iota. = Sxp_\alpha$ Sub=: 159 160
(162) 51,130,131,145,148,150 $\vdash$**COND**[**COND**[$lt_{o\alpha\alpha}[\iota\, X_{o\alpha}].\iota. = Sxp_\alpha]\ \bot.\, sing_{o(o\alpha)}. = Sxp$][**COND** $\top$ $\bot.\, sing. = Sxp$]. $sing\, X$

Sub=: 161 155
(163) 51,130,131,145,148,150 $\vdash \exists\, W_{o\alpha}$**COND**[**COND**[$lt_{o\alpha\alpha}[\iota\, X_{o\alpha}].\iota\, W]\ \bot.\, sing_{o(o\alpha)}\, W$][**COND** $\top\ \bot.\, sing\, W$]. $sing\, X$

EGen: [$= Sxp_\alpha$] 162
(164) 51,130,131,145,148 $\vdash \exists\, W_{o\alpha}$**COND**[**COND**[$lt_{o\alpha\alpha}[\iota\, X_{o\alpha}].\iota\, W]\ \bot.\, sing_{o(o\alpha)}\, W$][**COND** $\top\ \bot.\, sing\, W$]. $sing\, X$

RuleC: 149 163
(165) 51,130,131,145 $\vdash \exists\, W_{o\alpha}$**COND**[**COND**[$lt_{o\alpha\alpha}[\iota\, X_{o\alpha}].\iota\, W]\ \bot.\, sing_{o(o\alpha)}\, W$][**COND** $\top\ \bot.\, sing\, W$]. $sing\, X$

RuleC: 147 164
(166) 51,130,131,166 $\vdash \sim\ sing_{o(o\alpha)}\, X_{o\alpha}$ Case 2: 144
(167) 51 $\vdash sing_{o(o\alpha)}. = k_\alpha$ UI: k_α 56
(168) 51 $\vdash \forall\, B_{o\alpha}.\ \sim\ sing_{o(o\alpha)}\, X_{o\alpha} \supset .\, sing\, B \supset$ **COND**[**COND**[$lt_{o\alpha\alpha}[\iota\, X].\iota\, B]\ \bot.\, sing\, B$][**COND** $\top$ $\bot.\, sing\, B$][$sing\, X$] $= \top$

UI: $X_{o\alpha}$ 108
(169) 51 $\vdash \sim\ sing_{o(o\alpha)}\, X_{o\alpha} \supset .\, sing[= k_\alpha] \supset$ **COND**[**COND**[$lt_{o\alpha\alpha}[\iota\, X].\iota. = k]\ \bot.\, sing. = k$][**COND** $\top\ \bot.\, sing. = k$][$sing\, X$] $= \top$

UI: [$= k_\alpha$] 168
(170) 130,131,166,51 $\vdash sing_{o(o\alpha)}[= k_\alpha] \supset$ **COND**[**COND**[$lt_{o\alpha\alpha}[\iota\, X_{o\alpha}].\iota. = k]\ \bot.\, sing. = k$][**COND** $\top$ $\bot.\, sing. = k$][$sing\, X$] $= \top$

MP: 166 169
(171) 130,131,166,51 $\vdash$**COND**[**COND**[$lt_{o\alpha\alpha}[\iota\, X_{o\alpha}].\iota. = k_\alpha]\ \bot.\, sing_{o(o\alpha)}. = k$][**COND** $\top\ \bot.\, sing. = k$][$sing\, X$] $= \top$

MP: 167 170
(172) 51,130,131,166 $\vdash \top$ RuleP
(173) 51,130,131,166 $\vdash$**COND**[**COND**[$lt_{o\alpha\alpha}[\iota\, X_{o\alpha}].\iota. = k_\alpha]\ \bot.\, sing_{o(o\alpha)}. = k$][**COND** $\top\ \bot.\, sing. = k$]. $sing\, X$

Sub=: 172 171
(174) 51,130,131,166 $\vdash \exists\, W_{o\alpha}$**COND**[**COND**[$lt_{o\alpha\alpha}[\iota\, X_{o\alpha}].\iota\, W]\ \bot.\, sing_{o(o\alpha)}\, W$][**COND** $\top\ \bot.\, sing\, W$]. $sing\, X$

EGen: [$= k_\alpha$] 173
(175) 51,130,131 $\vdash \exists\, W_{o\alpha}$**COND**[**COND**[$lt_{o\alpha\alpha}[\iota\, X_{o\alpha}].\iota\, W]\ \bot.\, sing_{o(o\alpha)}\, W$][**COND** $\top\ \bot.\, sing\, W$]. $sing\, X$

Cases: 144 165 174
(176) 51,130,131,176 $\vdash \sim\sim$**COND**[**COND**[$lt_{o\alpha\alpha}[\iota\, X_{o\alpha}].\iota\, X]\ \bot.\, sing_{o(o\alpha)}\, X$][**COND** $\top\ \bot.\, sing\, X$]. $sing\, X$

Assume negation
(177) 51,130,131,176 $\vdash$**COND**[**COND**[$lt_{o\alpha\alpha}[\iota\, X_{o\alpha}].\iota\, X]\ \bot.\, sing_{o(o\alpha)}\, X$][**COND** $\top\ \bot.\, sing\, X$]. $sing\, X$

Neg: 176
(178) 51,130,131,176 $\vdash sing_{o(o\alpha)}\, X_{o\alpha} \vee \sim\ sing\, X$ RuleP
(179) 51,130,131,176,179 $\vdash sing_{o(o\alpha)}\, X_{o\alpha}$ Case 1: 178
(180) 130,131,176,179,51 $\vdash [\lambda\, z_{o\alpha}\ \exists\, zp_\alpha.\, z = [= zp]]\, X_{o\alpha}$ Sub=: 179 51
(181) 130,131,176,179,51 $\vdash \exists\, zp_\alpha.\, X_{o\alpha} = [= zp]$ Lambda: 180
(182) 51,130,131,176,179,182 $\vdash X_{o\alpha} = [= xp_\alpha]$ Choose: xp_α 181
(183) 51 $\vdash \forall\, B_{o\alpha}.\, sing_{o(o\alpha)}\, X_{o\alpha} \supset .\, sing\, B \supset$ **COND**[**COND**[$lt_{o\alpha\alpha}[\iota\, X].\iota\, B]\ \bot.\, sing\, B$][**COND** $\top\ \bot.\, sing\, B$][$sing\, X$] $= lt[\iota\, X].\iota\, B$

UI: $X_{o\alpha}$ 127
(184) 51 $\vdash sing_{o(o\alpha)}\, X_{o\alpha} \supset .\, sing\, X \supset$ **COND**[**COND**[$lt_{o\alpha\alpha}[\iota\, X].\iota\, X]\ \bot.\, sing\, X$][**COND** $\top\ \bot.\, sing\, X$][$sing\, X$] $= lt[\iota\, X].\iota\, X$

UI: $X_{o\alpha}$ 183

(185) 130,131,176,179,51 $\vdash sing_{o(o\alpha)}\ X_{o\alpha} \supset \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, X].\iota\, X]\ \bot.\ sing\, X][\mathbf{COND}\ \top\ \bot.\ sing\, X][sing\, X] = lt[\iota\, X].\iota\, X$

MP: 179 184

(186) 130,131,176,179,51 $\vdash \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, X_{o\alpha}].\iota\, X]\ \bot.\ sing_{o(o\alpha)}\ X][\mathbf{COND}\ \top\ \bot.\ sing\, X][sing\, X] = lt[\iota\, X].\iota\, X$

MP: 179 185

(187) 130,131,176,179,51 $\vdash lt_{o\alpha\alpha}[\iota\, X_{o\alpha}].\iota\, X$ Sub=: 177 186

(188) 51,130,131,176,179,182 $\vdash lt_{o\alpha\alpha}[\iota.\ =\ xp_\alpha].\iota.\ =\ xp$ Sub=: 187 182

(189) $\vdash \iota[=\ xp_\alpha]\ =\ xp$ Assert DESCR

(190) 51,130,131,176,179,182 $\vdash lt_{o\alpha\alpha}\ xp_\alpha\ xp$ Sub=: 188 189

(191) 51,130,131 $\vdash \sim\ lt_{o\alpha\alpha}\ xp_\alpha\ xp$ UI: xp_α 140

(192) 51,130,131,176,179,182 $\vdash \bot$ RuleP: 190 191

(193) 51,130,131,176,179 $\vdash \bot$ RuleC: 181 192

(194) 51,130,131,176,194 $\vdash \sim\ sing_{o(o\alpha)}\ X_{o\alpha}$ Case 2: 178

(195) 51 $\vdash \forall B_{o\alpha}.\ \sim\ sing_{o(o\alpha)}\ B \supset \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, X_{o\alpha}].\iota\, B]\ \bot.\ sing\, B][\mathbf{COND}\ \top\ \bot.\ sing\, B][sing\, X] = \bot$

UI: $X_{o\alpha}$ 89

(196) 51 $\vdash \sim\ sing_{o(o\alpha)}\ X_{o\alpha} \supset \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, X].\iota\, X]\ \bot.\ sing\, X][\mathbf{COND}\ \top\ \bot.\ sing\, X][sing\, X] = \bot$

UI: $X_{o\alpha}$ 195

(197) 130,131,176,194,51 $\vdash \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, X_{o\alpha}].\iota\, X]\ \bot.\ sing_{o(o\alpha)}\ X][\mathbf{COND}\ \top\ \bot.\ sing\, X][sing\, X] = \bot$

MP: 194 196

(198) 51,130,131,176,194 $\vdash \bot$ Sub=: 177 197

(199) 51,130,131,176 $\vdash \bot$ Cases: 178 193 198

(200) 51,130,131 $\vdash \sim \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, X_{o\alpha}].\iota\, X]\ \bot.\ sing_{o(o\alpha)}\ X][\mathbf{COND}\ \top\ \bot.\ sing\, X].\ sing\, X$

Indirect: 199

(201) 51,130,131 $\vdash \exists\, W_{o\alpha} \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, X_{o\alpha}].\iota\, W]\ \bot.\ sing_{o(o\alpha)}\ W][\mathbf{COND}\ \top\ \bot.\ sing\, W][sing\, X] \wedge \sim \mathbf{COND}[\mathbf{COND}[lt[\iota\, X].\iota\, X]\ \bot.\ sing\, X][\mathbf{COND}\ \top\ \bot.\ sing\, X].\ sing\, X$

Conj: 175 200

(202) 51,130,131,202 $\vdash \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, X_{o\alpha}].\iota\, Y_{o\alpha}]\ \bot.\ sing_{o(o\alpha)}\ Y][\mathbf{COND}\ \top\ \bot.\ sing\, Y].\ sing\, X$

Hyp

(203) 51,130,131,202,203 $\vdash \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, Y_{o\alpha}].\iota\, Z_{o\alpha}]\ \bot.\ sing_{o(o\alpha)}\ Z][\mathbf{COND}\ \top\ \bot.\ sing\, Z].\ sing\, Y$

Hyp

(204) 51,130,131,202,203,204 $\vdash \sim\ sing_{o(o\alpha)}\ Z_{o\alpha}$ Assume negation

(205) 51 $\vdash \forall B_{o\alpha}.\ \sim\ sing_{o(o\alpha)}\ B \supset \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, Y_{o\alpha}].\iota\, B]\ \bot.\ sing\, B][\mathbf{COND}\ \top\ \bot.\ sing\, B][sing\, Y] = \bot$

UI: $Y_{o\alpha}$ 89

(206) 51 $\vdash \sim\ sing_{o(o\alpha)}\ Z_{o\alpha} \supset \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, Y_{o\alpha}].\iota\, Z]\ \bot.\ sing\, Z][\mathbf{COND}\ \top\ \bot.\ sing\, Z][sing\, Y] = \bot$

UI: $Z_{o\alpha}$ 205

(207) 130,131,202,203,204,51 $\vdash \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, Y_{o\alpha}].\iota\, Z_{o\alpha}]\ \bot.\ sing_{o(o\alpha)}\ Z][\mathbf{COND}\ \top\ \bot.\ sing\, Z][sing\, Y] = \bot$

MP: 204 206

(208) 51,130,131,202,203,204 $\vdash \bot$ Sub=: 203 207

(209) 51,130,131,202,203 $\vdash sing_{o(o\alpha)}\ Z_{o\alpha}$ Indirect: 208

(210) 51,130,131,202,203,210 $\vdash \sim\ sing_{o(o\alpha)}\ Y_{o\alpha}$ Assume negation

(211) 51 $\vdash \forall B_{o\alpha}.\ \sim\ sing_{o(o\alpha)}\ B \supset \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, X_{o\alpha}].\iota\, B]\ \bot.\ sing\, B][\mathbf{COND}\ \top\ \bot.\ sing\, B][sing\, X] = \bot$

UI: $X_{o\alpha}$ 89

(212) $51 \vdash \sim sing_{o(o\alpha)}\, Y_{o\alpha} \supset \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, X_{o\alpha}].\iota\, Y] \perp.\, sing\, Y][\mathbf{COND} \top \perp.\, sing\, Y][sing\, X] = \perp$

UI: $Y_{o\alpha}$ 211

(213) $130,131,202,203,210,51 \vdash \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, X_{o\alpha}].\iota\, Y_{o\alpha}] \perp.\, sing_{o(o\alpha)}\, Y][\mathbf{COND} \top \perp.\, sing\, Y][sing\, X] = \perp$

MP: 210 212

(214) $51,130,131,202,203,210 \vdash \perp$ Sub=: 202 213

(215) $51,130,131,202,203 \vdash sing_{o(o\alpha)}\, Y_{o\alpha}$ Indirect: 214

(216) $51,130,131,202,203 \vdash sing_{o(o\alpha)}\, X_{o\alpha} \vee \sim sing\, X$ RuleP

(217) $51 \vdash \forall B_{o\alpha}.\, \sim sing_{o(o\alpha)}\, B \supset \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, X_{o\alpha}].\iota\, B] \perp.\, sing\, B][\mathbf{COND} \top \perp.\, sing\, B][sing\, X] = \perp$

UI: $X_{o\alpha}$ 89

(218) $51 \vdash \sim sing_{o(o\alpha)}\, Y_{o\alpha} \supset \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, X_{o\alpha}].\iota\, Y] \perp.\, sing\, Y][\mathbf{COND} \top \perp.\, sing\, Y][sing\, X] = \perp$

UI: $Y_{o\alpha}$ 217

(219) $51,130,131,202,203,219 \vdash sing_{o(o\alpha)}\, X_{o\alpha}$ Case 1: 216

(220) $130,131,202,203,219,51 \vdash [\lambda\, z_{o\alpha}\ \exists\, zp_{\alpha}.\, z = [=\ zp]]\, X_{o\alpha}$ Sub=: 219 51

(221) $130,131,202,203,219,51 \vdash \exists\, zp_{\alpha}.\, X_{o\alpha} = [=\ zp]$ Lambda: 220

(222) $51,130,131,202,203,219,222 \vdash X_{o\alpha} = [=\ xp_{\alpha}]$ Choose: xp_{α} 221

(223) $130,131,202,203,51 \vdash [\lambda\, z_{o\alpha}\ \exists\, zp_{\alpha}.\, z = [=\ zp]]\, Y_{o\alpha}$ Sub=: 215 51

(224) $130,131,202,203,51 \vdash \exists\, zp_{\alpha}.\, Y_{o\alpha} = [=\ zp]$ Lambda: 223

(225) $51,130,131,202,203,219,222,225 \vdash Y_{o\alpha} = [=\ yp_{\alpha}]$ Choose: yp_{α} 224

(226) $130,131,202,203,51 \vdash [\lambda\, z_{o\alpha}\ \exists\, zp_{\alpha}.\, z = [=\ zp]]\, Z_{o\alpha}$ Sub=: 209 51

(227) $130,131,202,203,51 \vdash \exists\, zp_{\alpha}.\, Z_{o\alpha} = [=\ zp]$ Lambda: 226

(228) $51,130,131,202,203,219,222,225,228 \vdash Z_{o\alpha} = [=\ zp_{\alpha}]$ Choose: zp_{α} 227

(229) $51 \vdash \forall B_{o\alpha}.\, sing_{o(o\alpha)}\, X_{o\alpha} \supset .\, sing\, B \supset \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, X].\iota\, B] \perp.\, sing\, B][\mathbf{COND} \top \perp.\, sing\, B][sing\, X] = lt[\iota\, X].\iota\, B$

UI: $X_{o\alpha}$ 127

(230) $51 \vdash sing_{o(o\alpha)}\, X_{o\alpha} \supset .\, sing\, Y_{o\alpha} \supset \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, X].\iota\, Y] \perp.\, sing\, Y][\mathbf{COND} \top \perp.\, sing\, Y][sing\, X] = lt[\iota\, X].\iota\, Y$

UI: $Y_{o\alpha}$ 229

(231) $130,131,202,203,219,51 \vdash sing_{o(o\alpha)}\, Y_{o\alpha} \supset \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, X_{o\alpha}].\iota\, Y] \perp.\, sing\, Y][\mathbf{COND} \top \perp.\, sing\, Y][sing\, X] = lt[\iota\, X].\iota\, Y$

MP: 219 230

(232) $130,131,202,203,219,51 \vdash \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, X_{o\alpha}].\iota\, Y_{o\alpha}] \perp.\, sing_{o(o\alpha)}\, Y][\mathbf{COND} \top \perp.\, sing\, Y][sing\, X] = lt[\iota\, X].\iota\, Y$

MP: 215 231

(233) $130,131,202,203,219,51 \vdash lt_{o\alpha\alpha}[\iota\, X_{o\alpha}].\iota\, Y_{o\alpha}$ Sub=: 202 232

(234) $51,130,131,202,203,219,222 \vdash lt_{o\alpha\alpha}[\iota.\ = xp_{\alpha}].\iota\, Y_{o\alpha}$ Sub=: 233 222

(235) $51,130,131,202,203,219,222,225 \vdash lt_{o\alpha\alpha}[\iota.\ = xp_{\alpha}].\iota.\ = yp_{\alpha}$ Sub=: 234 225

(236) $\vdash \iota[=\ xp_{\alpha}] = xp$ Assert DESCR

(237) $51,130,131,202,203,219,222,225 \vdash lt_{o\alpha\alpha}\, xp_{\alpha}.\iota.\ = yp_{\alpha}$ Sub=: 235 236

(238) $\vdash \iota[=\ yp_{\alpha}] = yp$ Assert DESCR

(239) $51,130,131,202,203,219,222,225 \vdash lt_{o\alpha\alpha}\, xp_{\alpha}\, yp_{\alpha}$ Sub=: 237 238

(240) $51 \vdash \forall B_{o\alpha}.\, sing_{o(o\alpha)}\, Y_{o\alpha} \supset .\, sing\, B \supset \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, Y].\iota\, B] \perp.\, sing\, B][\mathbf{COND} \top \perp.\, sing\, B][sing\, Y] = lt[\iota\, Y].\iota\, B$

UI: $Y_{o\alpha}$ 127

(241) $51 \vdash sing_{o(o\alpha)}\, Y_{o\alpha} \supset .\, sing\, Z_{o\alpha} \supset \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, Y].\iota\, Z] \perp.\, sing\, Z][\mathbf{COND} \top \perp.\, sing\, Z][sing\, Y] = lt[\iota\, Y].\iota\, Z$

UI: $Z_{o\alpha}$ 240

(242) 130,131,202,203,51 $\vdash sing_{o(o\alpha)}\ Z_{o\alpha} \supset \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, Y_{o\alpha}].\iota\, Z] \perp.\ sing\ Z][\mathbf{COND}\ \top \perp.\ sing\ Z][sing\ Y] = lt[\iota\, Y].\iota\, Z$

MP: 215 241

(243) 130,131,202,203,51 $\vdash \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, Y_{o\alpha}].\iota\, Z_{o\alpha}] \perp.\ sing_{o(o\alpha)}\ Z][\mathbf{COND}\ \top \perp.\ sing\ Z][sing\ Y] = lt[\iota\, Y].\iota\, Z$

MP: 209 242

(244) 130,131,202,203,51 $\vdash lt_{o\alpha\alpha}[\iota\, Y_{o\alpha}].\iota\, Z_{o\alpha}$ Sub=: 203 243

(245) 51,130,131,202,203,219,222,225 $\vdash lt_{o\alpha\alpha}[\iota.\ =\ yp_\alpha].\iota\, Z_{o\alpha}$ Sub=: 244 225

(246) 51,130,131,202,203,219,222,225,228 $\vdash lt_{o\alpha\alpha}[\iota.\ =\ yp_\alpha].\iota.\ =\ zp_\alpha$ Sub=: 245 228

(247) $\vdash \iota[=\ yp_\alpha]\ =\ yp$ Assert DESCR

(248) 51,130,131,202,203,219,222,225,228 $\vdash lt_{o\alpha\alpha}\ yp_\alpha.\iota.\ =\ zp_\alpha$ Sub=: 246 247

(249) $\vdash \iota[=\ zp_\alpha]\ =\ zp$ Assert DESCR

(250) 51,130,131,202,203,219,222,225,228 $\vdash lt_{o\alpha\alpha}\ yp_\alpha\ zp_\alpha$ Sub=: 248 249

(251) 51 $\vdash \forall B_{o\alpha}.\ sing_{o(o\alpha)}\ X_{o\alpha} \supset .\ sing\ B \supset \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, X].\iota\, B] \perp.\ sing\ B][\mathbf{COND}\ \top \perp.\ sing\ B][sing\ X] = lt[\iota\, X].\iota\, B$

UI: $X_{o\alpha}$ 127

(252) 51 $\vdash sing_{o(o\alpha)}\ X_{o\alpha} \supset .\ sing\ Z_{o\alpha} \supset \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, X].\iota\, Z] \perp.\ sing\ Z][\mathbf{COND}\ \top \perp.\ sing\ Z][sing\ X] = lt[\iota\, X].\iota\, Z$

UI: $Z_{o\alpha}$ 251

(253) 130,131,202,203,219,51 $\vdash sing_{o(o\alpha)}\ Z_{o\alpha}\ \supset\ \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, X_{o\alpha}].\iota\, Z]\ \perp.\ sing\ Z][\mathbf{COND}\ \top \perp.\ sing\ Z][sing\ X]\ =\ lt[\iota\, X].\iota\, Z$

MP: 219 252

(254) 130,131,202,203,219,51 $\vdash \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, X_{o\alpha}].\iota\, Z_{o\alpha}] \perp.\ sing_{o(o\alpha)}\ Z][\mathbf{COND}\ \top \perp.\ sing\ Z][sing\ X] = lt[\iota\, X].\iota\, Z$

MP: 209 253

(255) 51,130,131 $\vdash \forall\, y_\alpha\ \forall\, z_\alpha.\ lt_{o\alpha\alpha}\ xp_\alpha\ y \supset .\ lt\ y\ z \supset lt\ xp\ z$ UI: xp_α 143

(256) 51,130,131 $\vdash \forall\, z_\alpha.\ lt_{o\alpha\alpha}\ xp_\alpha\ yp_\alpha \supset .\ lt\ yp\ z \supset lt\ xp\ z$ UI: yp_α 255

(257) 51,130,131 $\vdash lt_{o\alpha\alpha}\ xp_\alpha\ yp_\alpha \supset .\ lt\ yp\ zp_\alpha \supset lt\ xp\ zp$ UI: zp_α 256

(258) 202,203,219,222,225,51,130,131 $\vdash lt_{o\alpha\alpha}\ yp_\alpha\ zp_\alpha \supset lt\ xp_\alpha\ zp$ MP: 239 257

(259) 51,130,131,202,203,219,222,225,228 $\vdash lt_{o\alpha\alpha}\ xp_\alpha\ zp_\alpha$ MP: 250 258

(260) $\vdash \iota[=\ zp_\alpha]\ =\ zp$ Assert DESCR

(261) 51,130,131,202,203,219,222,225,228 $\vdash lt_{o\alpha\alpha}\ xp_\alpha.\iota.\ =\ zp_\alpha$ Sub=: 259 260

(262) $\vdash \iota[=\ xp_\alpha]\ =\ xp$ Assert DESCR

(263) 51,130,131,202,203,219,222,225,228 $\vdash lt_{o\alpha\alpha}[\iota.\ =\ xp_\alpha].\iota.\ =\ zp_\alpha$ Sub=: 261 262

(264) 51,130,131,202,203,219,222,225,228 $\vdash lt_{o\alpha\alpha}[\iota.\ =\ xp_\alpha].\iota\, Z_{o\alpha}$ Sub=: 263 228

(265) 51,130,131,202,203,219,222,225,228 $\vdash lt_{o\alpha\alpha}[\iota\, X_{o\alpha}].\iota\, Z_{o\alpha}$ Sub=: 264 222

(266) 51,130,131,202,203,219,222,225,228 $\vdash \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, X_{o\alpha}].\iota\, Z_{o\alpha}]\ \perp.\ sing_{o(o\alpha)}\ Z][\mathbf{COND}\ \top \perp.\ sing\ Z].\ sing\ X$

Sub=: 265 254

(267) 51,130,131,202,203,219,222,225 $\vdash \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, X_{o\alpha}].\iota\, Z_{o\alpha}] \perp.\ sing_{o(o\alpha)}\ Z][\mathbf{COND}\ \top \perp.\ sing\ Z].\ sing\ X$

RuleC: 227 266

(268) 51,130,131,202,203,219,222 $\vdash \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, X_{o\alpha}].\iota\, Z_{o\alpha}] \perp.\ sing_{o(o\alpha)}\ Z][\mathbf{COND}\ \top \perp.\ sing\ Z].\ sing\ X$

RuleC: 224 267

(269) 51,130,131,202,203,219 $\vdash \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, X_{o\alpha}].\iota\, Z_{o\alpha}] \perp.\ sing_{o(o\alpha)}\ Z][\mathbf{COND}\ \top \perp.\ sing\ Z].\ sing\ X$

RuleC: 221 268

(270) 51,130,131,202,203,270 $\vdash \sim\ sing_{o(o\alpha)}\ X_{o\alpha}$ Case 2: 216

(271) 51 $\vdash\ \forall\, B_{o\alpha}.\ \sim\ sing_{o(o\alpha)}\ X_{o\alpha}\ \supset\ .\ sing\ B\ \supset\ \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, X].\iota\, B]\ \perp.\ sing\ B][\mathbf{COND}\ \top \perp.\ sing\ B][sing\ X]\ =\ \top$

UI: $X_{o\alpha}$ 108

(272) 51 $\vdash \sim\ sing_{o(o\alpha)}\ X_{o\alpha} \supset .\ sing\ Z_{o\alpha} \supset \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, X].\iota\, Z] \perp.\ sing\ Z][\mathbf{COND}\ \top \perp.\ sing\ Z][sing\ X] = \top$

UI: $Z_{o\alpha}$ 271

(273) 130,131,202,203,270,51 $\vdash sing_{o(o\alpha)}\, Z_{o\alpha} \supset \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, X_{o\alpha}].\iota\, Z] \perp.\, sing\, Z][\mathbf{COND}\ \top \perp.\, sing\, Z][sing\, X] = \top$

MP: 270 272

(274) 130,131,202,203,270,51 $\vdash \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, X_{o\alpha}].\iota\, Z_{o\alpha}] \perp.\, sing_{o(o\alpha)}\, Z][\mathbf{COND}\ \top \perp.\, sing\, Z][sing\, X] = \top$

MP: 209 273

(275) 51,130,131,202,203,270 $\vdash \top$ RuleP

(276) 51,130,131,202,203,270 $\vdash \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, X_{o\alpha}].\iota\, Z_{o\alpha}] \perp.\, sing_{o(o\alpha)}\, Z][\mathbf{COND}\ \top \perp.\, sing\, Z].\, sing\, X$

Sub=: 275 274

(277) 51,130,131,202,203 $\vdash \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, X_{o\alpha}].\iota\, Z_{o\alpha}] \perp.\, sing_{o(o\alpha)}\, Z][\mathbf{COND}\ \top \perp.\, sing\, Z].\, sing\, X$

Cases: 216 269 276

(278) 51,130,131,202 $\vdash \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, Y_{o\alpha}].\iota\, Z_{o\alpha}] \perp.\, sing_{o(o\alpha)}\, Z][\mathbf{COND}\ \top \perp.\, sing\, Z][sing\, Y] \supset \mathbf{COND}[\mathbf{COND}[lt[\iota\, X_{o\alpha}].\iota\, Z] \perp.\, sing\, Z][\mathbf{COND}\ \top \perp.\, sing\, Z].\, sing\, X$

Deduct: 277

(279) 51,130,131 $\vdash \mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, X_{o\alpha}].\iota\, Y_{o\alpha}] \perp.\, sing_{o(o\alpha)}\, Y][\mathbf{COND}\ \top \perp.\, sing\, Y][sing\, X] \supset .\mathbf{COND}[\mathbf{COND}[lt[\perp.\, sing\, Z][\mathbf{COND}\ \top \perp.\, sing\, Z][sing\, Y] \supset \mathbf{COND}[\mathbf{COND}[lt[\iota\, X].\iota\, Z] \perp.\, sing\, Z][\mathbf{COND}\ \top \perp.\, sing\, Z].\, sing\, X$

Deduct: 278

(280) 51,130,131 $\vdash \exists\, W_{o\alpha}\mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, X_{o\alpha}].\iota\, W] \perp.\, sing_{o(o\alpha)}\, W][\mathbf{COND}\ \top \perp.\, sing\, W][sing\, X] \wedge \sim \mathbf{COND}[\mathbf{COND}[lt[\iota\, X].\iota\, X] \perp.\, sing\, X][\mathbf{COND}\ \top \perp.\, sing\, X][sing\, X] \wedge.\mathbf{COND}[\mathbf{COND}[lt[\iota\, X].\iota\, Y_{o\alpha}] \perp.\, sing\, Y][\mathbf{COND}\ \top \perp.\, sing\, Y][sing\, X] \supset .\mathbf{COND}[\mathbf{COND}[lt[\iota\, Y].\iota\, Z_{o\alpha}] \perp.\, sing\, Z][\mathbf{COND}\ \top \perp.\, sing\, Z][sing\, Y] \supset \mathbf{COND}[\mathbf{COND}[lt[\iota\, X].\iota\, Z] \perp.\, sing\, Z][\mathbf{COND}\ \top \perp.\, sing\, Z].\, sing\, X$

Conj: 201 279

(281) 51,130,131 $\vdash \exists\, W_{o\alpha}[\lambda\, a_{o\alpha}\lambda\, b_{o\alpha}\mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, a].\iota\, b] \perp.\, sing_{o(o\alpha)}\, b][\mathbf{COND}\ \top \perp.\, sing\, b].\, sing\, a]\, X_{o\alpha}\, W \wedge \sim [\lambda\, a\lambda\, b\mathbf{COND}[\mathbf{COND}[lt[\iota\, a].\iota\, b] \perp.\, sing\, b][\mathbf{COND}\ \top \perp.\, sing\, b].\, sing\, a]\, X\, X \wedge.[\lambda\, a\lambda\, b\mathbf{COND}[\mathbf{COND}[lt[\iota\, a].\iota\, b] \perp.\, sing\, b][\mathbf{COND}\ \top \perp.\, sing\, b].\, sing\, a]\, X\, Y_{o\alpha} \supset .[\lambda\, a\lambda\, b\mathbf{COND}[\mathbf{COND}[lt[\iota\, a].\iota\, b] \perp.\, sing\, b][\mathbf{COND}\ \top \perp.\, sing\, b].\, sing\, a]\, Y\, Z_{o\alpha} \supset [\lambda\, a\lambda\, b\mathbf{COND}[\mathbf{COND}[lt[\iota\, a].\iota\, b] \perp.\, sing\, b][\mathbf{COND}\ \top \perp.\, sing\, b].\, sing\, a]\, X\, Z$

Lambda: 280

(282) 51,130,131 $\vdash \forall\, Z_{o\alpha}.\ \exists\, W_{o\alpha}[\lambda\, a_{o\alpha}\lambda\, b_{o\alpha}\mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, a].\iota\, b] \perp.\, sing_{o(o\alpha)}\, b][\mathbf{COND}\ \top \perp.\, sing\, b].\, sing\, a]\, X_{o\alpha}\, W \wedge \sim [\lambda\, a\lambda\, b\mathbf{COND}[\mathbf{COND}[lt[\iota\, a].\iota\, b] \perp.\, sing\, b][\mathbf{COND}\ \top \perp.\, sing\, b].\, sing\, a]\, X\, X \wedge.[\lambda\, a\lambda\, b\mathbf{COND}[\mathbf{COND}[lt[\iota\, a].\iota\, b] \perp.\, sing\, b][\mathbf{COND}\ \top \perp.\, sing\, b].\, sing\, a]\, X\, Y_{o\alpha} \supset .[\lambda\, a\lambda\, b\mathbf{COND}[\mathbf{COND}[lt[\iota\, a].\iota\, b] \perp.\, sing\, b][\mathbf{COND}\ \top \perp.\, sing\, b].\, sing\, a]\, Y\, Z \supset [\lambda\, a\lambda\, b\mathbf{COND}[\mathbf{COND}[lt[\iota\, a].\iota\, b] \perp.\, sing\, b][\mathbf{COND}\ \top \perp.\, sing\, b].\, sing\, a]\, X\, Z$

UGen: $Z_{o\alpha}$ 281

(283) 51,130,131 $\vdash \forall\, Y_{o\alpha}\ \forall\, Z_{o\alpha}.\ \exists\, W_{o\alpha}[\lambda\, a_{o\alpha}\lambda\, b_{o\alpha}\mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, a].\iota\, b] \perp.\, sing_{o(o\alpha)}\, b][\mathbf{COND}\ \top \perp.\, sing\, b].\, sing\, a]\, X_{o\alpha}\, W \wedge \sim [\lambda\, a\lambda\, b\mathbf{COND}[\mathbf{COND}[lt[\iota\, a].\iota\, b] \perp.\, sing\, b][\mathbf{COND}\ \top \perp.\, sing\, b].\, sing\, a]\, X\, X \wedge.[\lambda\, a\lambda\, b\mathbf{COND}[\mathbf{COND}[lt[\iota\, a].\iota\, b] \perp.\, sing\, b][\mathbf{COND}\ \top \perp.\, sing\, b].\, sing\, a]\, X\, Y \supset .[\lambda\, a\lambda\, b\mathbf{COND}[\mathbf{COND}[lt[\iota\, a].\iota\, b] \perp.\, sing\, b][\mathbf{COND}\ \top \perp.\, sing\, b].\, sing\, a]\, Y\, Z \supset [\lambda\, a\lambda\, b\mathbf{COND}[\mathbf{COND}[lt[\iota\, a].\iota\, b] \perp.\, sing\, b][\mathbf{COND}\ \top \perp.\, sing\, b].\, sing\, a]\, X\, Z$

UGen: $Y_{o\alpha}$ 282

(284) 51,130,131 $\vdash \forall\, X_{o\alpha}\ \forall\, Y_{o\alpha}\ \forall\, Z_{o\alpha}.\ \exists\, W_{o\alpha}[\lambda\, a_{o\alpha}\lambda\, b_{o\alpha}\mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, a].\iota\, b] \perp.\, sing_{o(o\alpha)}\, b][\mathbf{COND}\ \top \perp.\, sing\, b].\, sing\, a]\, X\, W \wedge \sim [\lambda\, a\lambda\, b\mathbf{COND}[\mathbf{COND}[lt[\iota\, a].\iota\, b] \perp.\, sing\, b][\mathbf{COND}\ \top \perp.\, sing\, b].\, sing\, a]\, X\, X \wedge.[\lambda\, a\lambda\, b\mathbf{COND}[\mathbf{COND}[lt[\iota\, a].\iota\, b] \perp.\, sing\, b][\mathbf{COND}\ \top \perp.\, sing\, b].\, sing\, a]\, X\, Y \supset .[\lambda\, a\lambda\, b\mathbf{COND}[\mathbf{COND}[lt[\iota\, a].\iota\, b] \perp.\, sing\, b][\mathbf{COND}\ \top \perp.\, sing\, b].\, sing\, a]\, Y\, Z \supset [\lambda\, a\lambda\, b\mathbf{COND}[\mathbf{COND}[lt[\iota\, a].\iota\, b] \perp.\, sing\, b][\mathbf{COND}\ \top \perp.\, sing\, b].\, sing\, a]\, X\, Z$

UGen: $X_{o\alpha}$ 283

(285) 51,130,131 $\vdash \exists\, R_{o(o\alpha)(o\alpha)}\ \forall\, X_{o\alpha}\ \forall\, Y_{o\alpha}\ \forall\, Z_{o\alpha}.\ \exists\, W_{o\alpha}\, R\, X\, W \wedge \sim R\, X\, X \wedge. R\, X\, Y \supset .\, R\, Y\, Z \supset R\, X\, Z$

EGen: $[\lambda\, a_{o\alpha}\lambda\, b_{o\alpha}\mathbf{COND}[\mathbf{COND}[lt_{o\alpha\alpha}[\iota\, a].\iota\, b] \perp.\, sing_{o(o\alpha)}\, b][\mathbf{COND}\top \perp.\, sing\, b].\, sing\, a]$ 284

(286) 51,130 $\vdash \exists\, R_{o(o\alpha)(o\alpha)}\ \forall\, X_{o\alpha}\ \forall\, Y_{o\alpha}\ \forall\, Z_{o\alpha}.\ \exists\, W_{o\alpha}\, R\, X\, W \wedge \sim R\, X\, X \wedge. R\, X\, Y \supset .\, R\, Y\, Z \supset R\, X\, Z$

RuleC: 130 285

(287) 51 $\vdash \exists\, r_{o\alpha\alpha}\ \forall\, x_{\alpha}\ \forall\, y_{\alpha}\ \forall\, z_{\alpha}[\ \exists\, w_{\alpha}\, r\, x\, w \wedge \sim r\, x\, x \wedge. r\, x\, y \supset .\, r\, y\, z \supset r\, x\, z] \supset \exists\, R_{o(o\alpha)(o\alpha)}\ \forall\, X_{o\alpha}\ \forall\, Y_{o\alpha}\ \forall\, Z_{o\alpha}.\ \exists\, W_{o\alpha}\, R\, X\, W \wedge \sim R\, X\, X \wedge. R\, X\, Y \supset .\, R\, Y\, Z \supset R\, X\, Z$

Deduct: 286

(288) $\vdash \exists\, r_{o\alpha\alpha}\ \forall\, x_{\alpha}\ \forall\, y_{\alpha}\ \forall\, z_{\alpha}[\ \exists\, w_{\alpha}\, r\, x\, w \wedge \sim r\, x\, x \wedge. r\, x\, y \supset .\, r\, y\, z \supset r\, x\, z] \supset \exists\, R_{o(o\alpha)(o\alpha)}\ \forall\, X_{o\alpha}\ \forall\, Y_{o\alpha}\ \forall\, Z_{o\alpha}.\ \exists\, W_{o\alpha}\, R\, X\, W \wedge \sim R\, X\, X \wedge. R\, X\, Y \supset .\, R\, Y\, Z \supset R\, X\, Z$

RuleC: 50 287