15–399: Recitation - Week 1

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January 19, 2005

1 Theorems in constructive logic

It is instructive to try to decide whether a given theorem is provable or not in constructive logic *without using inference rules or by exploiting other well-known theorems.* For each theorem given below, check whether it is provable or not.

1. `A => (B => C) <=> A & B => C` (from assignment 1)

   True. This theorem implies that proving $A_1 \supset (A_2 \supset \cdots (A_n \supset B) \cdots)$ *true* is tantamount to proving $(A_1 \wedge A_2 \wedge \cdots \wedge A_n) \supset B$ *true*. I think this rule is worth remembering.

2. `(A | B) => C <=> (A => C) & (B => C)` (from assignment 1)

   True.

3. `A => (B => A)`

   True. Use the theorem 1. This theorem implies that A *true* is stronger (*i.e.*, more difficult to prove and more informative) than $B \supset A$ *true*. Therefore, if your proof generates two hypotheses A *true* and $B \supset A$ *true* at some point, there is no need to use $B \supset A$ *true* afterwards. I think this rule is worth remembering; you might find it useful in building a big complicated proof.

4. `((A => B) | (A => C)) => (A => (B | C))`

   True. The lesson here is that if a disjuction `A | B` appears as a hypothesis, there is no other option than trying both branches, and that if it appears as a conclusion that we want to prove, you have to make a guess on which of `A` and `B` is true. Please try to prove it using Tutch and see what happens.

5. `((A => B) => C) => ((A | B) & (B => C))`

   False. The culprit is `A | B`. As it appears as a conclusion, you have to make a guess on which of `A` and `B` is true. Whichever you try, however, you can't prove it.

6. `((A => B) => C) <= ((A | B) & (B => C))`

   True. As `A | B` appears as a hypothesis, you have to try both branches (`A` and `B`); there is no other option.

7. `(A => B) => (B => C) => (C => D) => (A => D)`

   True.

8. `((A => B) => A) => A`

   False. See Assignment 2.

2 Proposition and judgment

Consider a statement 'the moon is made of cheese.' The statement is not yet an object of verification, or a proposition, since there is no way to check its truth. It becomes a proposition when an inference rule is given, for example, (written in a pedantic way) "'the moon is made of cheese' is true if 'the moon is greenish white and has holes in it' is true." Now we can attempt to verify the proposition, for example, by taking a picture of the moon. That is, we still do not know whether the proposition is true or not, but by virtue of the inference rule, we know at least what counts as a verification of it. If the picture indeed shows that the moon is greenish white and has holes in it, the inference rule makes evident the judgment "'the moon is made of cheese' is true." Now we know "'the moon is made of cheese' is true" by the proof consisting of the picture and the inference rule. Thus a proposition is an object of verification which may or may not be true, whereas a judgment is an object of knowledge which we either know or do not know.