10707
Deep Learning

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Sparse Coding

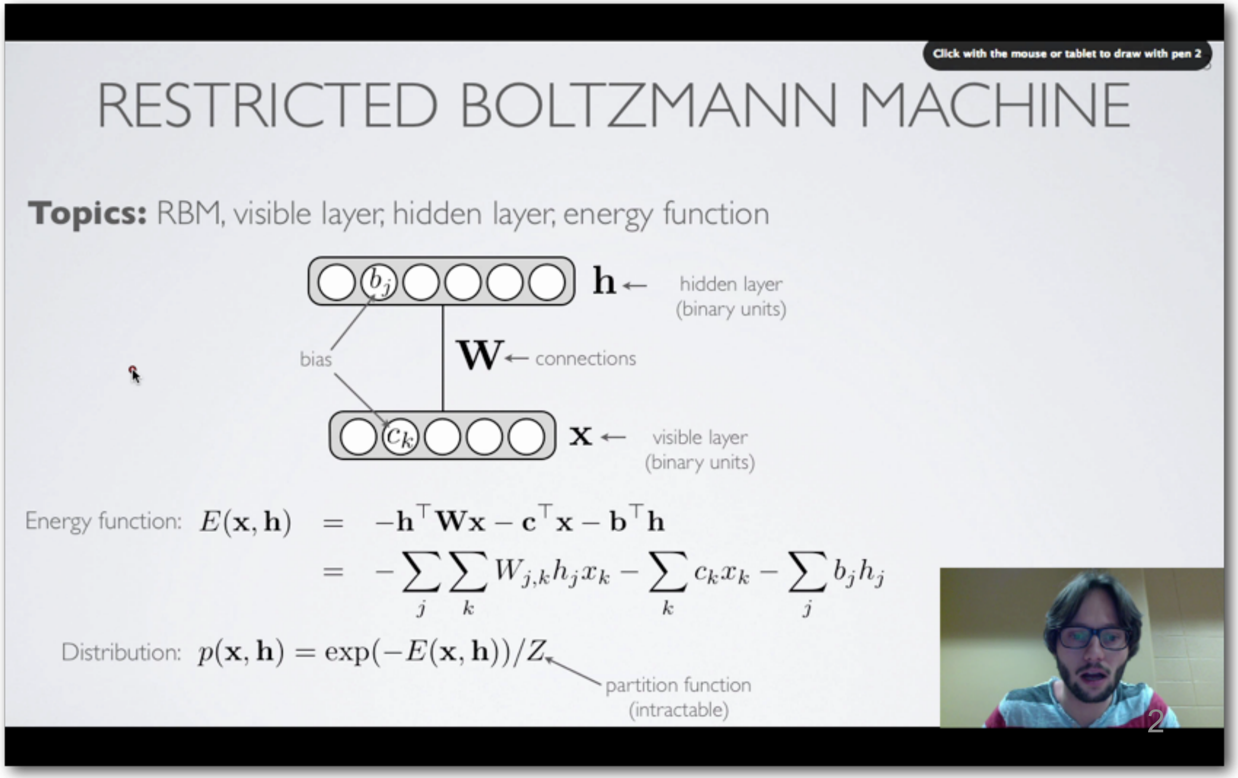
Neural Networks Online Course

- **Disclaimer**: Much of the material and slides for this lecture were borrowed from Hugo Larochelle's class on Neural Networks:

- Hugo's class covers many other topics: convolutional networks, neural language model, Boltzmann machines, autoencoders, sparse coding, etc.

- We will use his material for some of the other lectures.

http://info.usherbrooke.ca/hlarochelle/neural_networks



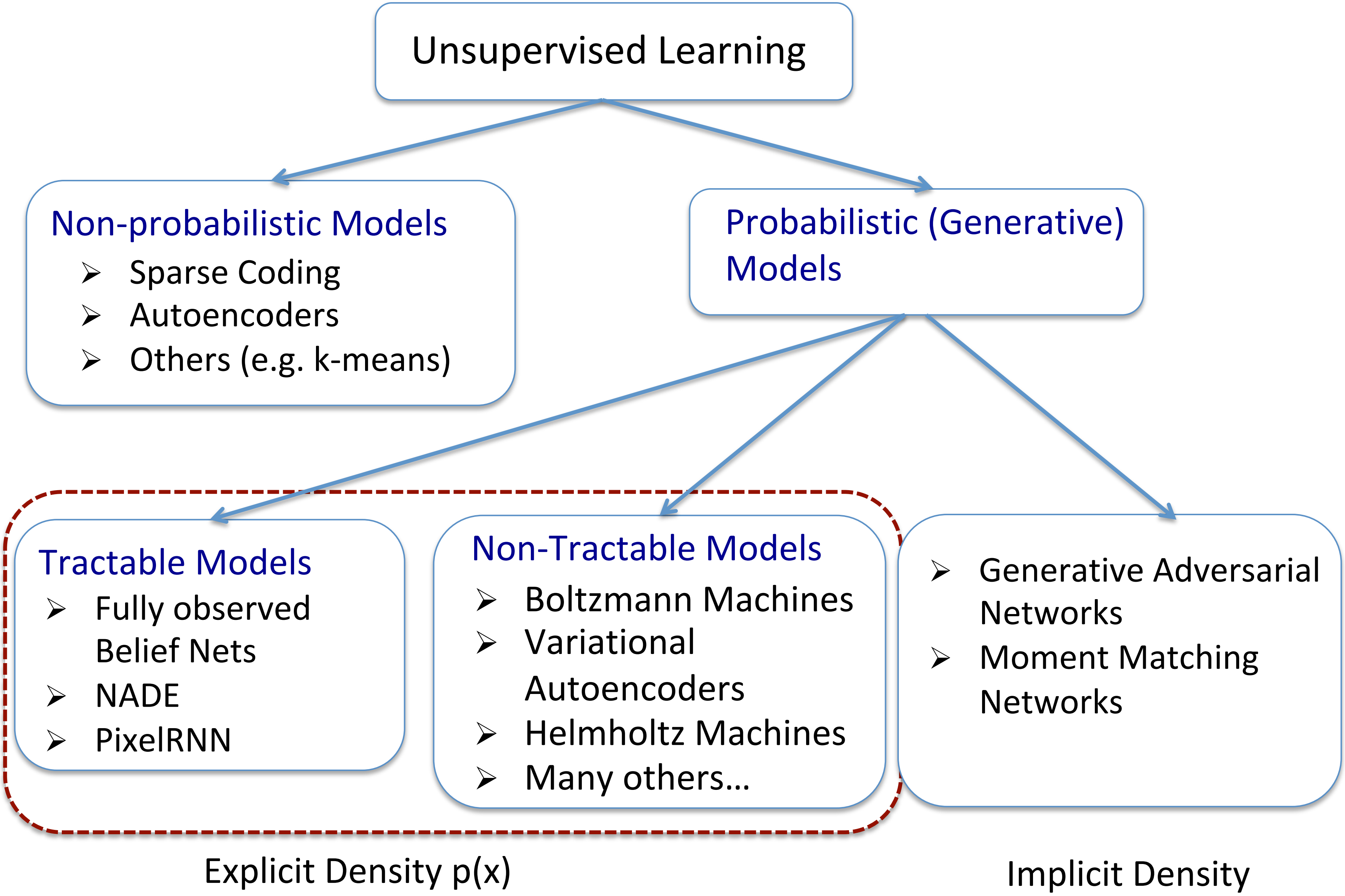
RESTRICTED BOLTZMANN MACHINE

Topics: RBM, visible layer, hidden layer, energy function

Energy function:
$$\begin{aligned} E(\mathbf{x},\mathbf{h}) &= -\mathbf{h}^\top \mathbf{W}\mathbf{x} - \mathbf{c}^\top \mathbf{x} - \mathbf{b}^\top \mathbf{h} \\ &= -\sum_j \sum_k W_{j,k} h_j x_k - \sum_k c_k x_k - \sum_j b_j h_j \end{aligned}$$

Distribution: $p(\mathbf{x},\mathbf{h}) = \exp(-E(\mathbf{x},\mathbf{h}))/Z$

partition function (intractable)



Unsupervised Learning

Non-probabilistic Models
- Sparse Coding
- Autoencoders
- Others (e.g. k-means)

Probabilistic (Generative) Models

Tractable Models
- Fully observed Belief Nets
- NADE
- PixelRNN

Non-Tractable Models
- Boltzmann Machines
- Variational Autoencoders
- Helmholtz Machines
- Many others…

- Generative Adversarial Networks
- Moment Matching Networks

Explicit Density p(x)

Implicit Density

Unsupervised Learning

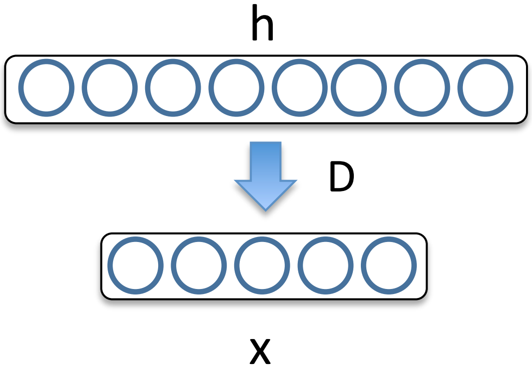
- Unsupervised learning: we only use the inputs $\mathbf{x}^{(t)}$ for learning
  - automatically extract meaningful features for your data
  - leverage the availability of unlabeled data
  - add a data-dependent regularizer to training ($-\log p(\mathbf{x}^{(t)})$)

- We will consider 3 models for unsupervised learning that will form the basic building blocks for deeper models:
  - Restricted Boltzmann Machines
  - Autoencoders
  - Sparse coding models

Sparse Coding

- Sparse coding (Olshausen & Field, 1996). Originally developed to explain early visual processing in the brain (edge detection).

- For each input $\mathbf{x}^{(t)}$ find a latent representation $\mathbf{h}^{(t)}$ such that:
  - it is sparse: the vector $\mathbf{h}^{(t)}$ has many zeros
  - we can good reconstruct the original input $\mathbf{x}^{(t)}$



Sparse Coding

- For each $\mathbf{x}^{(t)}$ find a latent representation $\mathbf{h}^{(t)}$ such that:
    - it is sparse: the vector $\mathbf{h}^{(t)}$ has many zeros
    - we can good reconstruct the original input $\mathbf{x}^{(t)}$

- In other words:

Reconstruction: $\widehat{\mathbf{x}}^{(t)}$

Sparsity vs.
reconstruction control

$$\min_{\mathbf{D}} \frac{1}{T} \sum_{t=1}^{T} \underbrace{\min_{\mathbf{h}^{(t)}} \frac{1}{2} ||\mathbf{x}^{(t)} - \mathbf{D}\ \mathbf{h}^{(t)}||_2^2}_{\text{Reconstruction error}} + \underbrace{\lambda ||\mathbf{h}^{(t)}||_1}_{\text{Sparsity penalty}}$$

Sparse Coding

- For each $\mathbf{x}^{(t)}$ find a latent representation $\mathbf{h}^{(t)}$ such that:
  - it is sparse: the vector $\mathbf{h}^{(t)}$ has many zeros
  - we can good reconstruct the original input $\mathbf{x}^{(t)}$

- In other words:

$$\min_{\mathbf{D}} \frac{1}{T} \sum_{t=1}^{T} \min_{\mathbf{h}^{(t)}} \frac{1}{2} ||\mathbf{x}^{(t)} - \mathbf{D}\,\mathbf{h}^{(t)}||_2^2 + \lambda ||\mathbf{h}^{(t)}||_1$$

  - we also constrain the columns of D to be of norm 1
  - otherwise, D could grow big while h becomes small to satisfy the L1 constraint

Sparse Coding

- For each $\mathbf{x}^{(t)}$ find a latent representation $\mathbf{h}^{(t)}$such that:
  - it is sparse: the vector $\mathbf{h}^{(t)}$ has many zeros
  - we can good reconstruct the original input $\mathbf{x}^{(t)}$

- In other words:

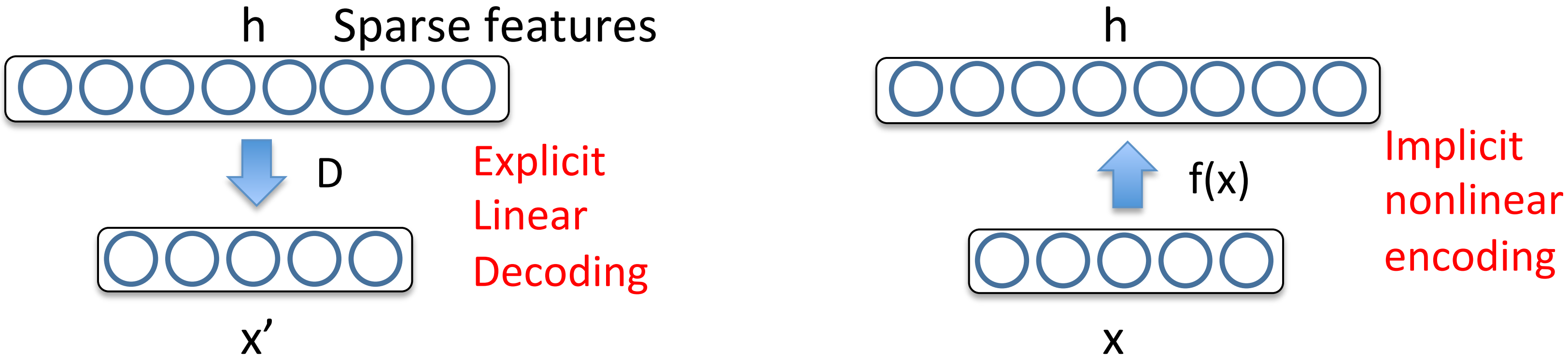
$$\min_{\mathbf{D}} \frac{1}{T} \sum_{t=1}^{T} \min_{\mathbf{h}^{(t)}} \frac{1}{2} ||\mathbf{x}^{(t)} - \mathbf{D}\ \mathbf{h}^{(t)}||_2^2 + \lambda ||\mathbf{h}^{(t)}||_1$$

  - D is equivalent to the autoencoder output weight matrix
  - However, $\mathbf{h}(\mathbf{x}^{(t)})$ is now a complicated function of $\mathbf{x}^{(t)}$
  - Encoder is the minimization problem:

$$\mathbf{h}(\mathbf{x}^{(t)}) = \arg\min_{\mathbf{h}^{(t)}} \frac{1}{2} ||\mathbf{x}^{(t)} - \mathbf{D}\ \mathbf{h}^{(t)}||_2^2 + \lambda ||\mathbf{h}^{(t)}||_1$$

Interpreting Sparse Coding

$$\min_{\mathbf{D}} \frac{1}{T} \sum_{t=1}^{T} \min_{\mathbf{h}^{(t)}} \frac{1}{2} ||\mathbf{x}^{(t)} - \mathbf{D}\,\mathbf{h}^{(t)}||_2^2 + \lambda ||\mathbf{h}^{(t)}||_1$$

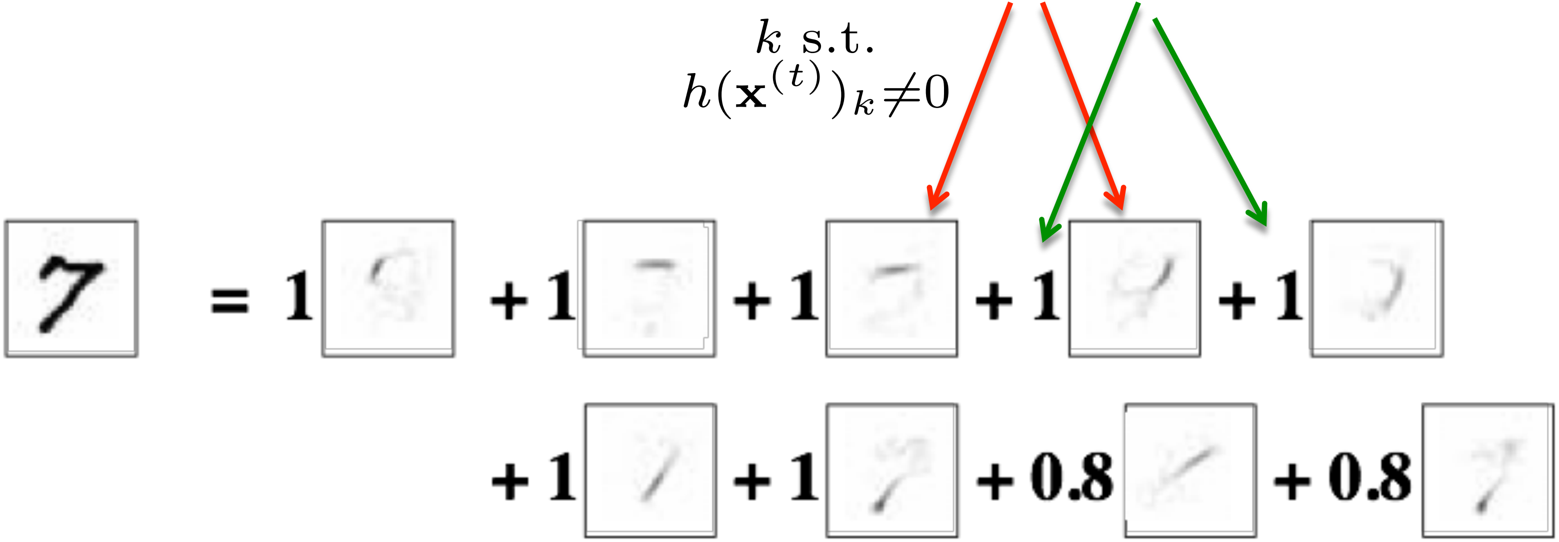


- Sparse, over-complete representation **h.**
- Encoding **h** = f(**x**) is implicit and nonlinear function of **x**.
- Reconstruction (or decoding) **x'** = Dh is linear and explicit.

Sparse Coding

- We can also write:

$$\widehat{\mathbf{x}}^{(t)} = \mathbf{D}\,\mathbf{h}(\mathbf{x}^{(t)}) = \sum_{\substack{k \text{ s.t.} \\ h(\mathbf{x}^{(t)})_k \neq 0}} \mathbf{D}_{\cdot,k}\, h(\mathbf{x}^{(t)})_k$$

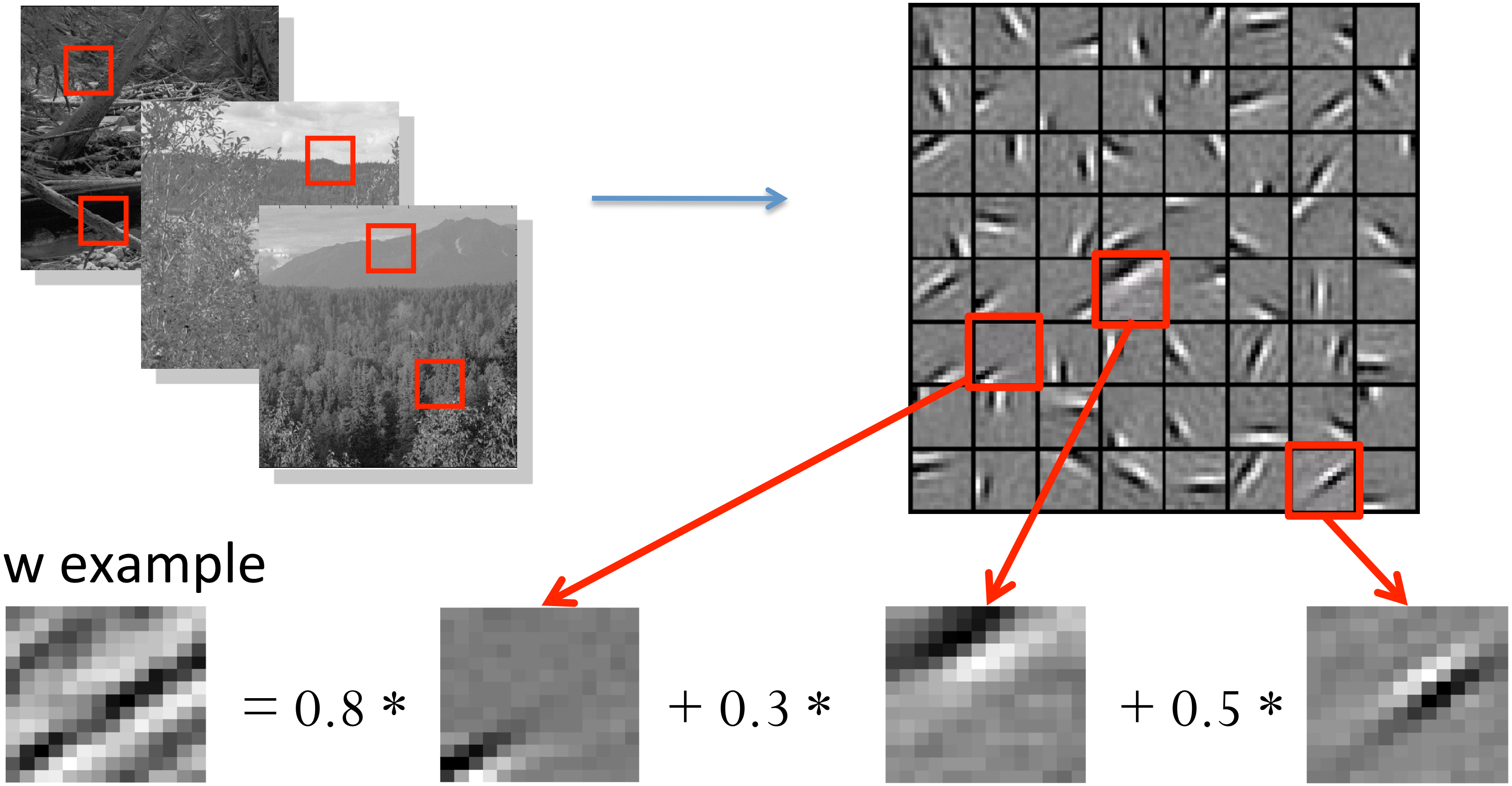


- D is often referred to as Dictionary
- In certain applications, we know what dictionary matrix to use
- In many cases, we have to learn it

Sparse Coding

Natural Images

Learned bases: “Edges”



New example

$$= 0.8 * \quad + 0.3 * \quad + 0.5 *$$

$$x = 0.8 * \phi_{36} + 0.3 * \phi_{42} + 0.5 * \phi_{65}$$

[0, 0, … **0.8**, …, **0.3**, …, **0.5**, …] = coefficients (feature representation)

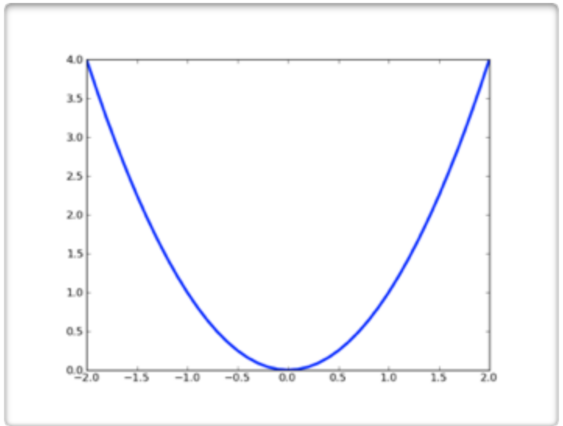
Slide Credit: Honglak Lee

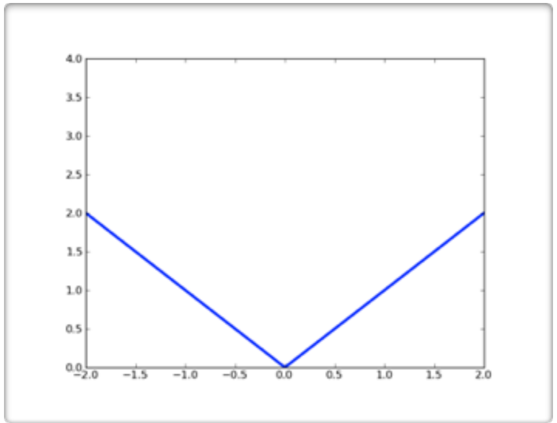
Inference

- Given dictionary D , how do we compute $\mathbf{h}(\mathbf{x}^{(t)})$?
  - ➢ We need to optimize:

$$l(\mathbf{x}^{(t)}) = \frac{1}{2}||\mathbf{x}^{(t)} - \mathbf{D}\,\mathbf{h}^{(t)}||_2^2 + \lambda||\mathbf{h}^{(t)}||_1$$

  - ➢ This is Lasso.





  - ➢ We could use a gradient descent method:

$$\nabla_{\mathbf{h}^{(t)}} l(\mathbf{x}^{(t)}) = \mathbf{D}^\top(\mathbf{D}\,\mathbf{h}^{(t)} - \mathbf{x}^{(t)}) + \lambda\,\mathrm{sign}(\mathbf{h}^{(t)})$$

Inference

- For a single hidden unit:

$$\frac{\partial}{\partial h_k^{(t)}} l(\mathbf{x}^{(t)}) = (\mathbf{D}_{\cdot,k})^\top (\mathbf{D}\,\mathbf{h}^{(t)} - \mathbf{x}^{(t)}) + \lambda\,\mathrm{sign}(h_k^{(t)})$$

  - issue: L1 norm not differentiable at 0
  - very unlikely for gradient descent to “land” on $h_k^{(t)} = 0$ (even if it’s the solution)

- Solution: if $h_k^{(t)}$ changes sign because of L1 norm gradient, clamp to 0.

Inference

- For a single hidden unit:

$$\frac{\partial}{\partial h_k^{(t)}} l(\mathbf{x}^{(t)}) = (\mathbf{D}_{\cdot,k})^\top (\mathbf{D}\ \mathbf{h}^{(t)} - \mathbf{x}^{(t)}) + \lambda\, \mathrm{sign}(h_k^{(t)})$$

- Solution: if $h_k^{(t)}$ changes sign because of L1 norm gradient, clamp to 0.

- Each hidden unit update would be performed as follows:
  - ➢ $h_k^{(t)} \Longleftarrow h_k^{(t)} - \alpha (\mathbf{D}_{\cdot,k})^\top (\mathbf{D}\ \mathbf{h}^{(t)} - \mathbf{x}^{(t)})$
  - ➢ If $\mathrm{sign}(h_k^{(t)}) \neq \mathrm{sign}(h_k^{(t)} - \alpha\, \lambda\, \mathrm{sign}(h_k^{(t)}))$ then $h_k^{(t)} \Longleftarrow 0$
  - ➢ Else $h_k^{(t)} \Longleftarrow h_k^{(t)} - \alpha\, \lambda\, \mathrm{sign}(h_k^{(t)})$

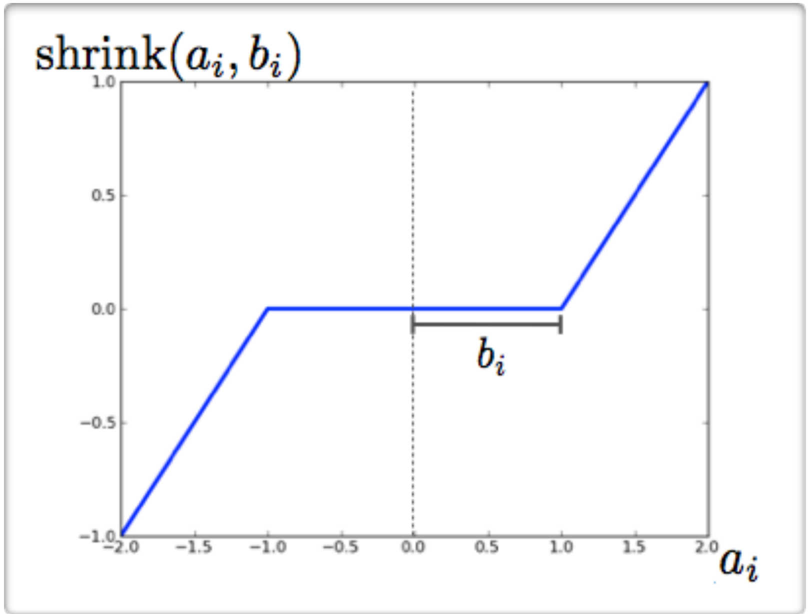
Update from reconstruction

Update sparsity term

ISTA Algorithm

- This process corresponds to the ISTA (Iterative Shrinkage and Thresholding) Algorithm:

```
➢ Initialize $\mathbf{h}^{(t)}$ (for example to 0)
➢ While $\mathbf{h}^{(t)}$ has not converged
    $\mathbf{h}^{(t)} \Longleftarrow \mathbf{h}^{(t)} - \alpha\,\mathbf{D}^\top(\mathbf{D}\,\mathbf{h}^{(t)} - \mathbf{x}^{(t)})$
    $\mathbf{h}^{(t)} \Longleftarrow \mathrm{shrink}(\mathbf{h}^{(t)}, \alpha\,\lambda)$
```



where

$$\mathrm{shrink}(\mathbf{a}, \mathbf{b}) = [\ldots, \mathrm{sign}(a_i)\ \max(|a_i| - b_i, 0), \ldots]$$

- Will converge if $\frac{1}{\alpha}$ is bigger than the largest eigenvalue of $\mathbf{D}^\top\mathbf{D}$

ISTA Algorithm

- ISTA updates all hidden units simultaneously
  - this is wasteful if many hidden units have already converged

- Idea: update only the “most promising” hidden unit
  - see coordinate descent algorithm in Learning Fast Approximations of Sparse Coding (Gregor and Lecun, 2010).
  - this algorithm has the advantage of not requiring a learning rate α

Dictionary Learning I

- Remember our optimization problem:

$$\min_{\mathbf{D}} \frac{1}{T} \sum_{t=1}^{T} \min_{\mathbf{h}^{(t)}} l(\mathbf{x}^{(t)}) = \min_{\mathbf{D}} \frac{1}{T} \sum_{t=1}^{T} \frac{1}{2} ||\mathbf{x}^{(t)} - \mathbf{D}\, \mathbf{h}(\mathbf{x}^{(t)})||_2^2 + \lambda ||\mathbf{h}(\mathbf{x}^{(t)})||_1$$

- Let us first assume that $\mathbf{h}(\mathbf{x}^{(t)})$ does not depend on D
  - ➢ We then minimize:

$$\min_{\mathbf{D}} \frac{1}{T} \sum_{t=1}^{T} \frac{1}{2} ||\mathbf{x}^{(t)} - \mathbf{D}\, \mathbf{h}(\mathbf{x}^{(t)})||_2^2$$

  - ➢ we must also constrain the columns of D to be of unit norm

Dictionary Learning I

- We can use projected gradient descent algorithm.

- While D has not converged:
  - Perform gradient update of D

$$\mathbf{D} \Longleftarrow \mathbf{D} + \alpha \frac{1}{T} \sum_{t=1}^{T} (\mathbf{x}^{(t)} - \mathbf{D}\,\mathbf{h}(\mathbf{x}^{(t)}))\,\mathbf{h}(\mathbf{x}^{(t)})^{\top}$$

  - Renormalize the columns of D
  - For each column of D:

$$\mathbf{D}_{\cdot,j} \Longleftarrow \frac{\mathbf{D}_{\cdot,j}}{||\mathbf{D}_{\cdot,j}||_2}$$

Dictionary Learning II

- An alternative method is to solve for each column $\mathbf{D}_{\cdot,j}$ in cycle.
  - ➢ setting the gradient for $\mathbf{D}_{\cdot,j}$ to zero, we have

$$\begin{aligned}
0 &= \frac{1}{T}\sum_{t=1}^{T}(\mathbf{x}^{(t)} - \mathbf{D}\,\mathbf{h}(\mathbf{x}^{(t)}))\,h(\mathbf{x}^{(t)})_j \\
0 &= \sum_{t=1}^{T}\left(\mathbf{x}^{(t)} - \left(\sum_{i\neq j}\mathbf{D}_{\cdot,i}\,h(\mathbf{x}^{(t)})_i\right) - \mathbf{D}_{\cdot,j}\,h(\mathbf{x}^{(t)})_j\right)\,h(\mathbf{x}^{(t)})_j \\
\sum_{t=1}^{T}\mathbf{D}_{\cdot,j}h(\mathbf{x}^{(t)})_j^2 &= \sum_{t=1}^{T}\left(\mathbf{x}^{(t)} - \left(\sum_{i\neq j}\mathbf{D}_{\cdot,i}\,h(\mathbf{x}^{(t)})_i\right)\right)\,h(\mathbf{x}^{(t)})_j \\
\mathbf{D}_{\cdot,j} &= \frac{1}{\sum_{t=1}^{T}h(\mathbf{x}^{(t)})_j^2}\sum_{t=1}^{T}\left(\mathbf{x}^{(t)} - \left(\sum_{i\neq j}\mathbf{D}_{\cdot,i}\,h(\mathbf{x}^{(t)})_i\right)\right)\,h(\mathbf{x}^{(t)})_j
\end{aligned}$$

  - ➢ Note that we don’t need to specify a learning rate to update D.

Dictionary Learning II

- An alternative method is to solve for each column $\mathbf{D}_{\cdot,j}$ in cycle.
  - We can rewrite

$$\begin{aligned}\mathbf{D}_{\cdot,j} &= \frac{1}{\sum_{t=1}^{T} h(\mathbf{x}^{(t)})_j^2} \sum_{t=1}^{T} \left(\mathbf{x}^{(t)} - \left(\sum_{i \neq j} \mathbf{D}_{\cdot,i}\, h(\mathbf{x}^{(t)})_i \right) \right) h(\mathbf{x}^{(t)})_j \\ &= \frac{1}{\sum_{t=1}^{T} h(\mathbf{x}^{(t)})_j^2} \left(\left(\sum_{t=1}^{T} \mathbf{x}^{(t)} h(\mathbf{x}^{(t)})_j \right) - \sum_{i \neq j} \mathbf{D}_{\cdot,i} \left(\sum_{t=1}^{T} h(\mathbf{x}^{(t)})_i h(\mathbf{x}^{(t)})_j \right) \right) \\ &= \frac{1}{A_{j,j}} (\mathbf{B}_{\cdot,j} - \mathbf{D}\ \mathbf{A}_{\cdot,j} + \mathbf{D}_{\cdot,j} A_{j,j})\end{aligned}$$

  - this way, we only need to store:

$$\mathbf{A} \Longleftarrow \textstyle\sum_{t=1}^{T} \mathbf{h}(\mathbf{x}^{(t)})\ \mathbf{h}(\mathbf{x}^{(t)})^\top$$

$$\mathbf{B} \Longleftarrow \textstyle\sum_{t=1}^{T} \mathbf{x}^{(t)}\ \mathbf{h}(\mathbf{x}^{(t)})^\top$$

Dictionary Learning II

- This leads to the following algorithm

```
• While D has not converged:
    ➢ for each column $\mathbf{D}_{\cdot,j}$ perform updates
        $\mathbf{D}_{\cdot,j} \Longleftarrow \frac{1}{A_{j,j}} (\mathbf{B}_{\cdot,j} - \mathbf{D}\ \mathbf{A}_{\cdot,j} + \mathbf{D}_{\cdot,j}\ A_{j,j})$
        $\mathbf{D}_{\cdot,j} \Longleftarrow \frac{\mathbf{D}_{\cdot,j}}{||\mathbf{D}_{\cdot,j}||_2}$
```

- This is referred to as a block-coordinate descent algorithm
  - a different block of variables are updated at each step
  - the “blocks” are the columns $\mathbf{D}_{\cdot,j}$

Learning Sparse Coding Model

- Putting it all together, we have the following algorithm, where learning alternates between inference and dictionary learning.

- While D has not converged:
  - find the sparse codes $\mathbf{h}(\mathbf{x}^{(t)})$ for all $\mathbf{x}^{(t)}$ in the training set with ISTA
  - Update the dictionary:

    $$\mathbf{A} \Longleftarrow \sum_{t=1}^{T} \mathbf{x}^{(t)}\ \mathbf{h}(\mathbf{x}^{(t)})^{\top}$$

    $$\mathbf{B} \Longleftarrow \sum_{t=1}^{T} \mathbf{h}(\mathbf{x}^{(t)})\ \mathbf{h}(\mathbf{x}^{(t)})^{\top}$$
  - run block-coordinate descent algorithm to update D

- Similar in spirit to EM algorithm

Online Learning

- This algorithm is “batch” (i.e. not online)
  - ➢ single update of the dictionary per pass on the training set
  - ➢ for large datasets, we’d like to update D after visiting each $\mathbf{x}^{(t)}$

- Solution: for each input $\mathbf{x}^{(t)}$
  - ➢ perform inference of $\mathbf{h}(\mathbf{x}^{(t)})$ for the current input
  - ➢ update running averages of the quantities required to update D:

$$\mathbf{B} \Longleftarrow \beta\, \mathbf{B} + (1-\beta)\, \mathbf{x}^{(t)}\ \mathbf{h}(\mathbf{x}^{(t)})^\top$$
$$\mathbf{A} \Longleftarrow \beta\ \mathbf{A} + (1-\beta)\ \mathbf{h}(\mathbf{x}^{(t)})\ \mathbf{h}(\mathbf{x}^{(t)})^\top$$

  - ➢ use current value of D as “warm start” to block-coordinate descent

Online Learning

- While D has not converged:
  - For each $\mathbf{x}^{(t)}$
    - Infer code $\mathbf{h}(\mathbf{x}^{(t)})$
    - Update the dictionary:

$$\mathbf{A} \Longleftarrow \beta\, \mathbf{A} + (1-\beta)\, \mathbf{h}(\mathbf{x}^{(T+1)})\, \mathbf{h}(\mathbf{x}^{(T+1)})^{\top}$$

$$\mathbf{B} \Longleftarrow \beta\, \mathbf{B} + (1-\beta)\, \mathbf{x}^{(t)}\, \mathbf{h}(\mathbf{x}^{(t)})^{\top}$$

    - while D hasn't converged
      - for each column of D perform gradient update

$$\mathbf{D}_{\cdot,j} \Longleftarrow \frac{1}{A_{j,j}} (\mathbf{B}_{\cdot,j} - \mathbf{D}\, \mathbf{A}_{\cdot,j} + \mathbf{D}_{\cdot,j}\, A_{j,j})$$

$$\mathbf{D}_{\cdot,j} \Longleftarrow \frac{\mathbf{D}_{\cdot,j}}{||\mathbf{D}_{\cdot,j}||_2}$$

Online Dictionary Learning for Sparse Coding.
Mairal, Bach, Ponce and Sapiro, 2009.

ZCA Preprocessing

- Before running a sparse coding algorithm, it is beneficial to remove "obvious" structure from the data
  - normalize such that mean is 0 and covariance is the identity (whitening)
  - this will remove 1st and 2nd order statistical structure

- ZCA preprocessing
  - let the empirical mean be $\widehat{\boldsymbol{\mu}}$ and the empirical covariance matrix be $\widehat{\Sigma} = \mathbf{U}\Lambda\mathbf{U}^\top$(in its eigenvalue/eigenvector representation)
  - ZCA transforms each input as follows:

$$\mathbf{x} \Longleftarrow \mathbf{U}\,\Lambda^{-\frac{1}{2}}\,\mathbf{U}^\top(\mathbf{x} - \widehat{\boldsymbol{\mu}})$$

ZCA Preprocessing

- After this transformation
    - ➢ the empirical mean is 0

$$
\begin{aligned}
& \frac{1}{T}\sum_t \mathbf{U}\,\Lambda^{-\frac{1}{2}}\,\mathbf{U}^\top(\mathbf{x}^{(t)} - \widehat{\boldsymbol{\mu}}) \\
= \quad & \mathbf{U}\,\Lambda^{-\frac{1}{2}}\,\mathbf{U}^\top \left(\left(\frac{1}{T}\sum_t \mathbf{x}^{(t)} \right) - \widehat{\boldsymbol{\mu}} \right) \\
= \quad & \mathbf{U}\,\Lambda^{-\frac{1}{2}}\,\mathbf{U}^\top \left(\widehat{\boldsymbol{\mu}} - \widehat{\boldsymbol{\mu}} \right) \\
= \quad & 0
\end{aligned}
$$

ZCA Preprocessing

- After this transformation
  - the empirical covariance matrix is the identity

$$
\begin{aligned}
&\frac{1}{T-1}\sum_t \left(\mathbf{U}\,\Lambda^{-\frac{1}{2}}\,\mathbf{U}^\top(\mathbf{x}^{(t)}-\widehat{\boldsymbol{\mu}})\right)\left(\mathbf{U}\,\Lambda^{-\frac{1}{2}}\,\mathbf{U}^\top(\mathbf{x}^{(t)}-\widehat{\boldsymbol{\mu}})\right)^\top \\
=\quad & \mathbf{U}\,\Lambda^{-\frac{1}{2}}\,\mathbf{U}^\top\left(\frac{1}{T-1}\sum_t(\mathbf{x}^{(t)}-\widehat{\boldsymbol{\mu}})(\mathbf{x}^{(t)}-\widehat{\boldsymbol{\mu}})\right)^\top\mathbf{U}\,\Lambda^{-\frac{1}{2}}\,\mathbf{U}^\top \\
=\quad & \mathbf{U}\,\Lambda^{-\frac{1}{2}}\,\mathbf{U}^\top\,\widehat{\Sigma}\,\mathbf{U}\Lambda^{-\frac{1}{2}}\,\mathbf{U}^\top \\
=\quad & \mathbf{U}\,\Lambda^{-\frac{1}{2}}\,\mathbf{U}^\top\,\mathbf{U}\,\Lambda\,\mathbf{U}^\top\,\mathbf{U}\,\Lambda^{-\frac{1}{2}}\,\mathbf{U}^\top \\
=\quad & \mathrm{I}
\end{aligned}
$$

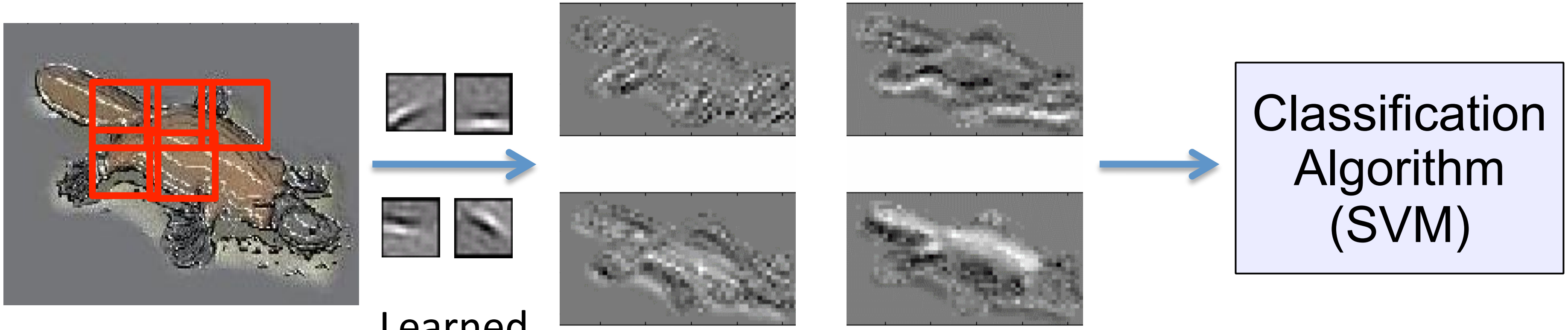
Feature Learning

- A sparse coding model can be used to extract features
  - given a labeled training set $\{(\mathbf{x}^{(t)}, y^{(t)})\}$
  - train sparse coding dictionary only on training inputs $\{\mathbf{x}^{(t)}\}$
  - this yields a dictionary $\mathbf{h}(\mathbf{x}^{(t)})$ from which to infer sparse codes
  - train your favorite classifier on transformed training set $\{(\mathbf{h}(\mathbf{x}^{(t)}), y^{(t)})\}$

- When classifying test input x
  - infer its sparse representation: $\mathbf{h}(\mathbf{x})$
  - feed it to the classifier

Image Classification

Evaluated on Caltech101 object category dataset.



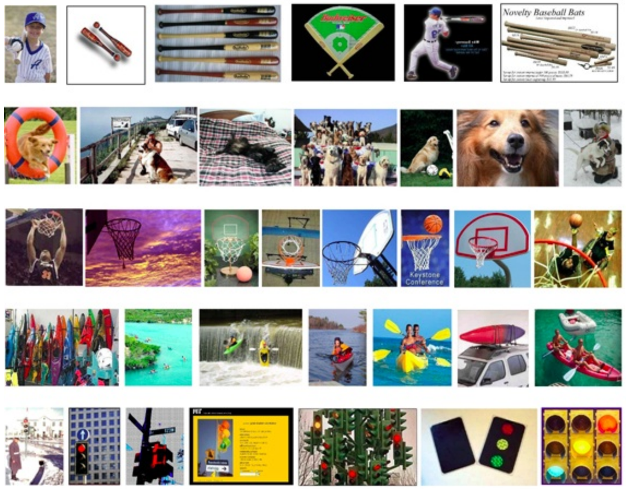
Classification Algorithm (SVM)

Input Image

Learned bases

Features (coefficients)

9K images, 101 classes

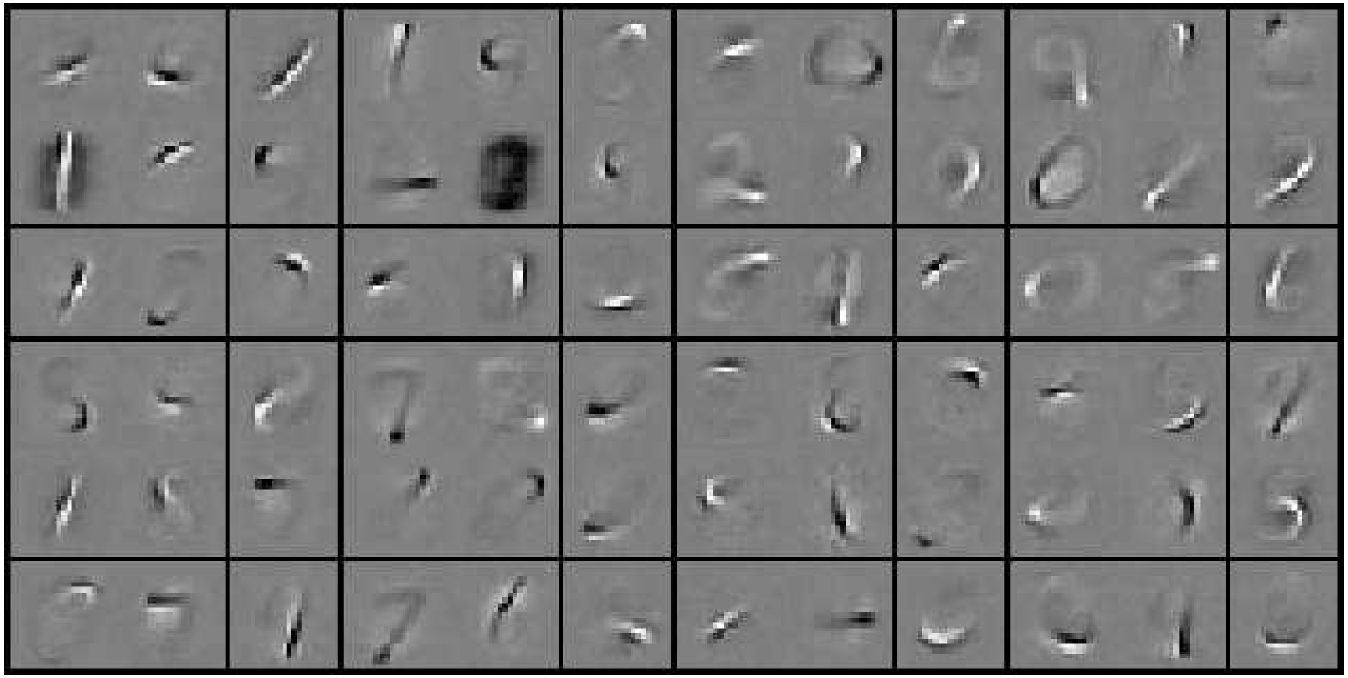


Algorithm	Accuracy
Baseline (Fei-Fei et al., 2004)	16%
PCA	37%
Sparse Coding	**47%**

Lee et al., NIPS 2006

Feature Learning

- Learned features on MNIST handwritten digits:



Self-taught Learning: Transfer Learning from
Unlabeled DataRaina, Battle, Lee, Packer and Ng.

Self-Taught Learning

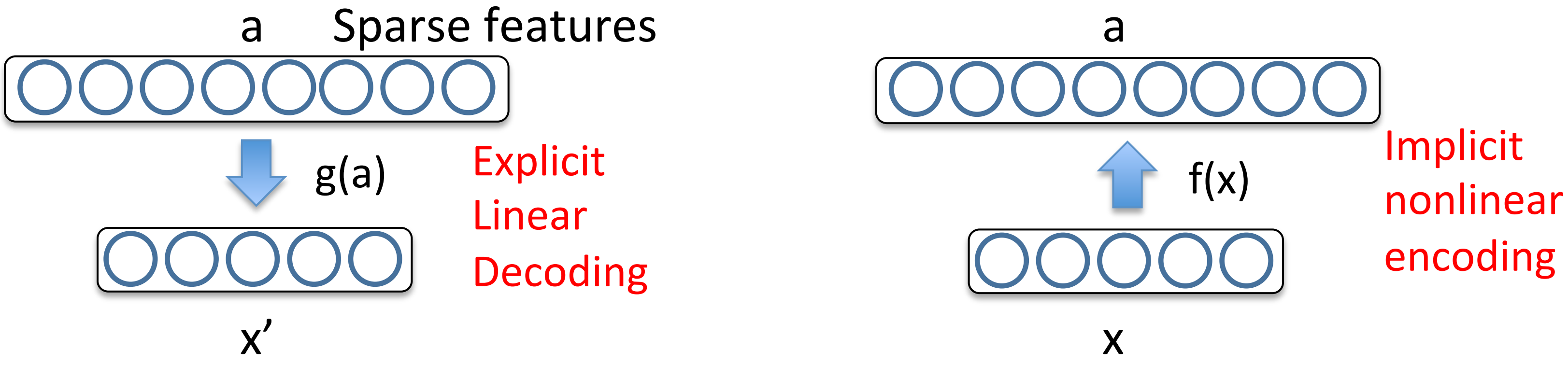
- Self-taught learning: when features trained on different input distribution
- Example:
  - train sparse coding dictionary on handwritten digits
  - use codes (features) to classify handwritten characters

Digits → English handwritten characters			
Training set size	Raw	PCA	Sparse coding
100	**39.8%**	25.3%	39.7%
500	54.8%	54.8%	**58.5%**
1000	61.9%	64.5%	**65.3%**

Self-taught Learning: Transfer Learning from Unlabeled DataRaina, Battle, Lee, Packer and Ng.

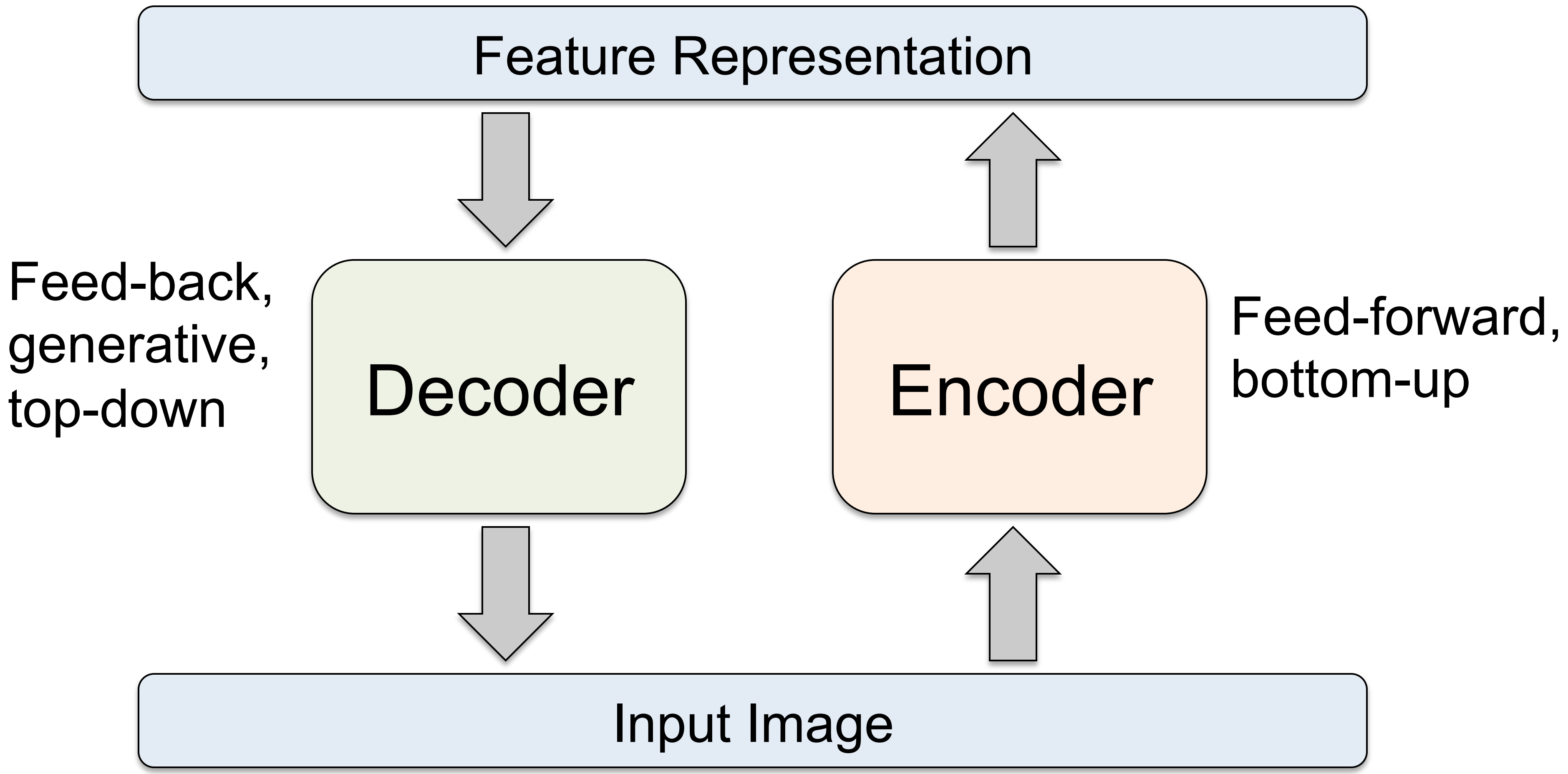
Interpreting Sparse Coding

$$\min_{\mathbf{a},\boldsymbol{\phi}} \sum_{n=1}^{N} \left\| \mathbf{x}_n - \sum_{k=1}^{K} a_{nk} \boldsymbol{\phi}_k \right\|_2^2 + \lambda \sum_{n=1}^{N} \sum_{k=1}^{K} |a_{nk}|$$



- Sparse, over-complete representation **a.**
- Encoding **a** = f(**x**) is implicit and nonlinear function of **x**.
- Reconstruction (or decoding) **x'** = g(**a**) is linear and explicit.

Autoencoder



Feature Representation

Feed-back,
generative,
top-down

Decoder

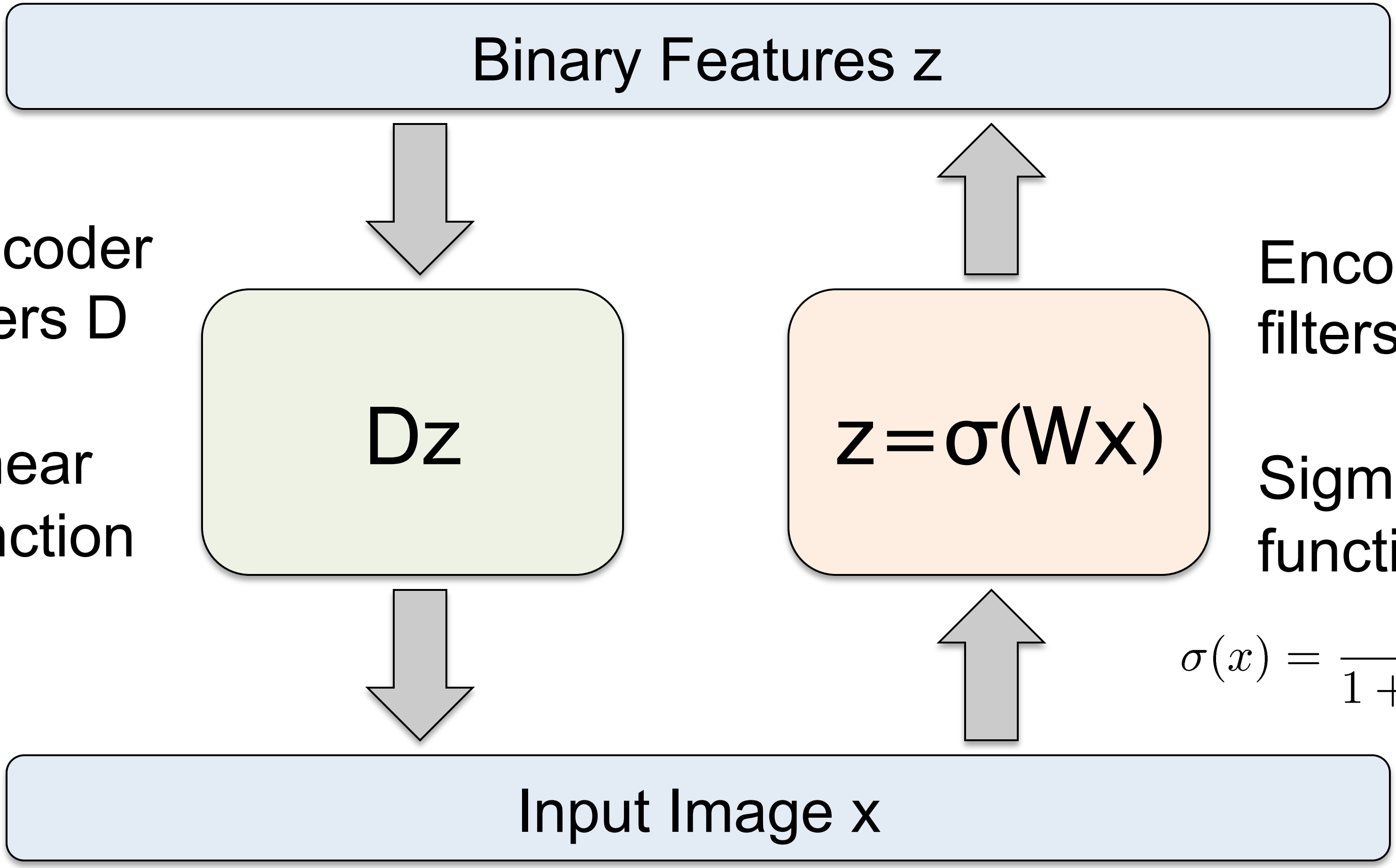
Encoder

Feed-forward,
bottom-up

Input Image

- Details of what goes insider the encoder and decoder matter!
- Need constraints to avoid learning an identity.

Autoencoder



Binary Features z

Decoder filters D

Linear function

Dz

$z=\sigma(Wx)$

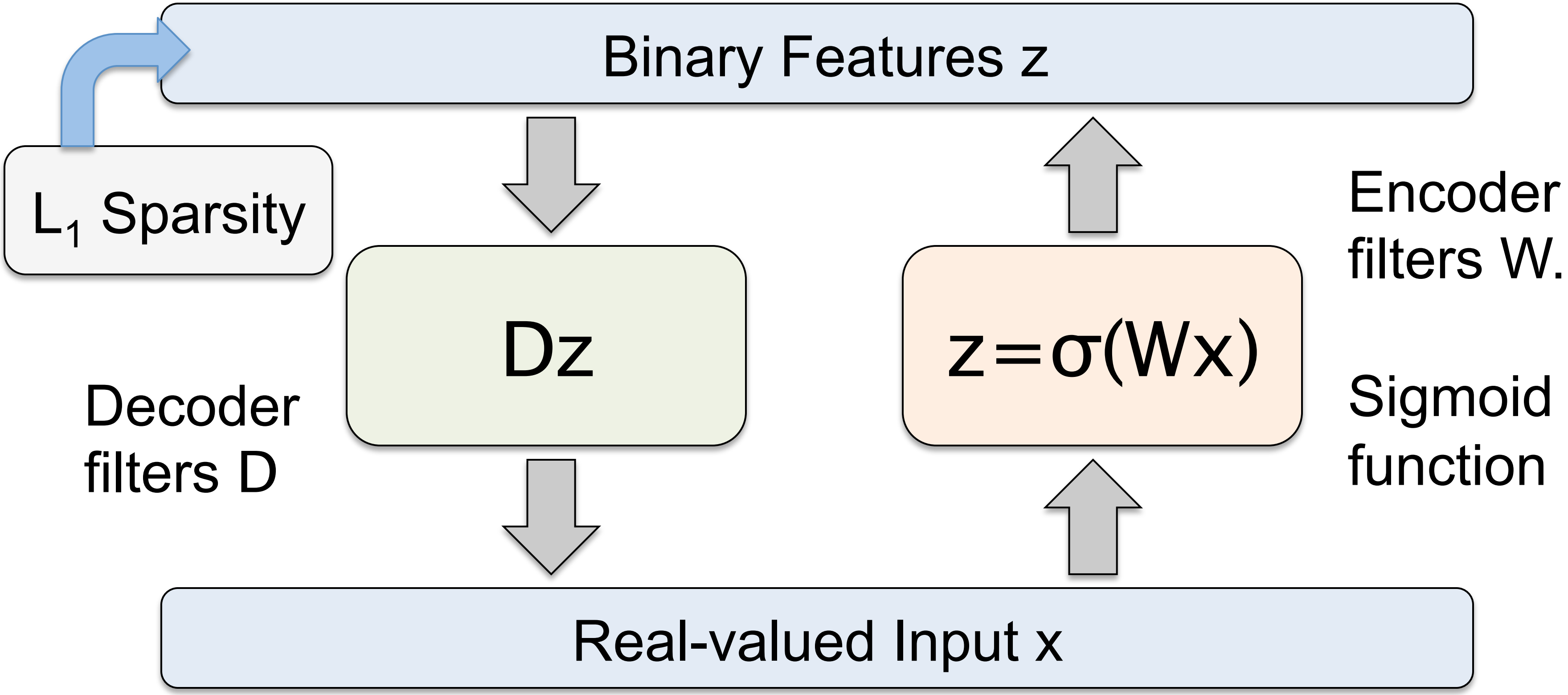
Encoder filters W.

Sigmoid function

$$\sigma(x) = \frac{1}{1+\exp(-x)}$$

Input Image x

Predictive Sparse Decomposition



Binary Features z

L_1 Sparsity

Dz

Decoder filters D

z=σ(Wx)

Encoder filters W.

Sigmoid function

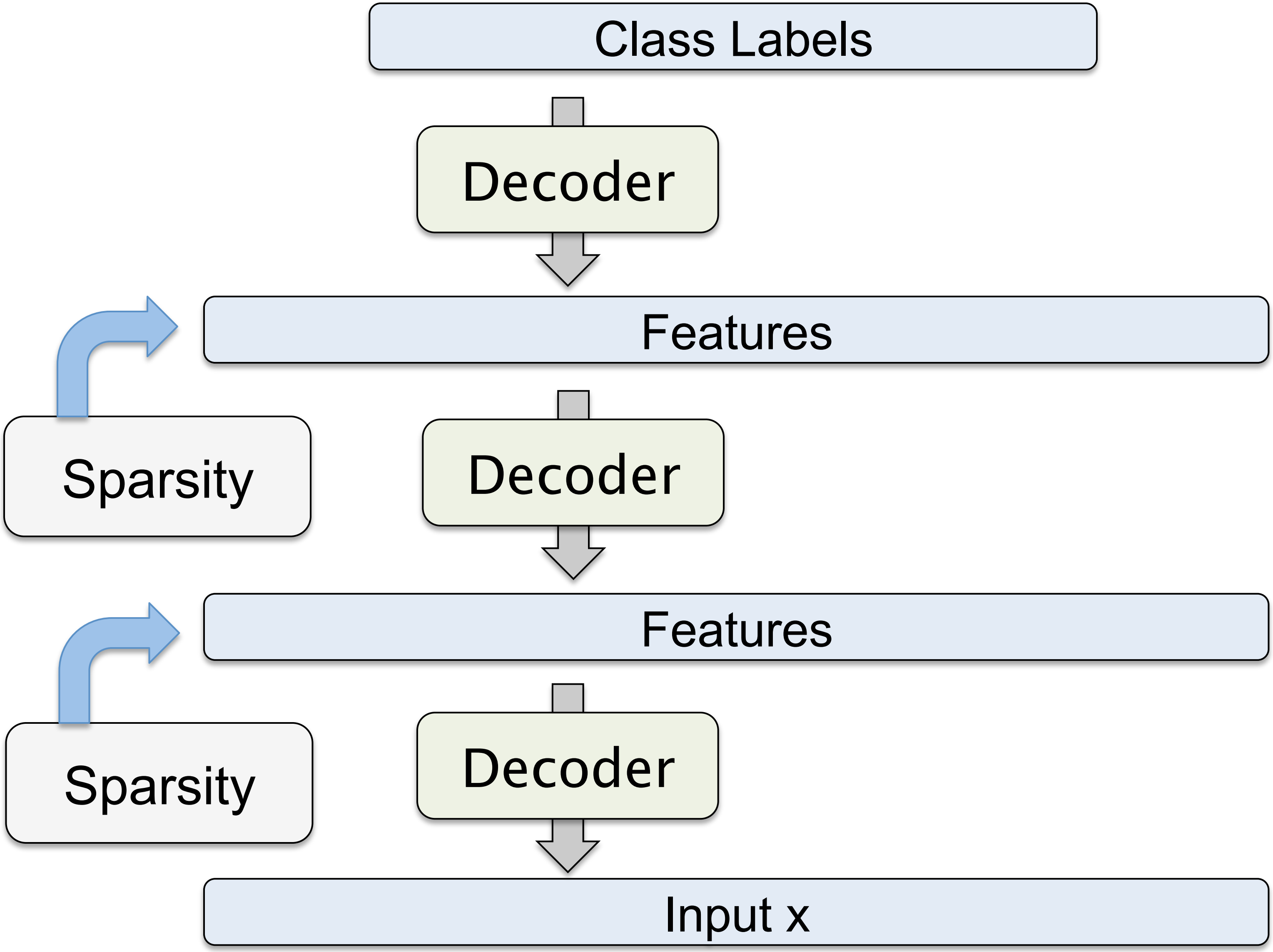
Real-valued Input x

At training time

$$\min_{D,W,\mathbf{z}} \underbrace{||D\mathbf{z}-\mathbf{x}||_2^2+\lambda|\mathbf{z}|_1}_{\text{Decoder}} + \underbrace{||\sigma(W\mathbf{x})-\mathbf{z}||_2^2}_{\text{Encoder}}$$

Kavukcuoglu et al., ‘09

Stacked Sparse Coding?



Class Labels

Decoder

Features

Sparsity

Decoder

Features

Sparsity

Decoder

Input x

Modeling Image Patches

- Natural image patches:
  - small image regions extracted from an image of nature (forest, grass, ...)

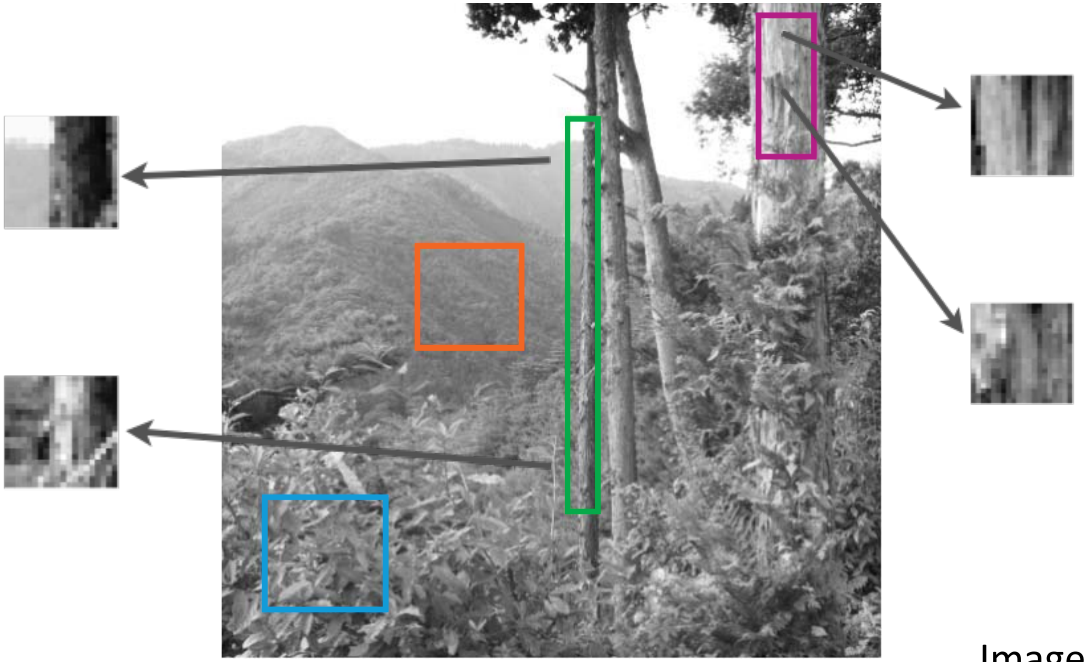
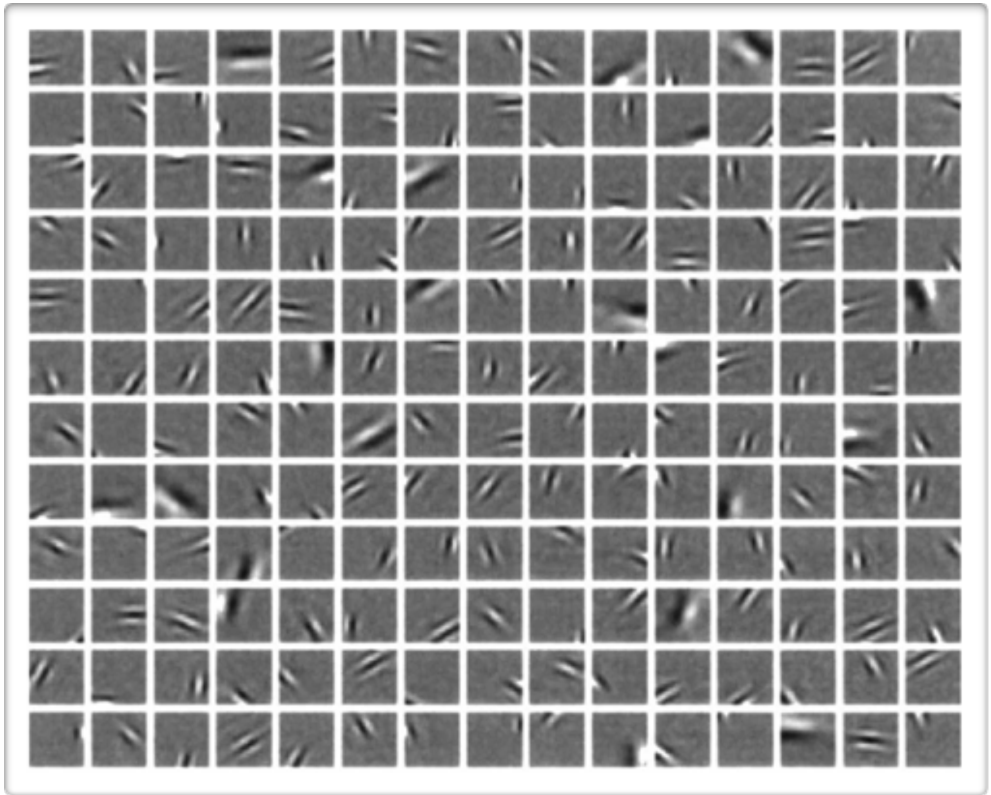


Image taken from:
Emergence of complex cell properties
by learning to generalize in natural scenes.
Karklin and Lewicki, 2009

Relationship to V1

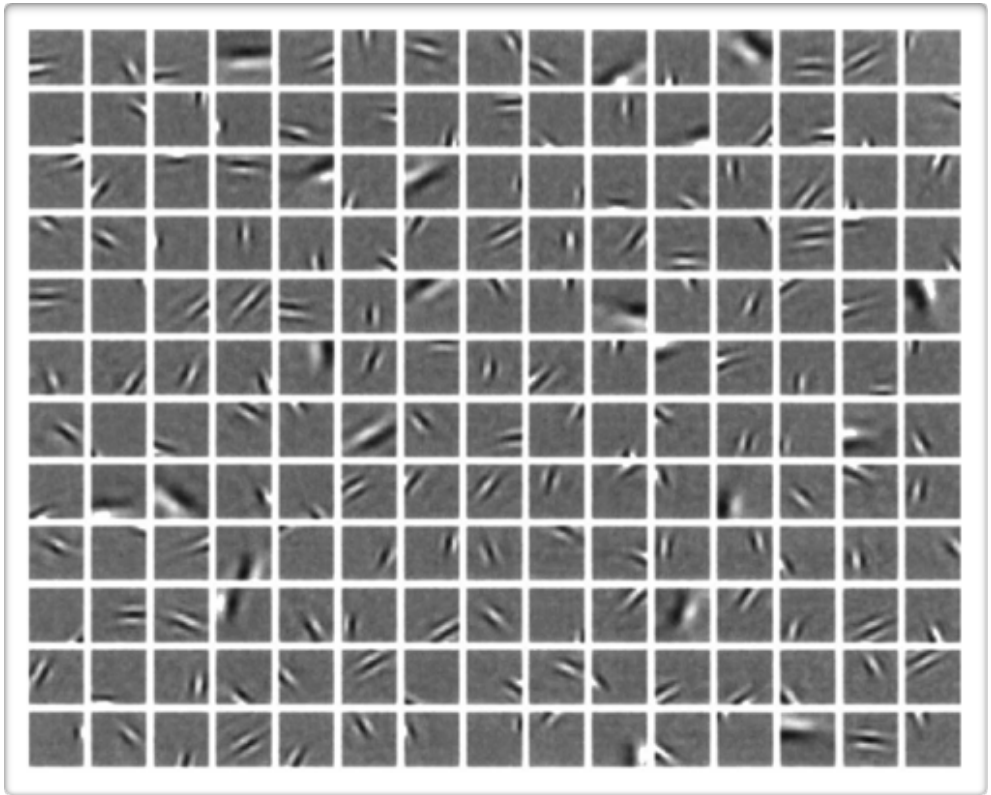
- When trained on natural image patches
  - the dictionary columns ("atoms") look like edge detectors
  - each atom is tuned to a particular position, orientation and spatial frequency
  - V1 neurons in the mammalian brain have a similar behavior



Emergence of simple-cell receptive field properties by learning a sparse code of natural images.Olshausen and Field, 1996.

Relationship to V1

- Suggests that the brain might be learning a sparse code of visual stimulus
  - ➢ Since then, many other models have been shown to learn similar features
  - ➢ they usually all incorporate a notion of sparsity



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