Description

This seminar examines the sum-of-squares method with an emphasis on recent developments and open directions for research. We will cover use of this method in designing algorithms for both worst-case optimization problems such as Max-Cut and Coloring and average-case problems arising in machine learning and algorithmic statistics such as learning mixtures of Gaussians and robust statistics. We will also cover techniques for proving lower-bounds on this method and its connection to restricted algorithmic frameworks such as Statistical Query Model, the Low-Degree Likelihood Ratio Method and Extended Formulations.

Lectures: 4-6:50 pm, Tuesdays on Zoom. Link on Diderot. First meeting: Sep 1.

Lecture Format Lectures will be held on zoom. Videos of the lectures will be uploaded online but there might be some delay. We expect most to attend live - this is best for maintaining energy and interaction in the course - the videos are meant primarily for those who are in different time-zones. **If you are not in a US time zone or cannot attend lectures live, please let me know immediately**. That way, we can decide on policies that can help alleviate some of the issues early on.

Office Hours: Pravesh: TBD, Peter: Mon 2-3pm.

HWs and Gradescope: We will use gradescope to manage HW submissions. Please let us know as soon as possible if you hvae trouble signing up. **If there's some circumstance that prevents you from submitting a HW before the deadline, please email Pravesh/Peter as soon as possible.**

Material: We will use Diderot to provide interactive lecture notes, manage discussions and questions and make announcements.

Prerequisites

We will assume "mathematical maturity" and familiarity with topics that are typically covered in basic classes such as: eigenvectors and eigenvalues, linear programming, LP duality and Farkas lemma, outline of ellipsoid method, basics of probability (expectation, variance, tail bounds), basic spectral graph theory (graphs and their adjacency matrices, relation between spectrum of adjacency matrix and random

walk). Most of these ideas are covered in classes such as Theorist's Toolkit. Graduate level classes in Complexity Theory and Algorithm Design will help appreciate the content better. Homework 0 is meant to communicate the sort of facts that we will assume without proof in this class and assume sufficient proficiency and familiarity with.

Syllabus

The following is a rough plan for the class. This plan has way more material that we can cover but we will use what is not covered in the class in order to choose topics for expository lecture notes (see "Evaluation"). We intend to focus more on material that has "open ends" and could easily spawn research projects.

1. Basics - SoS, SDPs, Solving, Proofs, Bit Complexity
2. Worst-Case Algorithm Design -
   (a) Rounding 2nd Moments: Ex: Max-Cut, RPR^2 framework , KMS-coloring, Rounding CSPs, UGC and Raghavendra Theorem, Small-Set Expansion
   (b) Triangle Inequality +2nd Moments: Arora-Rao-Vazirani Sparsest Cut, ...
   (c) Global Correlation Rounding: Max-Bisection, Unique Games via ABS Decomposition, 2-to-4 Norm
   (d) Reweightings: Polynomial Maximization, Convergence of SoS on Unit Sphere
3. Average-case Algorithm Design -
   (a) "Low-entropy roundings" via certifiability proofs, Ex: robust moment estimation, clustering mixtures, community detection
   (b) Rounding via Entropy surrogates, Ex: random polynomial maximization, tensor decomposition, dictionary learning, list-decodable learning,
4. Lower-bounds:
   (a) Maximum "Entropy" paradigm and Satisfying Hard Constraints: Grigoriev's XOR/Tseitin Lower bound, Perfect Matching
   (b) Matrix Polynomials: Alon-Bopanna, Lovasz-Theta
   (c) Symmetry and Explicit Eigendecompositions: Knapsack
   (d) Local Gram-Schmidt Method: Random CSPs

(e) Pseudo-calibration: Clique, Tensor PCA

(f) Geometry of Sphere vs the Hypercube: Feige-Schechtman, Khot-Vishnoi Instances,

5. Connections:

(a) Local Algorithms

(b) Low Degree Likelihood Ratio

(c) Linear/Semidefinite Extended Formulations

Evaluation

We will have 4-6 homeworks including homework 0. These will account for 60 percent of your grade. At the end of October, we will decide on a list of topics that fit our goals but won't be covered in the lectures. Each student will pick a topic and (individually) write expository lecture notes - similar to the ones that we post - on their chosen topic. These lecture notes will account for 40 percent of your grade. The goal of this part is to get a glimpse of ideas beyond what we will have time to cover. The grading of these notes will be done by your peers in the class (they will be provided with an official "review" form) and will be based on clarity and usefulness in learning the new topic.

Your Health and Well Being

This is a somewhat intense course. One major goal is that you will have fun taking it and learning this new cool material (and perhaps chalking out related research projects) as much as I hope to have fun teaching it. This certainly involves you taking care of yourself. Do your best to maintain a healthy lifestyle this semester by eating well, exercising, getting enough sleep and taking some time to relax. This will help you achieve your goals and cope with stress.

If you or anyone you know experiences any academic stress, difficult life events, or feelings like anxiety or depression, we strongly encourage you to seek support. Counseling and Psychological Services (CaPS) is here to help: call 412-268-2922 and visit http://www.cmu.edu/counseling/. Consider reaching out to a friend, faculty or family member you trust for help getting connected to the support that can help.

Accommodations for Students with Disabilities

If you have a disability and are registered with the Office of Disability Resources, please **talk to me to discuss your needs as early in the semester as possible**. I will work with you to ensure that accommodations are provided as appropriate. If you suspect that you may have a disability and would benefit from accommodations but are not yet registered with the Office of Disability Resources, I encourage you to contact them at access@andrew.cmu.edu.