Discrete Mathematics Exam

Name________________________________

This exam will help us to evaluate your knowledge of basic discrete mathematics, which at CMU is taught in course 21-127 (the prefix 21 specifies the Mathematics department; the prefix 15 specifies the Computer Science department). This course is a prerequisite for 15-211, although students who score highly enough on this exam may take 21-127 at the same time as 15-211. Please complete this exam, even if you are not considering taking 15-211 in the fall.

The purpose of this exam is for you to demonstrate (from long term memory) your general knowledge of Discrete Mathematics. Doing so will help us to place you in the right courses. The purpose of taking this exam is NOT for you to get the highest score possible. Therefore, do not look-up material in a book or on the web, and do not talk to anyone. In fact, there is no stigma for scoring a zero on this exam, if you have never studied this material before at the appropriate level. Finally, you should finish this exam in one sitting. If you have no idea what a question is asking, or what the right answer is, please just write question mark in the provided space (there is a penalty for guessing wrong).

Return only this page (not the exam itself). Carefully separate it from the exam when you finish.

Fill in the letter corresponding to the best answer for question; write "?" if you are unsure.

Answers:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

Briefly describe any formal discrete mathematics course you took in high school, or at a college level:

Notation:

`x*y`	the product of x and y
`sqrt(x)`	the positive square root of x
`log(x)`	log of x to the base 2
`n!`	n factorial
`x^y`	x to the power y
`gcd(x,y)`	the greatest common divisor of x and y
`x = y (mod n)`	is x is congruent to y modulo n
`(n choose k)`	is the binomial coefficient

1. Convert the hexadecimal number *abc* to its decimal representation

```
a) 2748
b) 3258
c) 2763
d) 3021
```

2. Suppose that we are to make a list of the perfect cubes: 1, 8, 27, 64, 125, ... If we have the perfect cube n then the minimum we have to go up the list of positive integers from n to find the next cube is

```
a) n
b) 2*sqrt(n) + 1
c) 3*(n^(2/3)) + 3*(n^(1/3)) + 1
d) none of the above
```

3. Predict the next term in the sequence below

1, 1, 7, 25, 61

```
a) 181
b) 111
c) 86
d) 121
```

4. Convert 1.25 from decimal to binary

```
a) 1.11001
b) 1.01
c) 1.001
d) 1.1
```

5. The last digit of 3^729 is

```
a) 1
b) 3
c) 9
d) 7
```

6. If the integer n is given in octal (base 8) notation d(k)...d(0) then n is divisible by 7 if and only if

```
a) d(0) = 7
b) d(k) * ... * d(0) is divisible by 7
c) d(k) + ... + d(0) is divisible by 7
d) d(k) is divisible by 7
```

7. A madman arranges 2n people in a circle. Given a number M, which may be larger than 2n but must be greater than 0. He will count out M people clockwise, kill the M+1st, and drag the body from the circle. He will then continue clockwise in the same way until everyone is dead. He requires you give him the number M so that the people in the circle are killed in the natural order 1,2,3,... or he will kill you also. You should give him the number (assuming that you want to live)

```
a) 1
b) 2
c) 2^(2n)
d) none of the above
```

8. Let a and b be two positive integers such that a > b. Then gcd(a, b) = gcd(a - b, b)

```
a) True
b) False
```

9. From the set {0, 4, 5, 6, 7, 8, 9}. How many 4-digit numbers with distinct digits can be created?

```
a) 840
b) 720
c) 420
d) 620
```

10. How many permutations of the digits 0, 1, 2, ..., 9 either start with a 3 or end with a 7, or both?

```
a) 9! + 9! - 8!
b) 9! * 9! - 8!
c) (9! * 9!)/8!
d) none of the above
```

11. Two dice are tossed. Find the probability that the sum of the dice is a prime

```
a) 17/36
b) 1/2
c) 15/36
d) none of the above
```

12. If we have the inequality m/n < p/q with p,q,m,n >0 then for the mediant fraction (m+p)/(n+q) we have m/n < (m+p)/(n+q) < p/q.

```
a) True
b) False
```

13. The fraction a/b in lowest terms has a finite ternary (base 3) expansion if and only if

```
a) b is a power of 3
b) b is a power of 2
c) gcd(a,b) = 3
d) none of the above
```

14. Let G = (V, E) be an undirected graph with 7 vertices; 3 of them of degree two and 4 of degree one. G has no self-loops and no multiple edges. Is this graph is connected?

```
a) True
b) False
```

15. Given any n consecutive integers a, a + 1, ..., a + n - 1 one of them is divisible by n.

```
a) True
b) False
```

16. Which if the following is not a one-to-one map to the set of natural numbers?

```
a) the set of integers
b) the set of rationals
c) the set of real numbers
d) the set of primes
```

17. Let G = (V, E) be an undirected graph with n vertices. G has no self-loops and no multiple edges. Suppose graph G is not connected. What is the maximum number of edges in G?

```
a) n-1
b) n(n-1)/2
c) (n-1)(n-2)/2
d) none of the above
```

18. Let a(n) satisfies the following recurrence

a(n) = a(n-1) + (-1)^n

a(0) = 0

then a(n) is

```
a) 0
b) (-1)^n - 1
c) -1
d) ((-1)^n – 1)/2
```

19. If p(x) and q(x) are both polynomials of degree N>0 and

p(0) = q(0), p(1) = q(1), p(2) = q(2), ... , p(N) = q(N)

then

```
a) p(x) = q(x)
b) p(x) - q(x) has degree n
c) p(x) * q(x) has degree n
d) none of the above
```

20. Let c be a real number. Then (x – c) divides the polynomial p(x) if and only if

```
a) p(c) = sin(c)
b) p(c) = 0
c) p(c) = 1
d) none of the above
```