

HOMEWORK 8
Due: 5:00pm, Thursday April 6

Feature: As before, if your homework is typeset (as opposed to handwritten), you will receive 1 bonus point.

1. **(On log-space reductions.)** In class we discussed that a good model for $O(\log n)$ -space decision algorithms is the following: On input x of length n , one should consider “pseudocode”, wherein: (a) n is “known”; (b) the code has only a *constant* number of “variables” (where a “variable” can hold an integer polynomially-bounded in n , or an $O(\log n)$ -bit string); (c) basic arithmetic can be performed on variables; (d) no “memory allocation” is allowed; (e) input lookups (like “ $b := x[i]$ ”) are allowed. Under this mental model, the extension to $O(\log n)$ -space algorithms with (write-once) output is very simple: the pseudocode may have “print” statements.

In this problem, you should describe $O(\log n)$ -space algorithms in this “pseudocode model”. Since your code will ideally be relatively easy to understand, you are not asked to prove it correct, but you should include “comments” in your pseudocode to explain it, and also a paragraph explaining the main idea of the code, if necessary.

- (a) (6 points.) Give a log-space algorithm that takes a directed graph in adjacency matrix format and outputs the same graph in adjacency list format. Also give a log-space algorithm that goes from adjacency list format to adjacency matrix format. (You might want to make some reasonable decisions and comments about the precise nature of your “adjacency matrix/list” formats.)
- (b) (4 points.) Theorem 7.32 of Sipser describes an explicit polynomial-time reduction from 3SAT to CLIQUE. Give explicit pseudocode for this reduction, showing that it is computable in log-space.

2. **(Strong connectivity.)** Let STRONGLY-CONNECTED be the language of all $\langle G \rangle$ such that G is a strongly connected digraph (meaning every vertex can reach every other vertex). Show that STRONGLY-CONNECTED is NL-complete, as follows:

- (a) (5 points.) Show STRONGLY-CONNECTED \in NL.
- (b) (5 points.) Show STRONGLY-CONNECTED is NL-complete (under log-space reductions, of course).

3. **(Equal partition.)** Consider the following task: The input is a list of n integers x_1, \dots, x_n , each x_i satisfying $0 \leq x_i < n^{10}$. The task is to accept if and only if there is a subset $S \subseteq \{1, 2, \dots, n\}$ such that

$$\sum_{i \in S} x_i = \sum_{i \notin S} x_i.$$

- (a) (0 points.) Show that this problem is in P.
- (b) (10 points.) Show (more strongly) that this problem is in NL.
- (c) (0 points.) Do you think this problem is in L?

4. (**HORN-SAT is P-complete.**) (10 points.) Recall the HORN-SAT problem from Homework #4, problem 3(b). Therein you showed it is in P. Now show it is P-complete, by showing it is P-hard (under log-space reductions). (Hint: you might like to use a reduction in which you have pairs of variables that are supposed to stand for the negations of each other.)