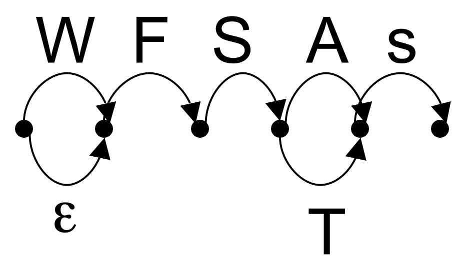
Language and Statistics II

Lecture 7: 

Noah Smith

Finite-State Technology

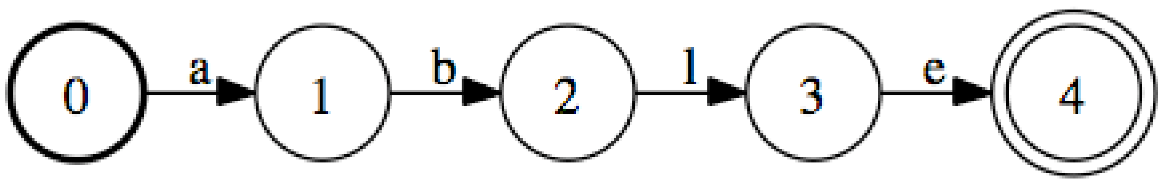
- Formally well-understood
  - Regular languages, rational relations
  - Generalizes n-grams, HMMs
- Many applications in NL technologies
  - Speech recognition
  - Lexical, morphological processing
  - Information extraction
  - Translation (!)
  - Parsing (!)
- Several toolkits
- Often determinizable: **very fast**

Finite-State Automata (Recognizers)

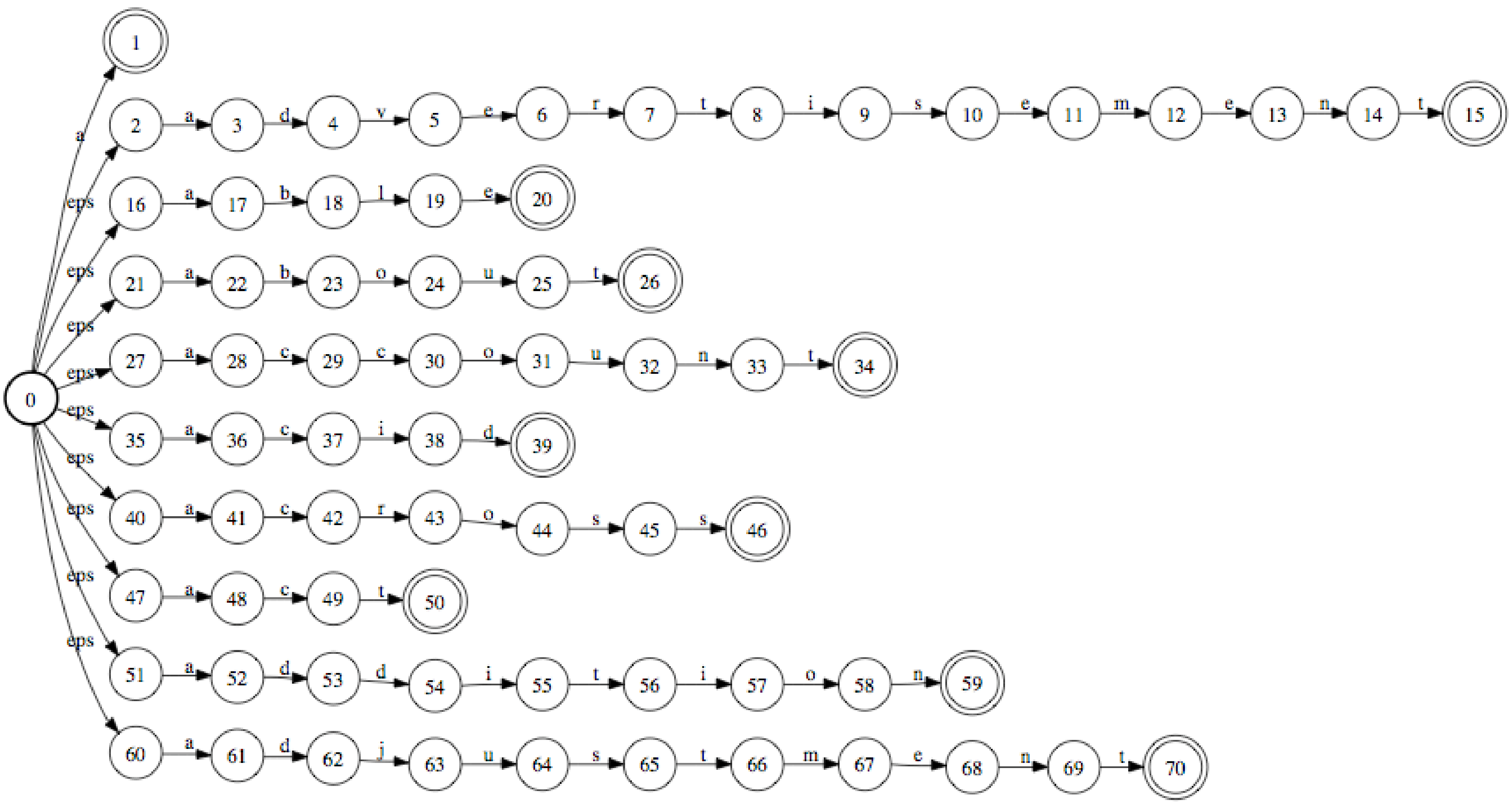
- Automaton that recognizes **regular** language
- Implementation of a **regular** expression
- Regular languages are closed under numerous operations
  - Concatenation, union, intersection, Kleene *, difference, reverse, complement, …
- Correspond to regular grammars (type 3 in Chomsky hierarchy)
- Pumping lemma: necessary condition for a language to be regular

FSM as a Dictionary

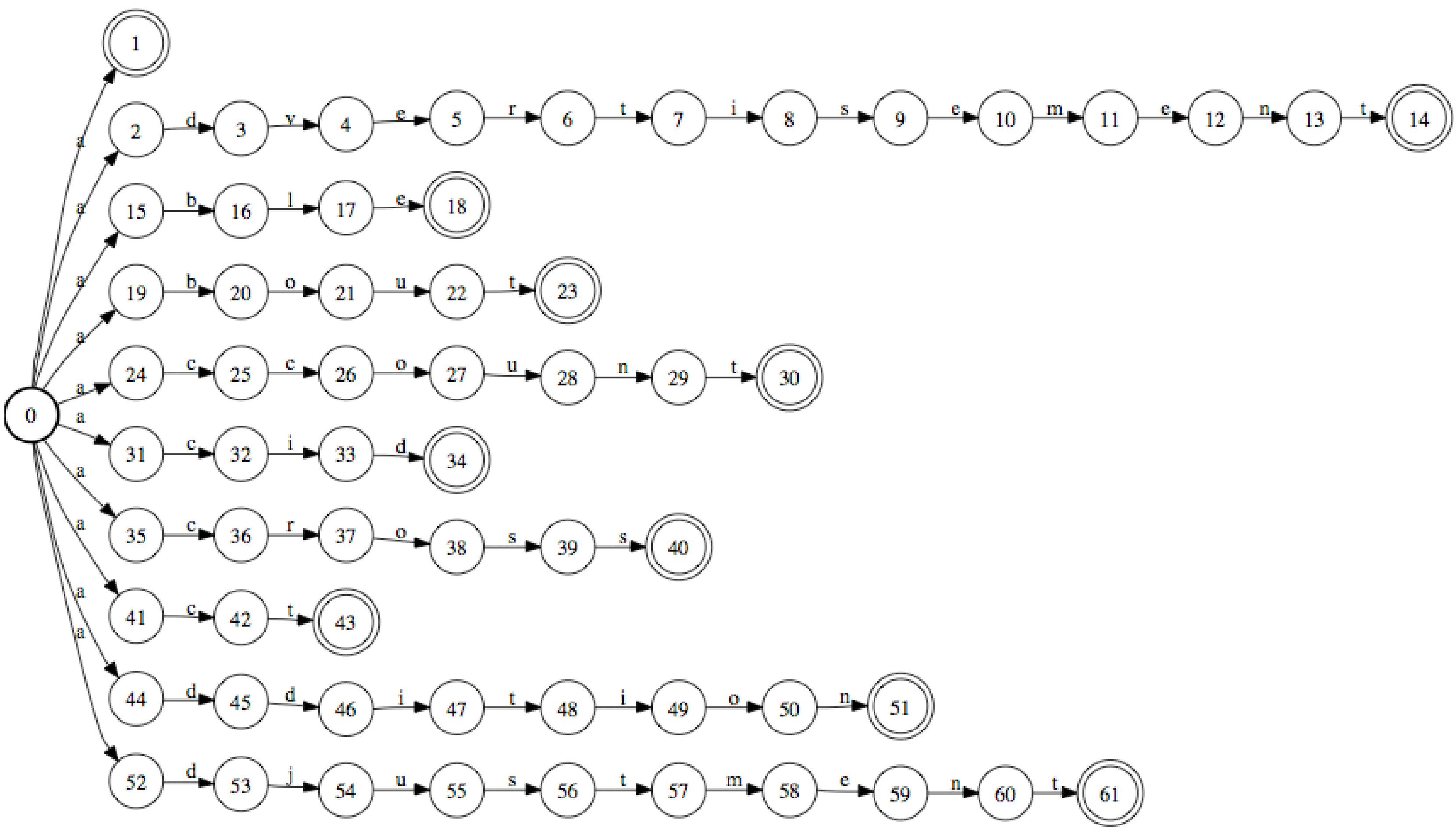
- Example: 850 words in “Basic English”
- Each word is an FSM



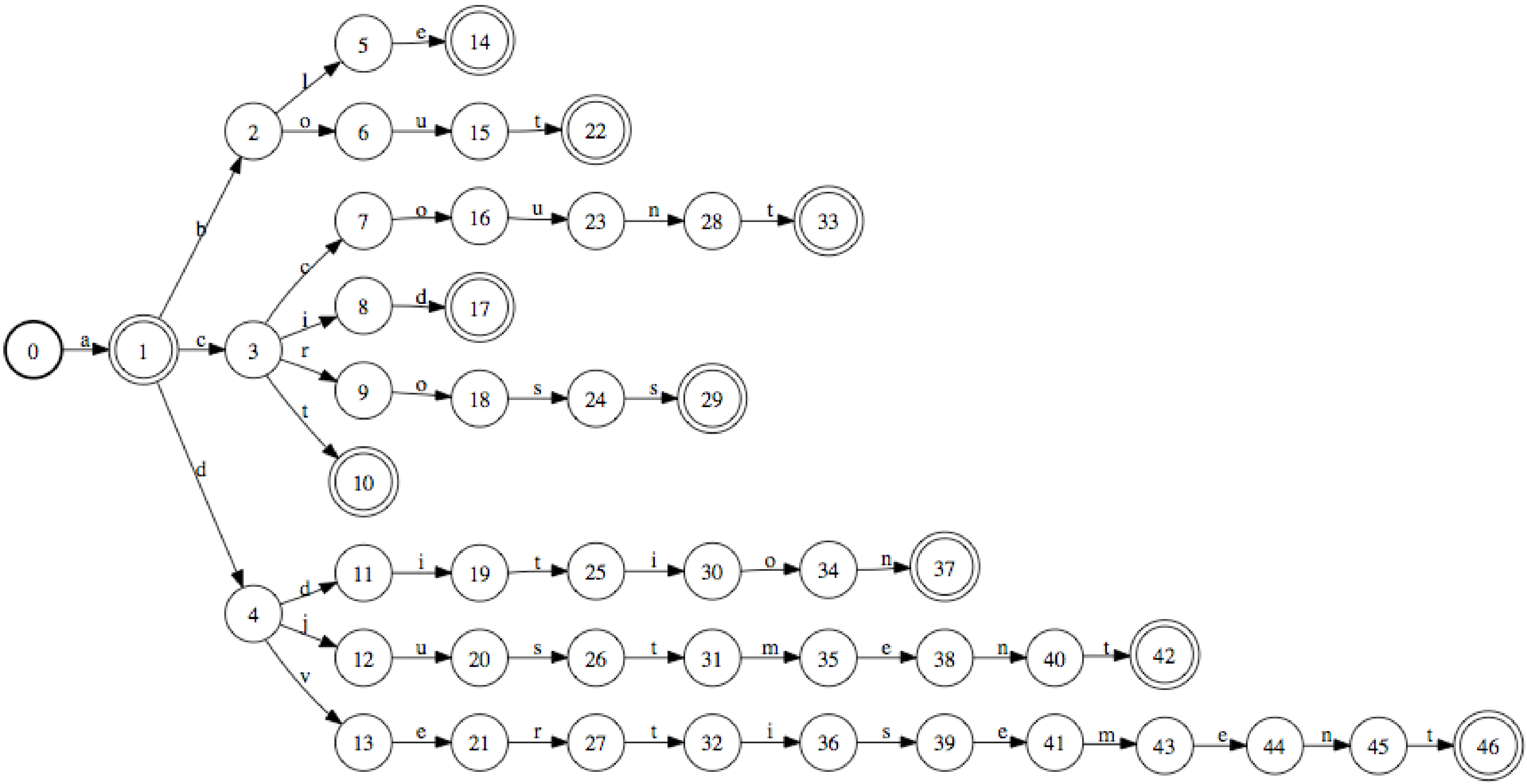
Ten-Word Dictionary



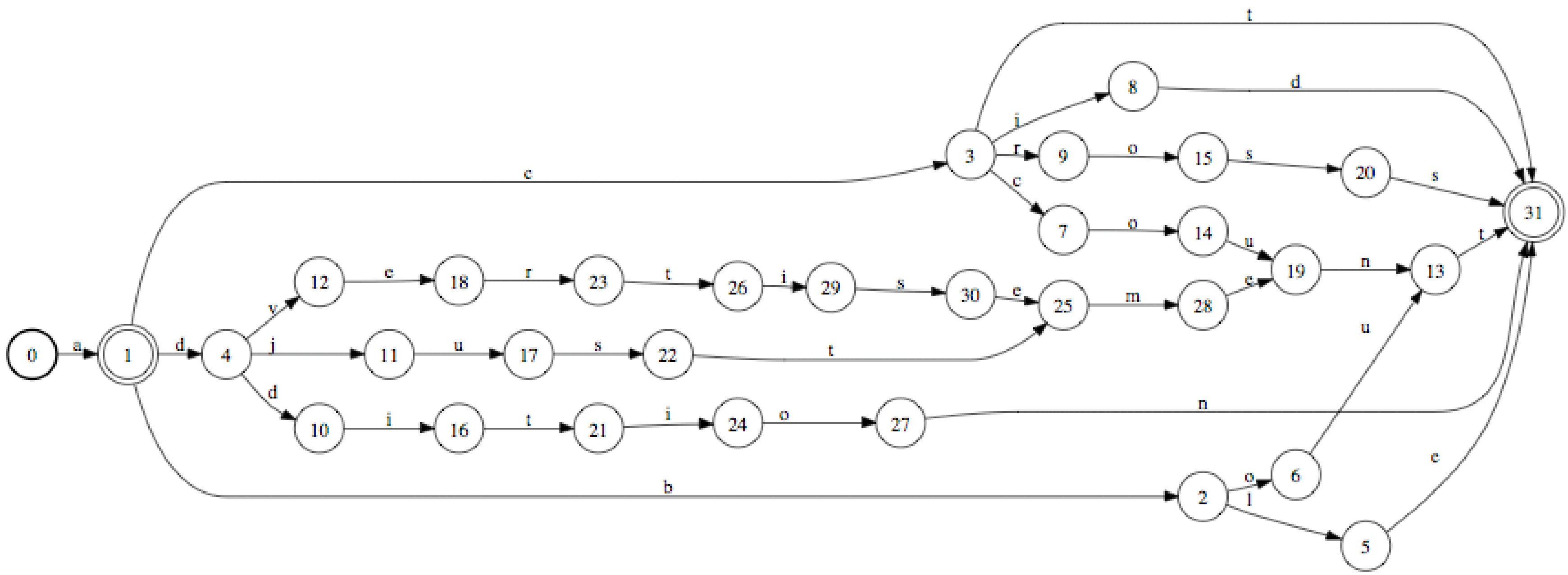
Remove ε-transitions



Determinize



Minimize



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Full 850-Word Dictionary

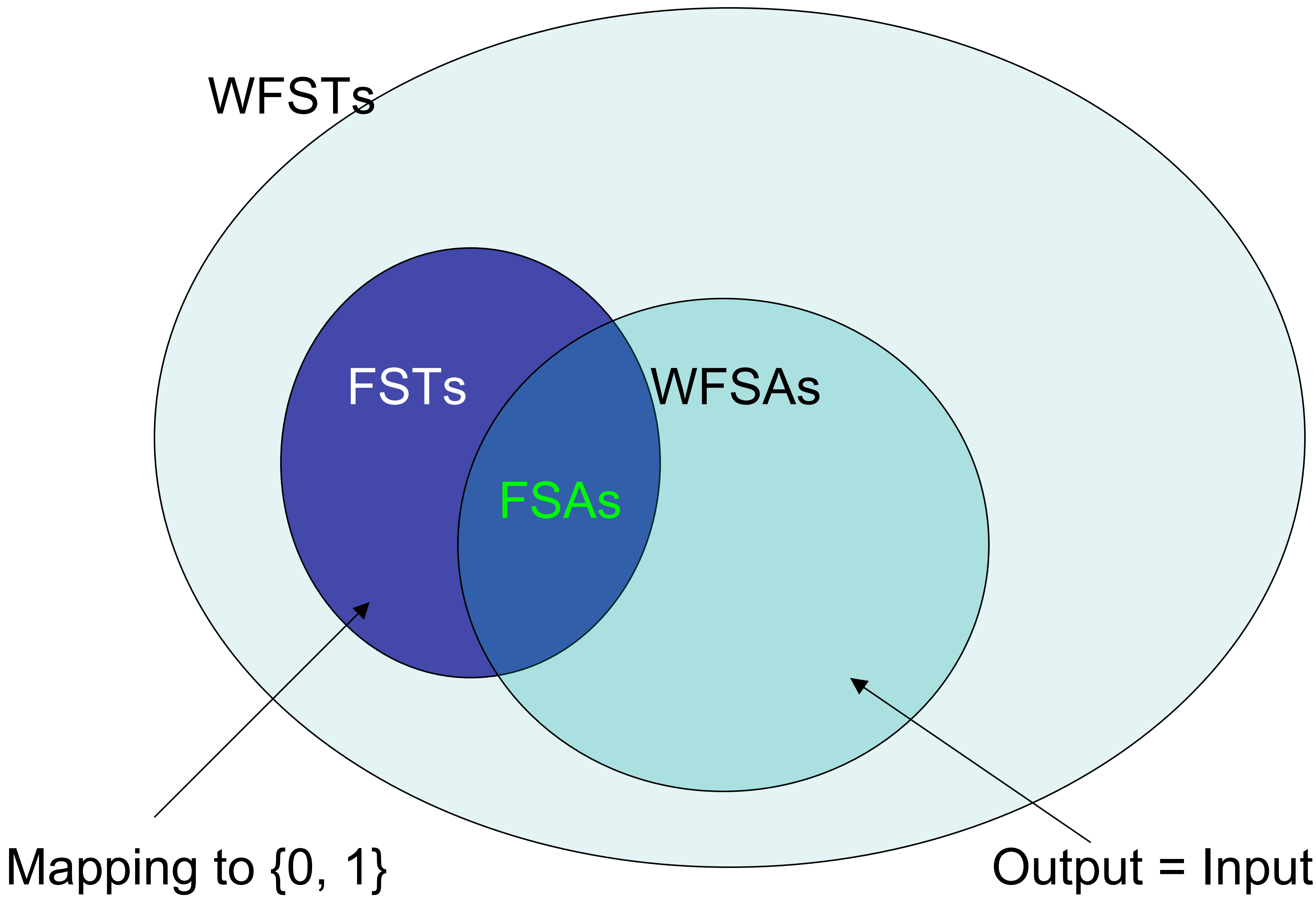
	states	*final states*	*arcs*
Union	5303	850	5302
Remove ε-transitions	4454	850	4453
Determinize	2609	848	2608
Minimize	744	42	1535

Algorithms

- Removing ε-transitions
- Determinization
- Minimization

Generalizations

- FS Recognizer is a function from $\Sigma^* \rightarrow \{0,1\}$
  - Meaning: fsa(s) = 1 $\Leftrightarrow$ s is in the language
- Other **rational relations** …
  - FS Transducer: $\Sigma^* \rightarrow \Delta^*$
  - Weighted FSA: $\Sigma^* \rightarrow \mathbb{R}$
  - Weighted FST: $\Sigma^* \rightarrow \Delta^* \times \mathbb{R}$
- WFSAs and WFSTs can be considered **probabilistic** (but don't have to be)


WFSTs
FSTs
WFSAs
FSAs
Mapping to {0, 1}
Output = Input

Finite-State Transducers

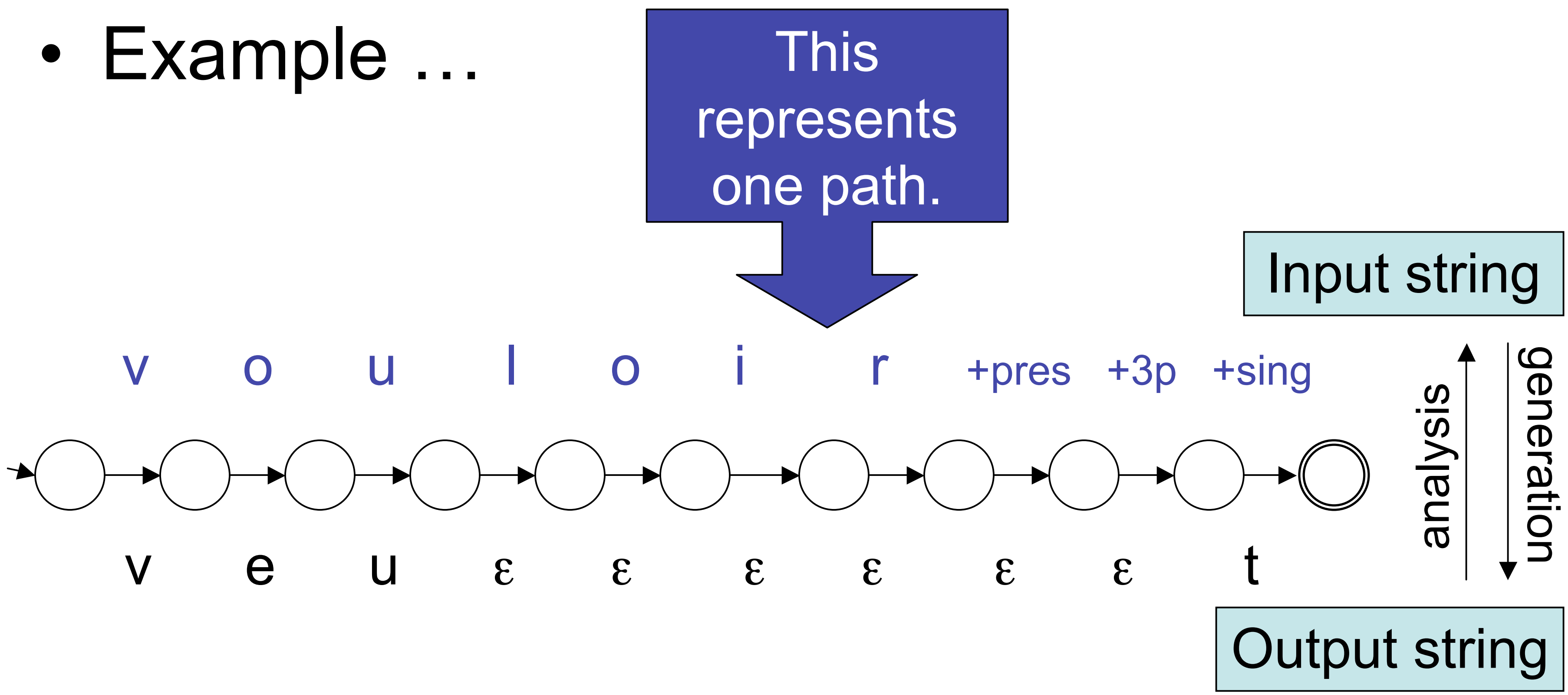
- Input alphabet Σ
- Output alphabet Δ
- Set of states Q
- Initial state q_0
- Final states $F \subseteq Q$
- Arcs, $Q \times \Sigma \times Q \times \Delta^*$

Sequential: arcs are functions from Q to $\Sigma \times Q \times \Delta^*$.

***p*-subsequential**: deterministic, **except** each final state has at most *p* output strings after each final state.

Finite-State Transducers

- Biggest application: morphology
  - Xerox tools: 20+ languages
- Example …



Ambiguity and Optionality

- leaves ➔

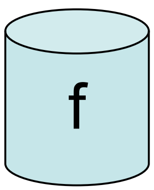
  {leaf +N +pl, leave +V +pres +3p +sing}
- advice +maker ➔

  {advisor, adviser}
- inter+ nation +al +ize +ation ➔
  {internationali<u>z</u>ation, internationali<u>s</u>ation}

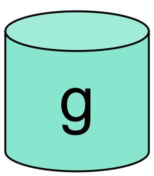
Also, Phonology

- Mapping between pronunciation (phonemes or phonetic symbols) and lexical entries (morpheme sequences or orthography).
- Optionality even more necessary here!

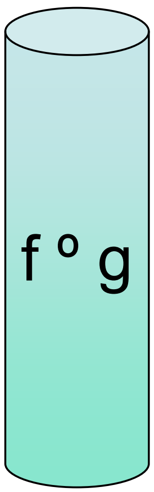
FST Composition



{vouloir → veux,
vouloir → veut,
vouloir → voulons,
vouloir → voulez,
vouloir → veulent,
…}



{veux → vœ,
veut → vœ,
voulons → vulõ,
voulez → vule,
veulent → vœl,
…}



{vouloir → vœ,
vouloir → vœ,
vouloir → vulõ,
vouloir → vule,
vouloir → vœl,
…}

FST Composition

- Formally, $(x, z) \in f \circ g$ iff
  there exists y such that $(x, y) \in f$ and $(y, z) \in g$.

- Set and relation:
  $(x, z) \in f \circ g$ iff $x \in f$ and $(x, z) \in g$
- Relation and set:
  $(x, z) \in f \circ g$ iff $(x, z) \in f$ and $z \in g$
- Set and set (intersection):
  $x \in f \circ g$ iff $x \in f$ and $x \in g$

Basically, treat sets as identity relations.

Why?

- String into transducer (to compute f(s)): string is a set of size one
- Feed set of strings to transducer in parallel!
- Filter a relation by the outputs (compose with a filter set)
- Building a morphological lexicon: define lexicon by a FSA (set), rules by a bunch of transducers (relation)

FST Projection

- Can strip off input or output symbols … get a FSA. (Might want to determinize.)

Weighted FSAs

- Instead of

  Is it grammatical (possible)?

  we might ask,

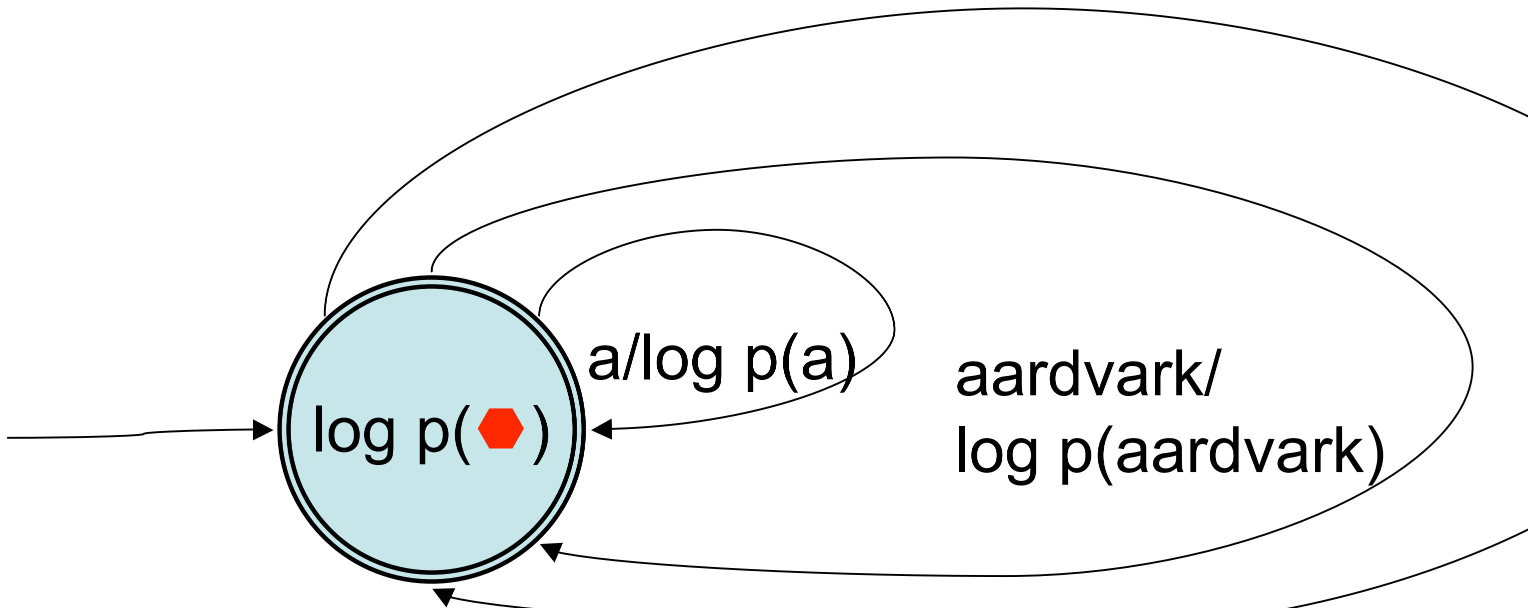
  How grammatical (likely) is it?

- Examples:
  - N-gram models
  - HMMs
  - Acoustic lattices
  - Perfect hash

Weighted Finite-State Acceptors

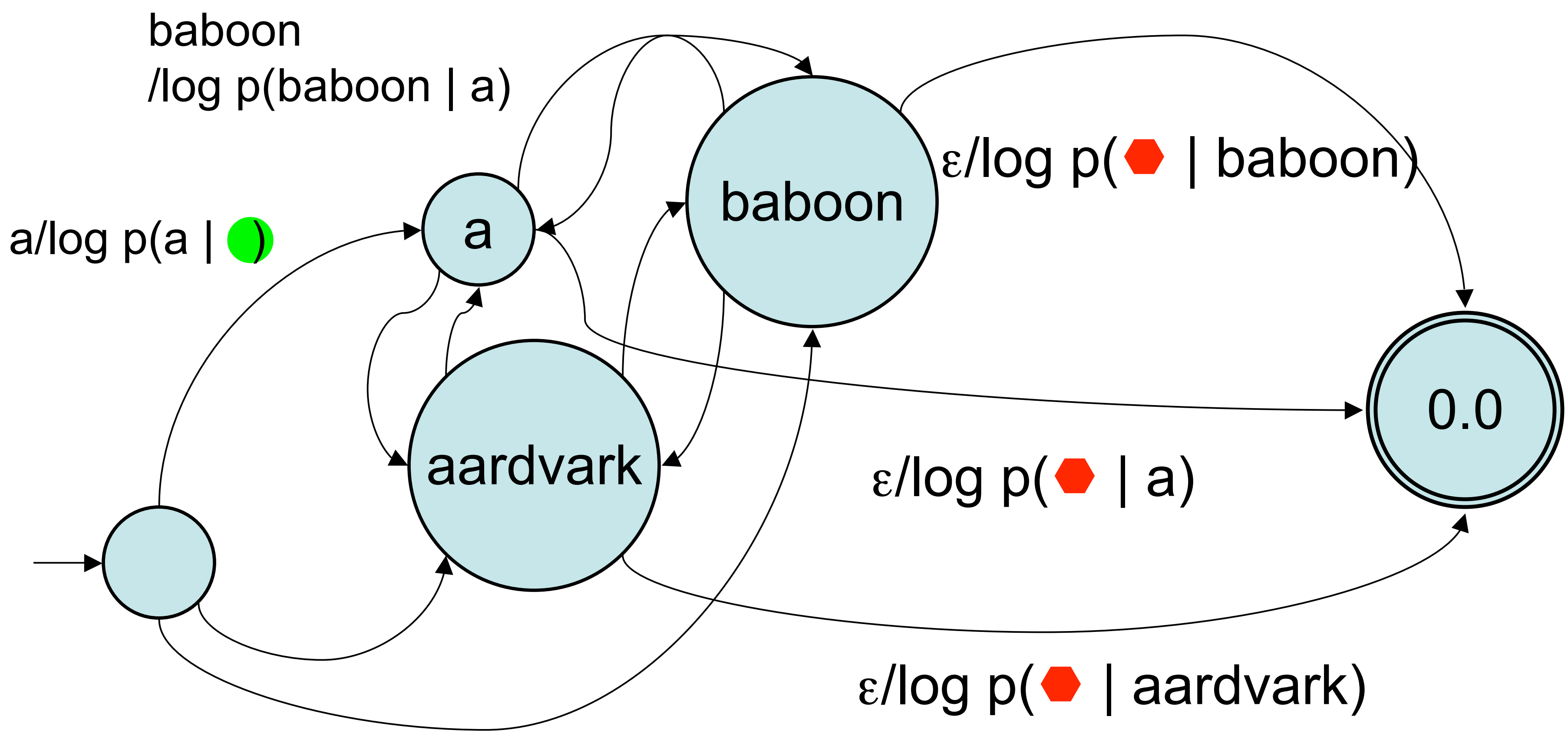
- Alphabet Σ
- Set of states Q
- Initial weight function, $\pi : Q \rightarrow \mathbb{R}$
- Final weight function, $\xi : Q \rightarrow \mathbb{R}$
- Arcs in $Q \times \Sigma \times Q \times \mathbb{R}$

Unigram model as a WFSA



One state.
One arc for every word.

Bigram model as a WFSA



|Σ| + 2 states.
One arc for every bigram.

Bigram HMM as a WFSA

- Alphabet Σ (HMM's alphabet)
- Set of states Q (HMM's states)
- Initial weight function, $\pi : Q \rightarrow \mathbb{R}$

  (0 for start state, $-\infty$ for others)
- Final weight function, $\xi : Q \rightarrow \mathbb{R}$

  $\log \gamma(⬣ \mid q)$
- Arcs in $Q \times \Sigma \times Q \times \mathbb{R}$

  $\log \gamma(q' \mid q) + \log \eta(s \mid q)$

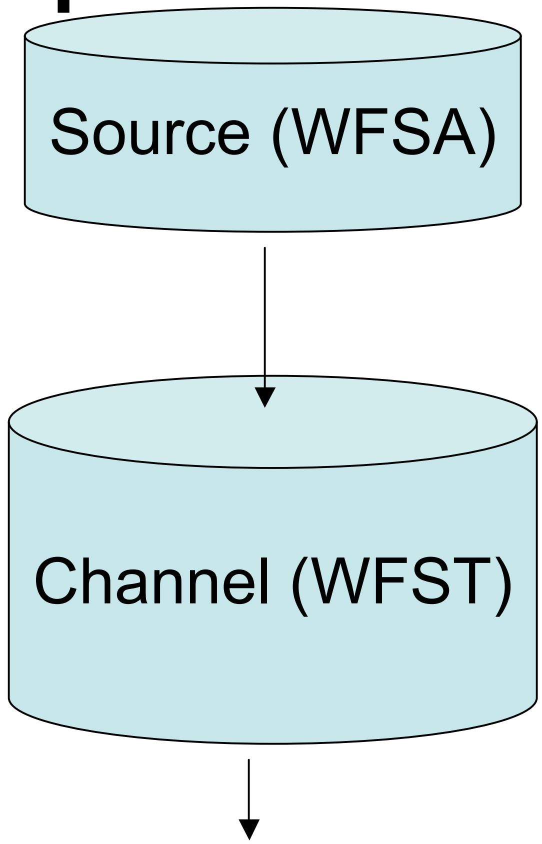
Can you tell how to build a WFSA from a **trigram** HMM?

WFSA as a Log-Linear Model

- WFSAs assign weights to **paths** through the network.
- Can exponentiate, normalize, and interpret this as a joint p(path, output) or conditional p(path | output).
- Feature schemata:
  - Initial state is q
  - Stop state is q
  - Number of times arc (q, q', s) was crossed

Weighted FSTs

- Weighted relation on $\Sigma^* \times \Delta^*$
- Like FSTs, closed under **composition**
- Examples:
  - Spelling correction
  - Morphological disambiguation
  - Edit distance
  - Machine translation
  - Speech recognition



Toolkits (links on course page)

- FSM libraries (AT&T)
  - Free binaries
  - Implements pretty much everything you need to build weighted and unweighted FS recognizers and transducers … except training!
- Xerox FS toolkit
  - Web demo; software can be purchased
  - No weights
- RWTH FSA toolkit
  - Newer, open-source
  - Not sure what's implemented

Interesting analogy

- (weighted) Regexps
- Regexp compiler
- (W)FSTs
- Determinization, minimization, …
- Composition
- Inversion

- Source code
- Compiler
- Object code
- Optimization
- Composition
- Inversion

Summary

- FSAs, FSTs, WFSAs, WFSTs as formal systems, with reference to some key operations, algorithms, and toolkits.
- Next time(s): examples of WFSTs, and learning WFSTs from data
  - Speech and MT
  - Semirings
  - Parameter estimation
  - Grammatical inference