Robot Motion Planning

http://voronoi.sbp.ri.cmu.edu/~motion

Howie Choset

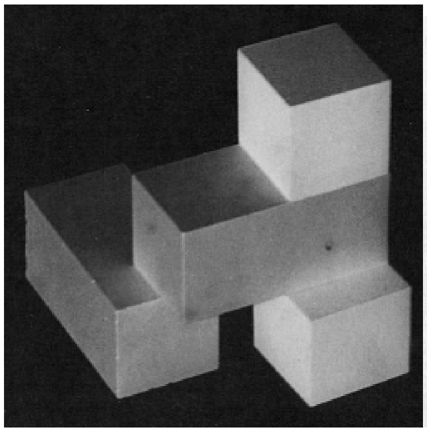
http://voronoi.sbp.ri.cmu.edu/~choset

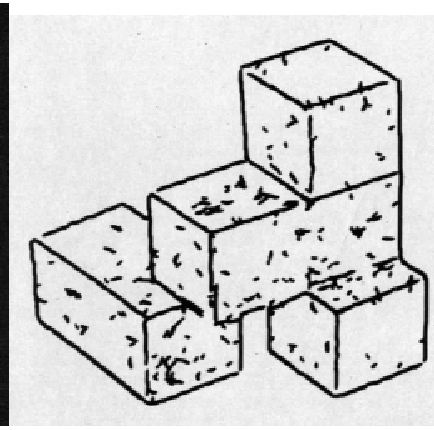
Things Digital Computers Do Well

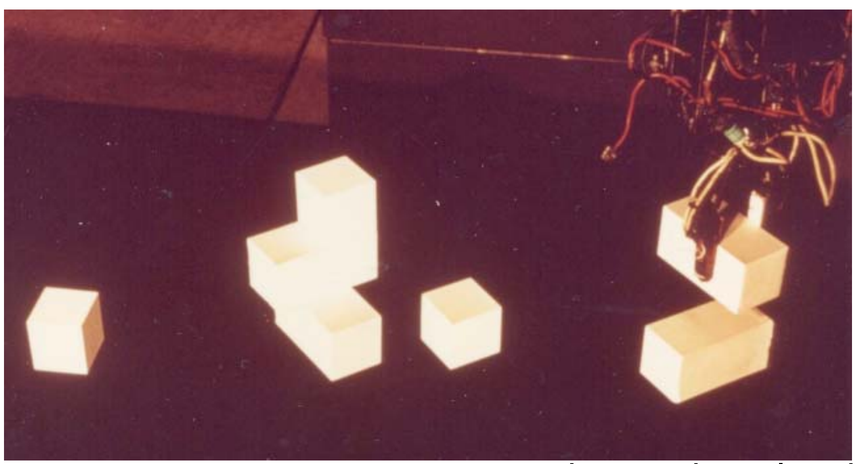
- Arithmetic
  - ALU capable of billions of calculations / sec
- Search
  - systematic exploration of discrete possibilities
- Storage and Retrieval of Data
  - manage huge databases of information
  - storage capacities increasing rapidly and getting cheaper every year…

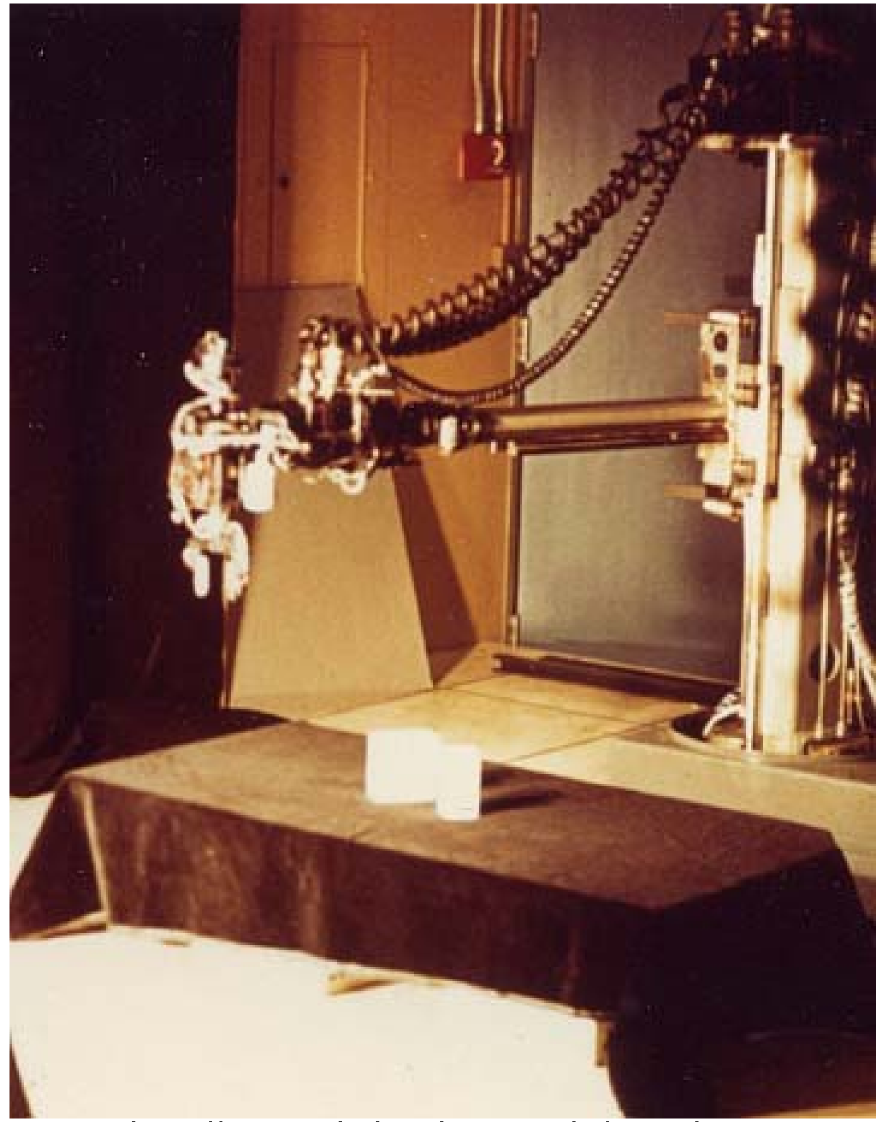
Historical Search-based AI

Blocks World (1960s)



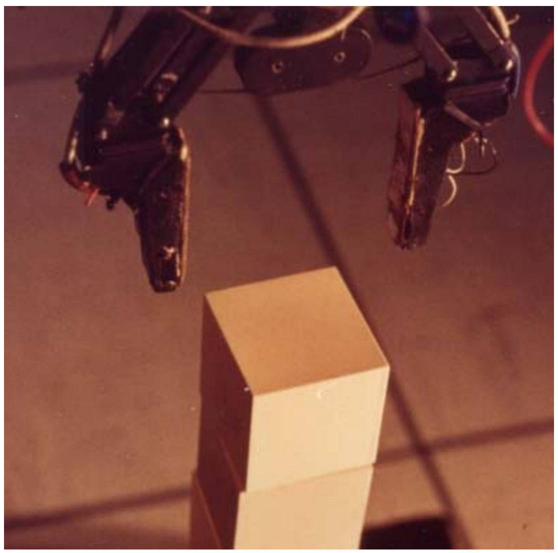


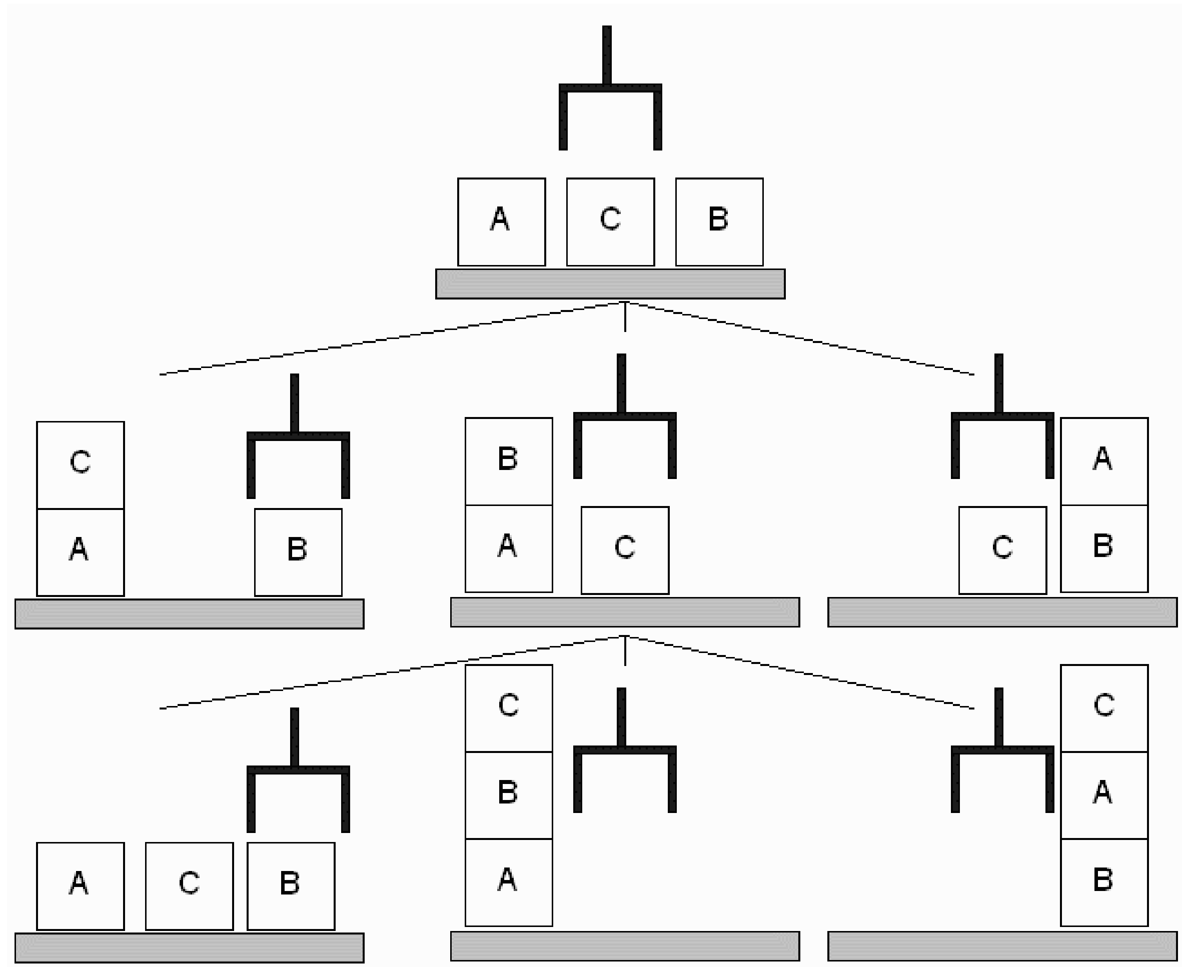




Historical Search-based AI

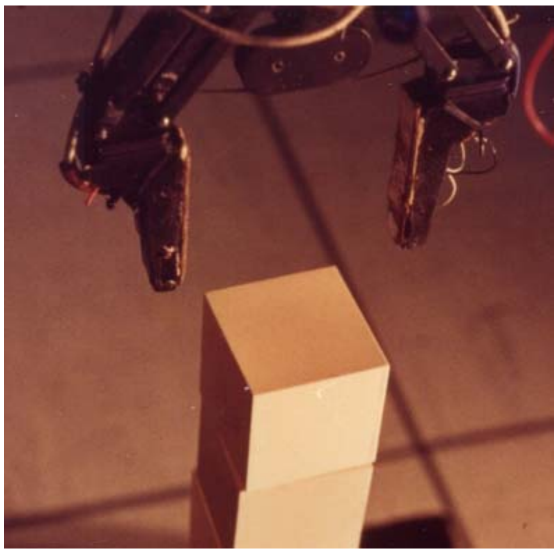
STRIPS Action Planning (1960s)

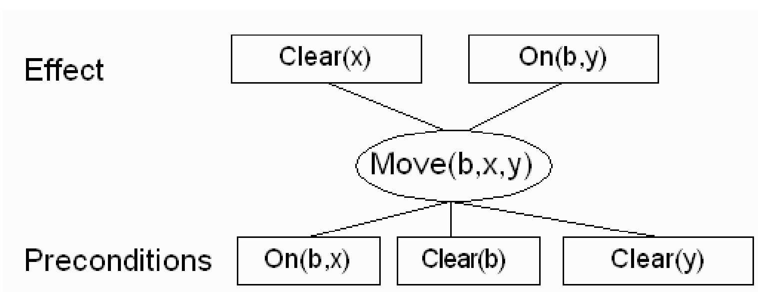


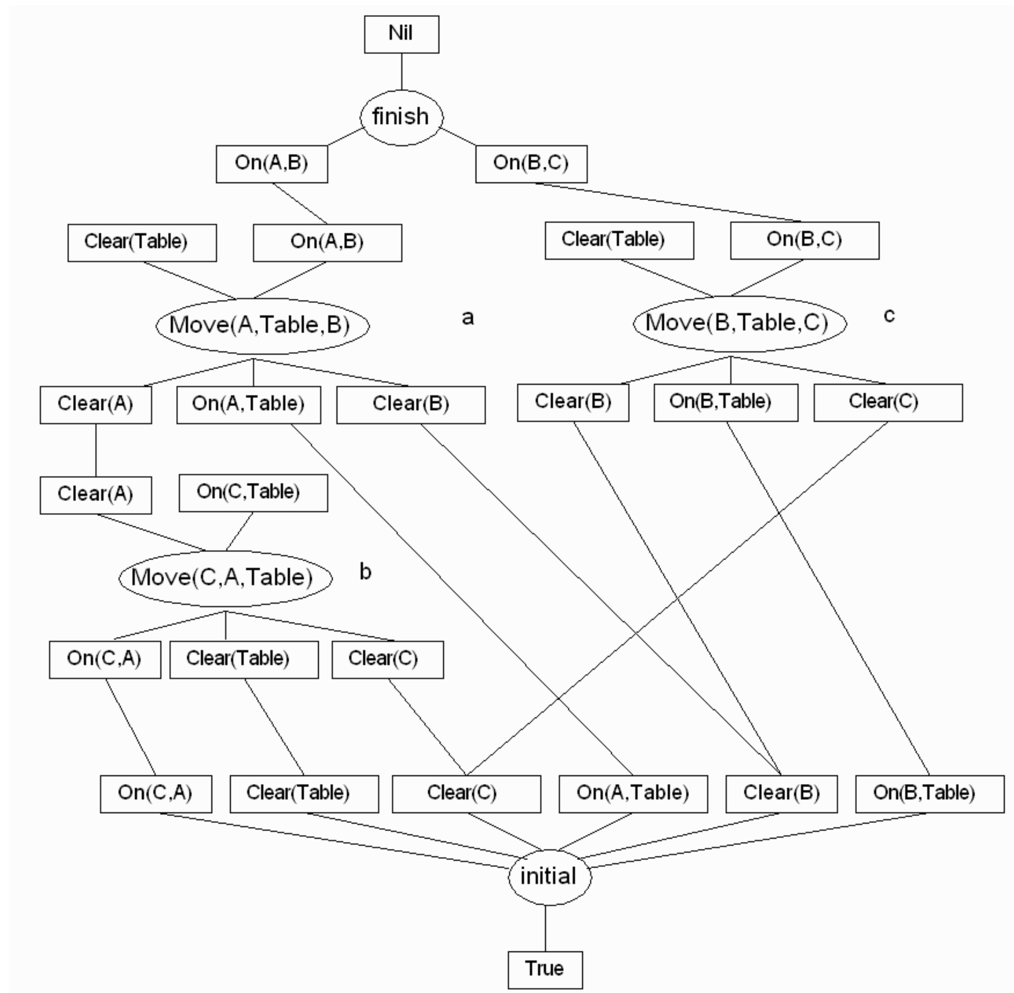


Historical Search-based AI

STRIPS Action Planning (1960s)

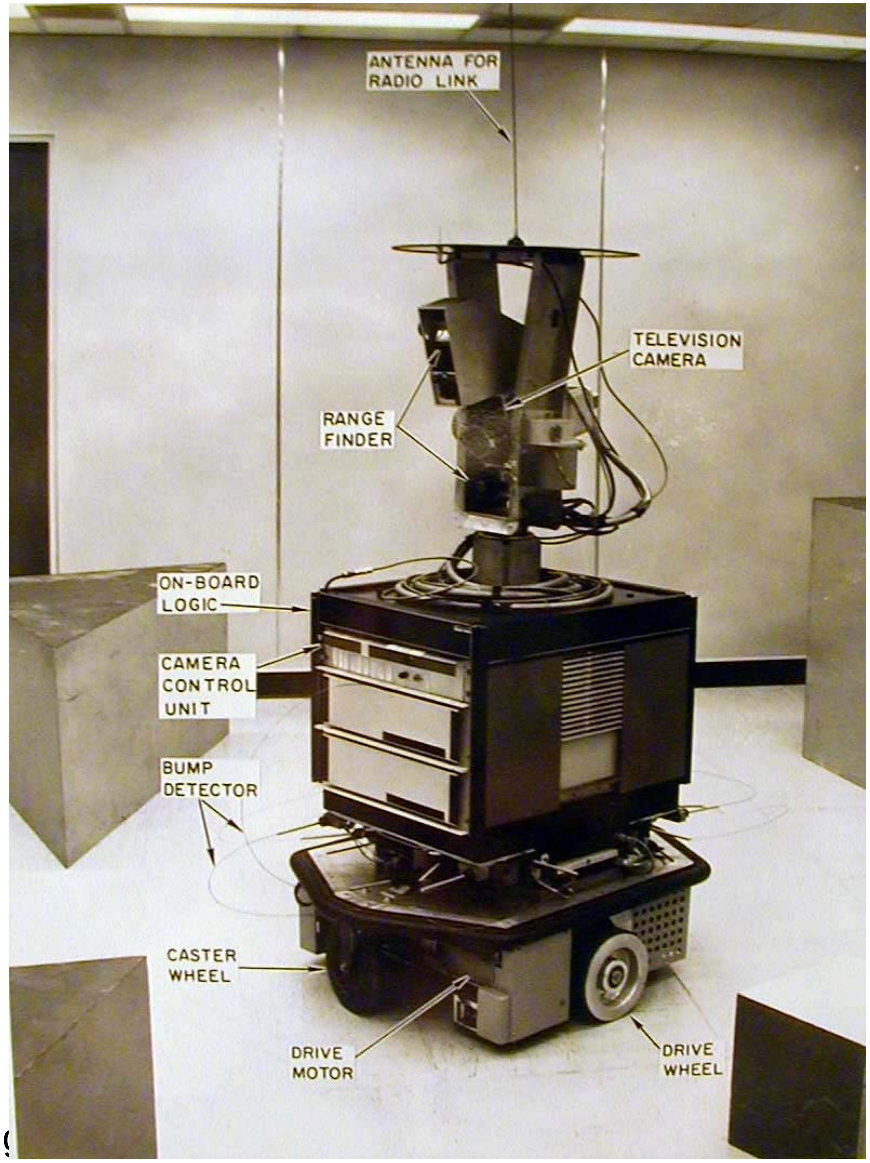






Historical Search-based AI

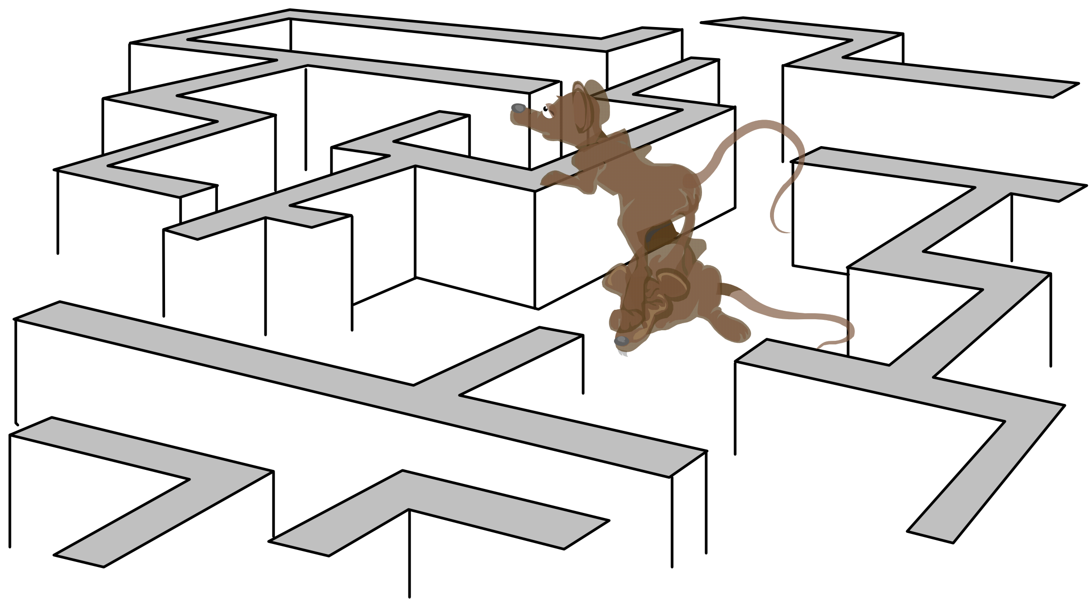
- Shakey the Robot **(SRI, 1966 – 1972)**
- Triangulating range-finder for sensing obstacles
- STRIPS based A* planner for navigating to a goal
- Wireless radio and video camera



What is Motion Planning?

- Determining where to go

  ...more than a search (or a geometric search)



Live Motion Planning Experiments

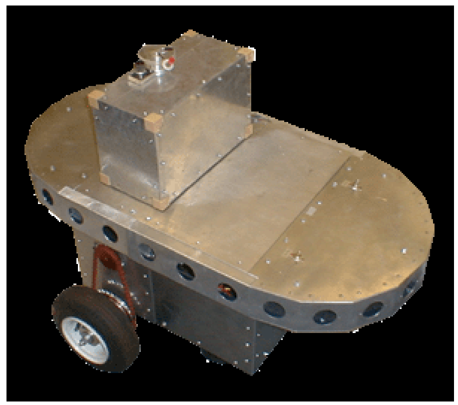
- Person 1 walks through some obstacles
- Person 1, looking at Person 2, directs Person 2 through obstacles
- Person 1, looking at Person 2 with eyes closed, directs Person 2 through obstacles
- Person 1, looking at a map and not Person 2, whose eyes are still closed, directs Person 2 through obstacles
- Person 1, looking at an object and Person 2, whose eyes are closed, directs Person 2 to grab an obstacle
- How do we do the last experiment with a map?

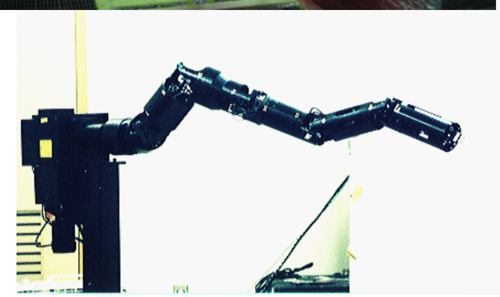
What did we assume?

- Perfect sensors?
  - What information
  - Uncertainty
- Perfect control?
  - What controls?
  - Uncertainty
- Perfect thinking?
  - Knowledge of the world? Complete?
  - Processing the world? Everything?

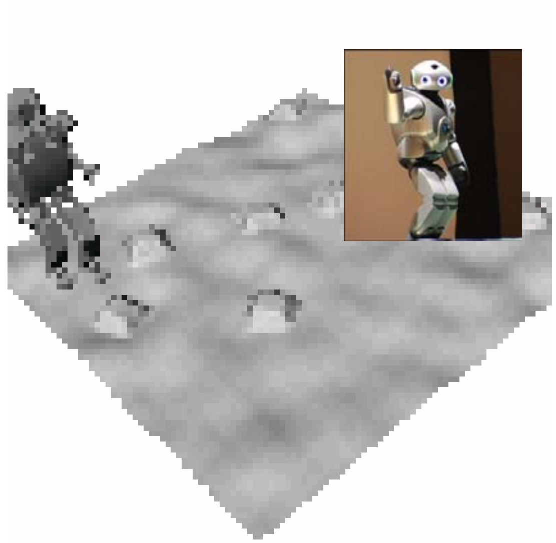
What else?

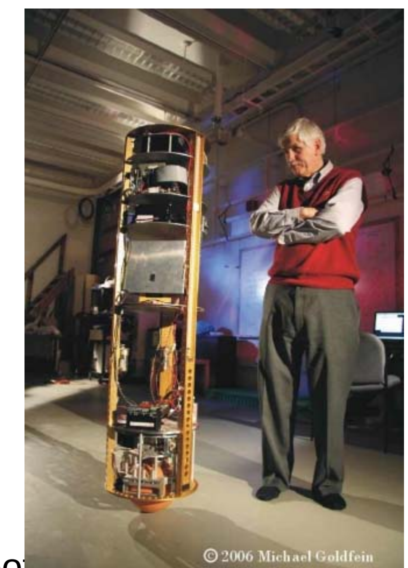
Robots

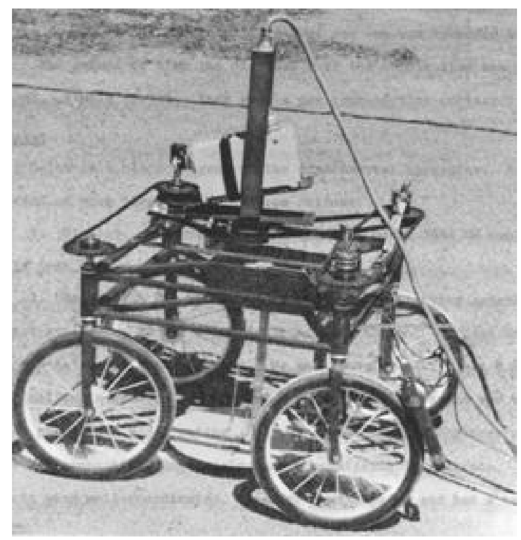




Robots

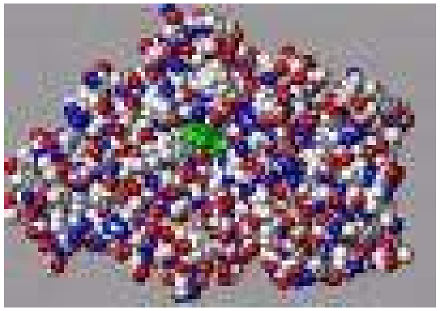
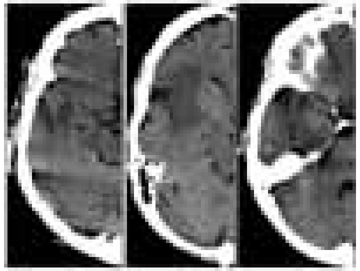






Robots




Trends in Robotics/Motion Planning

Classical Robotics (mid-70's)
- exact models
- no sensing necessary

Reactive Paradigm (mid-80's)
- no models
- relies heavily on good sensing

Hybrids (since 90's)
- model-based at higher levels
- reactive at lower levels

Probabilistic Robotics (since mid-90's)
- seamless integration of models and sensing
- inaccurate models, inaccurate sensors

Copied from G. Hager

Overview

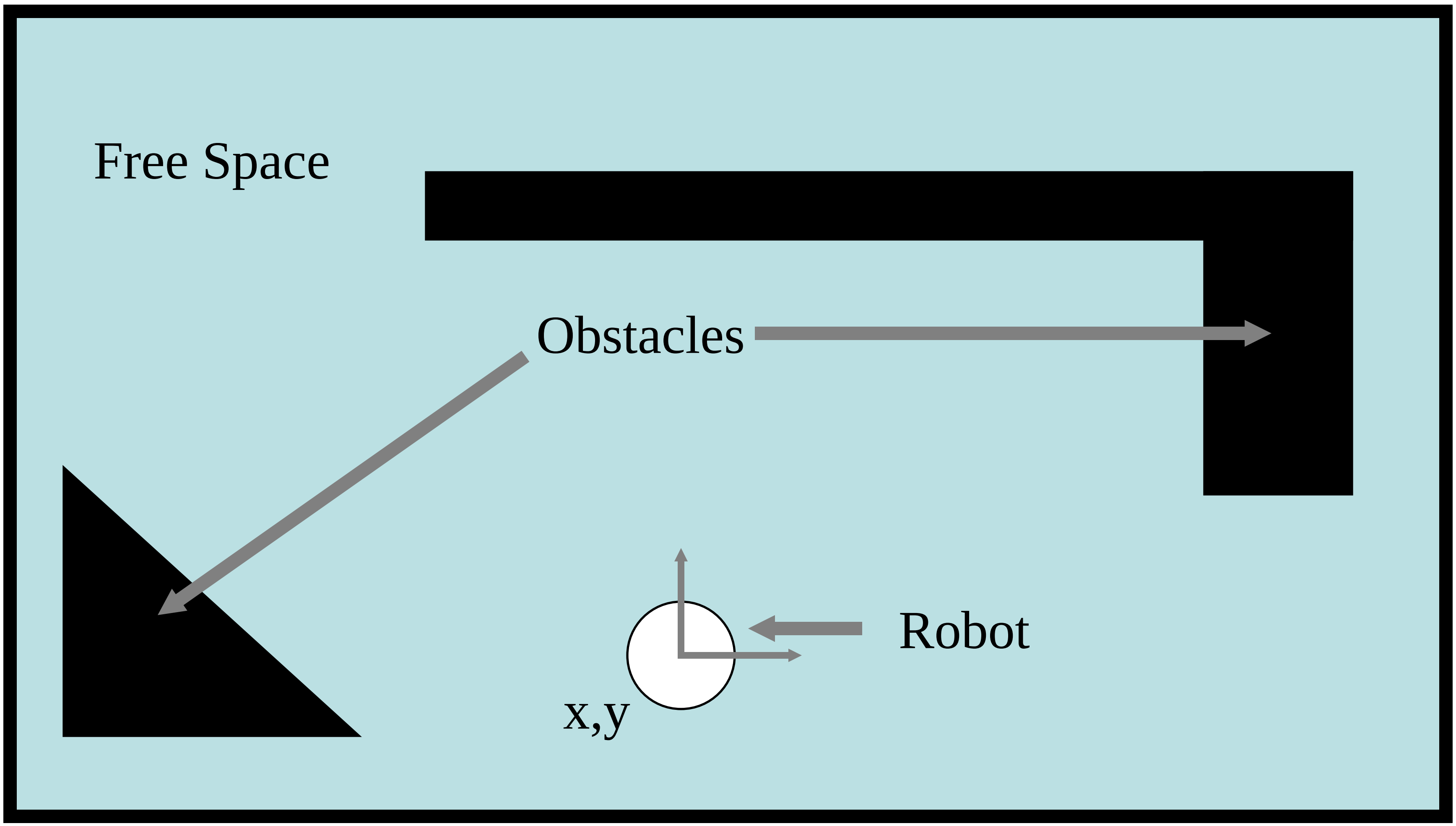
- Planning tasks
  - Navigation
  - Coverage
  - Localization
  - Mapping
- Properties of the robot
  - degrees of freedom,
  - holonomic or not,
  - kinematic vs. dynamic
- Properties of Algorithms
  - Optimality
  - Computational complexity
  - Completeness
    - Resolution completeness
    - Probabilistic completeness
  - Online vs. offline
  - Sensor-based vs. not
  - Feedback or not

Mathematical Rigor

Symbol	Meaning
$\exists$	there exists
$\forall$	for all
∞	infinity
$\in$	element
$\notin$	not in
s.t.	such that
$\mathbb{R}$	real numbers
$\mathbb{R}^m$	m-dimensioned real numbers
$\bigcup$	union
$\bigcap$	intersection
$\setminus$	set difference
$\Rightarrow$	implies. $p \to q$ is p implies q
$\Leftarrow$	implies. $q \to p$ is q implies p
$\Longleftrightarrow$	if and only if
S^1	a circle
∇	gradient
D	differential or distance to closest obstacle (depending on context)
d_i	distance to obstacle i in either the workspace or configuration space (depending on context)
$d(x, y)$	distance between the two points x and y
Null	null space

J	Jacobian
Γ	Christoffel symbol
RM	roadmap
$\mathcal{W}$	workspace
$\mathcal{Q}$	configuration space
$\mathcal{Q}_{\text{free}}$	free space
$x(k)$	state at time k
$\|\|x\|\|$	norm of x
$\subseteq$	subset of
$\subset$	strict subset of
$\text{cl}(A)$	closure of A
T^n	n-dimensional torus
S^n	n-dimensional sphere in $\mathbb{R}^{n+1}$
$SO(n)$	special orthogonal group
$SE(n)$	special Euclidean group
$B_\epsilon(q)$	open ball of radius ϵ centered at q
Df	differential of f
∇f	gradient of f
∇	affine connection
$\nabla_{Y_1} Y_2$	covariant derivative of Y_2 with respect to Y_1
C^0	continuous
C^n	n times differentiable
$\langle x, y \rangle$	inner product of x and y
$\mathcal{I}$	identity matrix
$\text{atan2}(y, x)$	returns angle to (x, y) in the plane in range $[-\pi, \pi)$
$T_x\mathcal{M}$	tangent space of $\mathcal{M}$ at x
$T\mathcal{M}$	tangent bundle of $\mathcal{M}$
$[f, g]$	Lie bracket of vector fields f, g
$\overline{\text{Lie}}(\mathcal{G})$	the Lie algebra of a set of vector fields $\mathcal{G}$
$\overline{\mathcal{D}}$	involutive closure of the distribution $\mathcal{D}$
$\mathcal{U}_\pm$	control set positively spanning $\mathbb{R}^m$
$\mathcal{U}_+$	control set spanning $\mathbb{R}^m$
$\langle Y_1 : Y_2 \rangle$	the symmetric product of vector fields Y_1 and Y_2

Example of a World (and Robot)

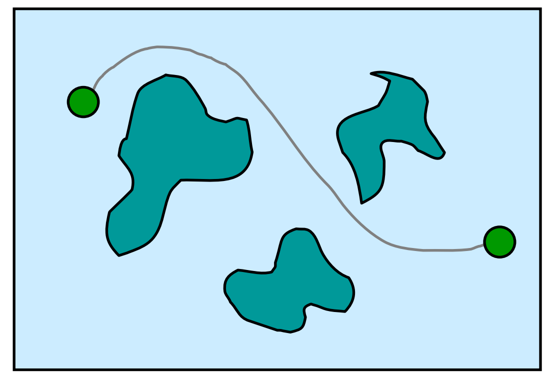


Basic Path Planning

Problem Statement:

Compute a continuous sequence of collision-free robot configurations connecting the initial and goal configurations

2D EXAMPLE:



Geometry of environment

Geometry and kinematics of robot

Initial and goal configurations

→ Path Planner → Collision-free path

Motion Planning Statement

If **W** denotes the robot's workspace,

And $\mathbf{WO_i}$ denotes the i'th obstacle,

Then the robot's free space, $\mathbf{W_{free}}$, is defined as:

$$\mathbf{W_{free} = W - (\bigcup WO_i)}$$

And a path $\mathbf{c} \in \mathbf{C^0}$ is $\mathbf{c : [0,1] \rightarrow W_{free}}$

where $\mathbf{c(0)}$ is $\mathbf{q_{start}}$ and $\mathbf{c(1)}$ is $\mathbf{q_{goal}}$

Topics

- Bug Algorithms
- Curve Following
- Sensors
- Configuration Space for Round Mobile Robot
- Potential Functions
- Graph Search (A* D*)
- Pixel Maps
- Configuration Space for non-Round Robots
- Roadmaps
- Coverage
- Sample-based Methods
- Kalman Filtering (for Localization, SLAM)
- Bayesian Techniques (for Localization, SLAM)
- Dynamics and Non-holonomic Constraints, if time permits

Homework Assignments

- HW 1: Getting started
- HW 2: Bug Algorithm
- HW 3: Two-dimensional Potential Function
- HW 4: A*
- HW 5: Visibility Graph
- HW 6: Voronoi Diagram
- HW 7: Probabilistic Roadmap

Homework Ground Rules

- Make a web site, with scanned in solutions and images/videos showing your work
- Get your own display programs
- Due at 9pm on the due dates (mail to TA)
- Worth 100 points
- Up to 30 bonus points for best assignment at the discretion of the TA

Class Project

- On a real robot
  - Subject to negotiation
  - Find a friend with a robot
- Implement a Bayesian or Kalman Filter or Comparable mapping method
- Videos, Possible Live Demos, Class Presentation
- Proposal (can work in groups)

Grading

- 70% Homework
- 30% Project

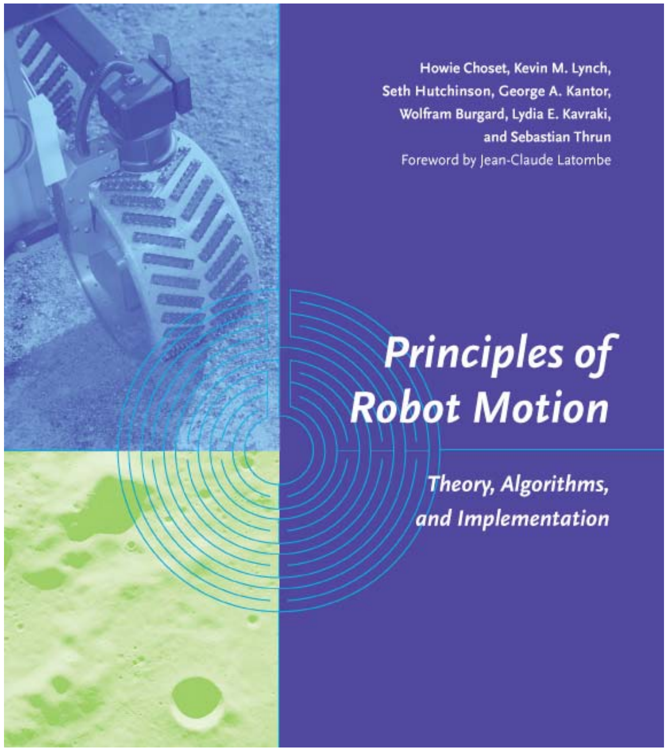
Book

http://motionplanning.com

Principles of Robot Motion: Theory, Algorithms, and Implementations

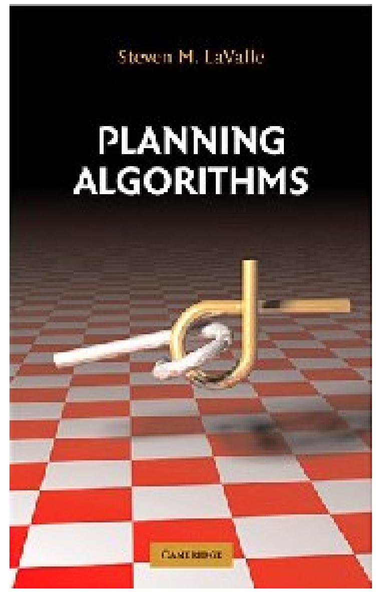
H. Choset, K. M. Lynch, S. Hutchinson, G. Kantor, W. Burgard, L. E. Kavraki and S. Thrun,

MIT Press, Boston, 2005.



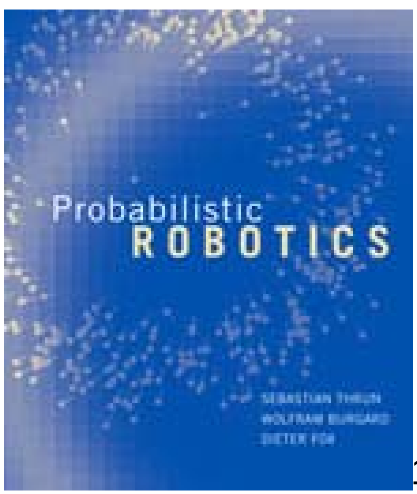
Other Books

Robot Motion Planning, Jean-Claude Latombe, Kluwer, 1991.

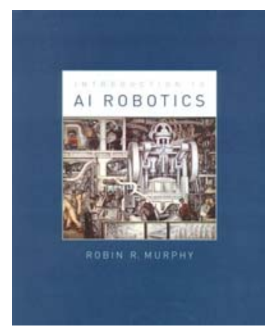


Planning Algorithms
Steven Lavalle, Cambridge University Prress, 2006

Free download:
http://planning.cs.uiuc.edu/



Probabilistic Robotics
S. Thrun, W. Burgard, D. Fox
MIT Press, 2006



An Introduction to AI/Robotics
Robin Murphy
MIT Press, 2006

My Goals for Class

- Teach people motion planning
- Learn about motion planning
  - From teaching, book
  - From class
- See impressive demonstrations
- Understand the right “level” and “capability” of motion planning
- Have fun (Demos are fun)