

Conflict-Driven Clause Learning

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Automated Reasoning and Satisfiability

September 13, 2023

The Satisfiability (SAT) problem

$$\begin{aligned} & (\chi_5 \vee \chi_8 \vee \bar{\chi}_2) \wedge (\chi_2 \vee \bar{\chi}_1 \vee \bar{\chi}_3) \wedge (\bar{\chi}_8 \vee \bar{\chi}_3 \vee \bar{\chi}_7) \wedge (\bar{\chi}_5 \vee \chi_3 \vee \chi_8) \wedge \\ & (\bar{\chi}_6 \vee \bar{\chi}_1 \vee \bar{\chi}_5) \wedge (\chi_8 \vee \bar{\chi}_9 \vee \chi_3) \wedge (\chi_2 \vee \chi_1 \vee \chi_3) \wedge (\bar{\chi}_1 \vee \chi_8 \vee \chi_4) \wedge \\ & (\bar{\chi}_9 \vee \bar{\chi}_6 \vee \chi_8) \wedge (\chi_8 \vee \chi_3 \vee \bar{\chi}_9) \wedge (\chi_9 \vee \bar{\chi}_3 \vee \chi_8) \wedge (\chi_6 \vee \bar{\chi}_9 \vee \chi_5) \wedge \\ & (\chi_2 \vee \bar{\chi}_3 \vee \bar{\chi}_8) \wedge (\chi_8 \vee \bar{\chi}_6 \vee \bar{\chi}_3) \wedge (\chi_8 \vee \bar{\chi}_3 \vee \bar{\chi}_1) \wedge (\bar{\chi}_8 \vee \chi_6 \vee \bar{\chi}_2) \wedge \\ & (\chi_7 \vee \chi_9 \vee \bar{\chi}_2) \wedge (\chi_8 \vee \bar{\chi}_9 \vee \chi_2) \wedge (\bar{\chi}_1 \vee \bar{\chi}_9 \vee \chi_4) \wedge (\chi_8 \vee \chi_1 \vee \bar{\chi}_2) \wedge \\ & (\chi_3 \vee \bar{\chi}_4 \vee \bar{\chi}_6) \wedge (\bar{\chi}_1 \vee \bar{\chi}_7 \vee \chi_5) \wedge (\bar{\chi}_7 \vee \chi_1 \vee \chi_6) \wedge (\bar{\chi}_5 \vee \chi_4 \vee \bar{\chi}_6) \wedge \\ & (\bar{\chi}_4 \vee \chi_9 \vee \bar{\chi}_8) \wedge (\chi_2 \vee \chi_9 \vee \chi_1) \wedge (\chi_5 \vee \bar{\chi}_7 \vee \chi_1) \wedge (\bar{\chi}_7 \vee \bar{\chi}_9 \vee \bar{\chi}_6) \wedge \\ & (\chi_2 \vee \chi_5 \vee \chi_4) \wedge (\chi_8 \vee \bar{\chi}_4 \vee \chi_5) \wedge (\chi_5 \vee \chi_9 \vee \chi_3) \wedge (\bar{\chi}_5 \vee \bar{\chi}_7 \vee \chi_9) \wedge \\ & (\chi_2 \vee \bar{\chi}_8 \vee \chi_1) \wedge (\bar{\chi}_7 \vee \chi_1 \vee \chi_5) \wedge (\chi_1 \vee \chi_4 \vee \chi_3) \wedge (\chi_1 \vee \bar{\chi}_9 \vee \bar{\chi}_4) \wedge \\ & (\chi_3 \vee \chi_5 \vee \chi_6) \wedge (\bar{\chi}_6 \vee \chi_3 \vee \bar{\chi}_9) \wedge (\bar{\chi}_7 \vee \chi_5 \vee \chi_9) \wedge (\chi_7 \vee \bar{\chi}_5 \vee \bar{\chi}_2) \wedge \\ & (\chi_4 \vee \chi_7 \vee \chi_3) \wedge (\chi_4 \vee \bar{\chi}_9 \vee \bar{\chi}_7) \wedge (\chi_5 \vee \bar{\chi}_1 \vee \chi_7) \wedge (\chi_5 \vee \bar{\chi}_1 \vee \chi_7) \wedge \\ & (\chi_6 \vee \chi_7 \vee \bar{\chi}_3) \wedge (\bar{\chi}_8 \vee \bar{\chi}_6 \vee \bar{\chi}_7) \wedge (\chi_6 \vee \chi_2 \vee \chi_3) \wedge (\bar{\chi}_8 \vee \chi_2 \vee \chi_5) \end{aligned}$$

Does there exist an assignment satisfying all clauses?

Search for a satisfying assignment (or proof none exists)

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SAT Solver Paradigms Overview

DPLL: Aims at finding a small search-tree by selecting effective splitting variables (e.g. via looking ahead).

Strength: Effective on small, hard formulas.

Weakness: Expensive.



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Conflict-driven clause learning (CDCL): Makes fast decisions and converts conflicts into learned clauses.

Strength: Effective on large, “easy” formulas.

Weakness: Hard to parallelize.



Conflict-driven Clause Learning: Overview

- Most successful architecture
- Superior on industrial benchmarks
- Brute-force?
 - Addition conflict clauses
 - Fast unit propagation
- Complete local search (for a refutation)?
- State-of-the-art (sequential) CDCL solvers:
Kissat, CaDiCaL, Glucose, CryptoMiniSAT

Clause Learning

Data-structures

Heuristics

Clause Management

Conflict-Clause Minimization

Recent Advances and Conclusions

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Recent Advances and Conclusions

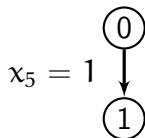
Conflict-driven SAT solvers: Search and Analysis

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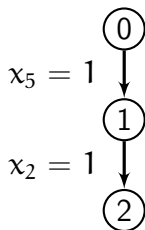
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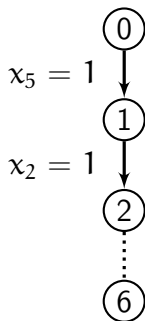
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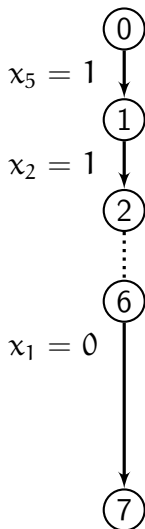
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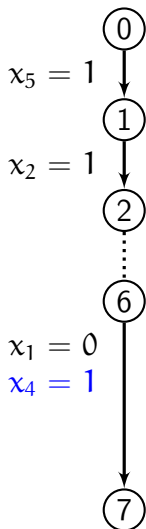
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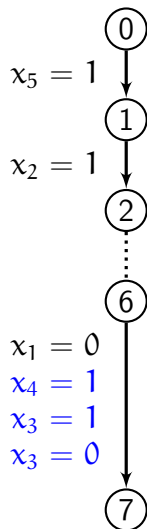
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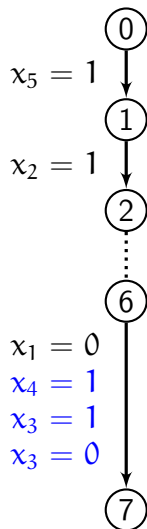
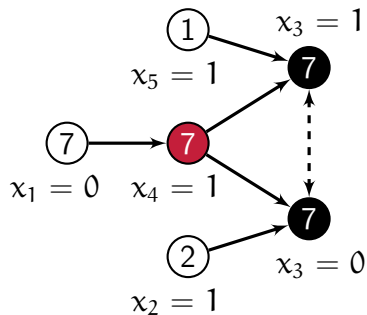
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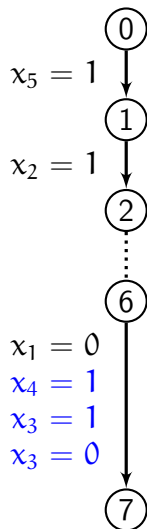
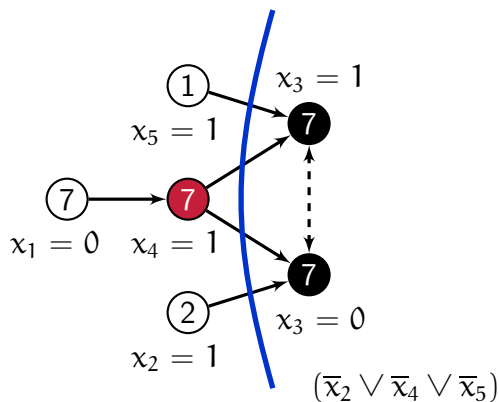
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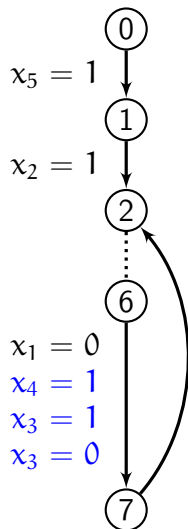
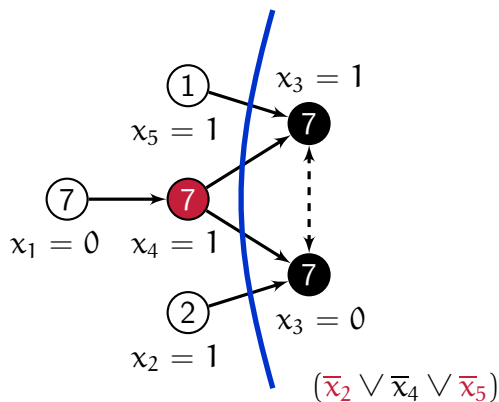
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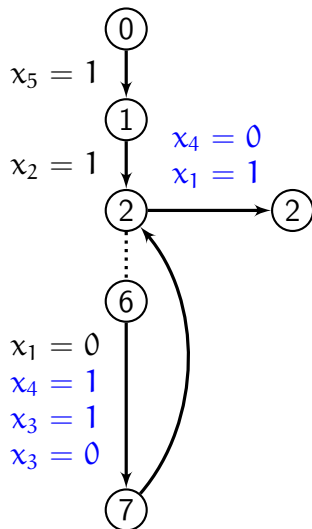
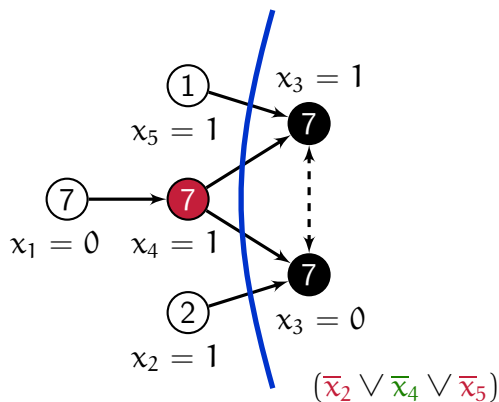
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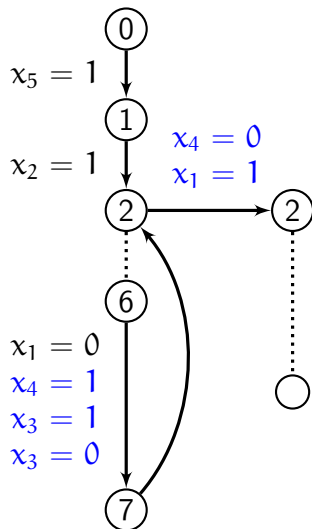
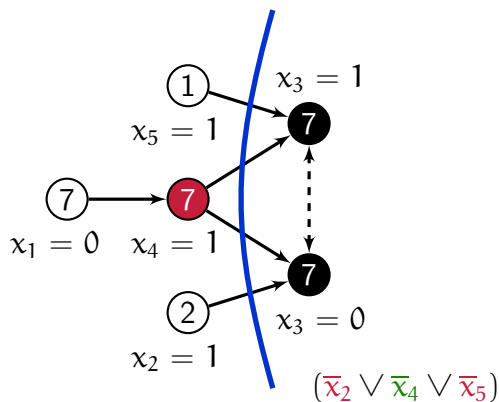
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Implication graph [Marques-SilvaSakallah '96]

CDCL in a nutshell:

1. Main loop combines **efficient** problem simplification with **cheap**, but effective decision heuristics; (> 90% of time)
2. Reasoning kicks in if the current state is **conflicting**;
3. The current state is analyzed and turned into a **constraint**;
4. The constraint is **added** to the problem, the heuristics are **updated**, and the algorithm (partially) **restarts**.

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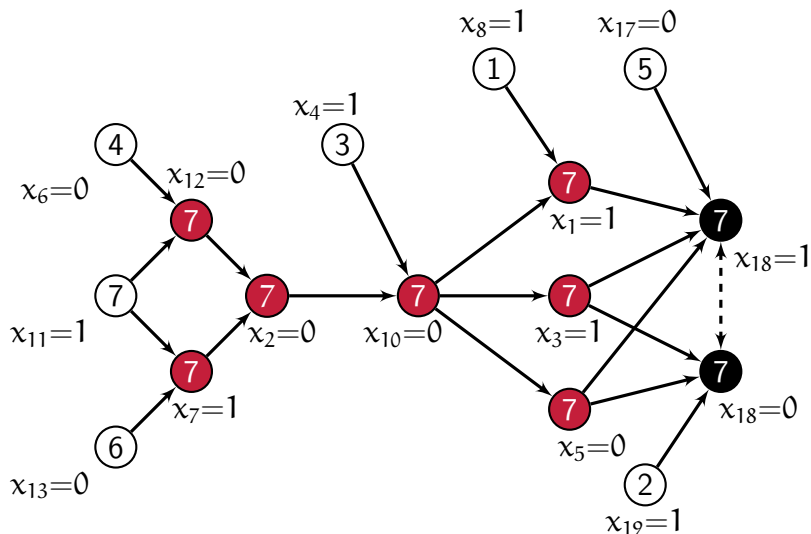
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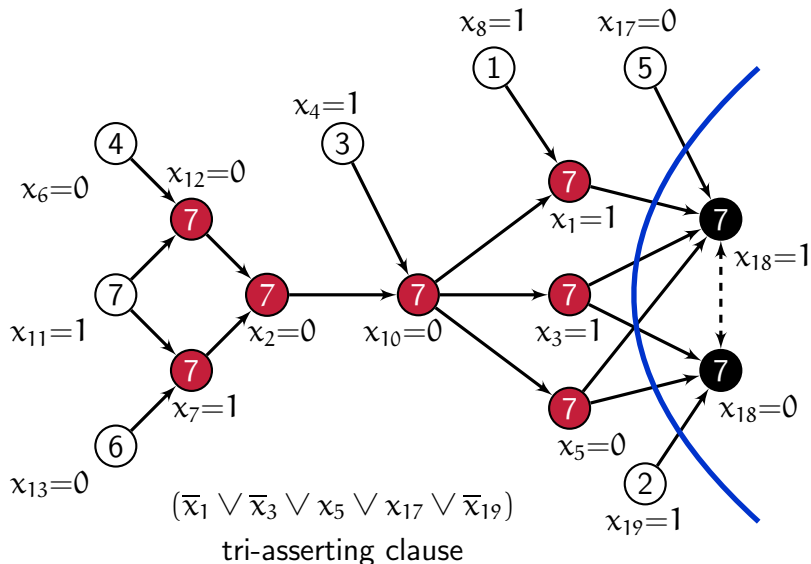
However, it has three weaknesses:

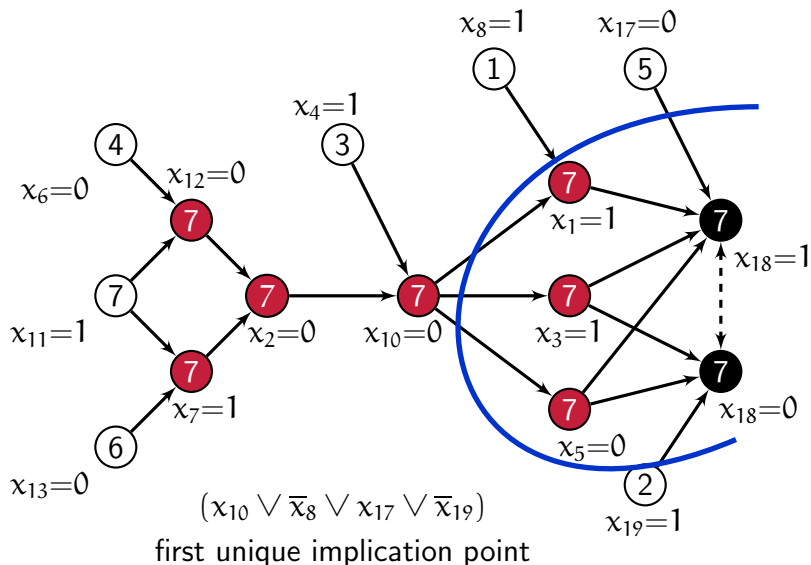
- CDCL is notoriously hard to **parallelize**;
- the **representation** impacts CDCL performance; and
- CDCL has **exponential runtime** on some “simple” problems.

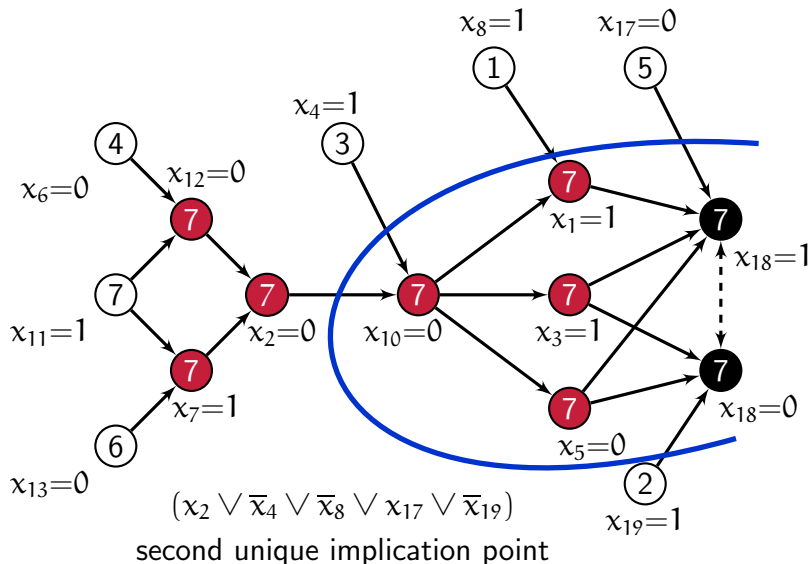
Conflict-driven Clause Learning: Pseudo-code

```
1: while TRUE do
2:    $l_{\text{decision}} := \text{Decide} ()$ 
3:   If no  $l_{\text{decision}}$  then return satisfiable
4:    $\Gamma := \text{Simplify} (\Gamma(l_{\text{decision}} \leftarrow 1))$ 
5:   while  $\Gamma$  contains  $C_{\text{falsified}}$  do
6:      $C_{\text{conflict}} := \text{Analyze} (C_{\text{falsified}})$ 
7:     If  $C_{\text{conflict}} = \perp$  then return unsatisfiable
8:     BackTrack ( $C_{\text{conflict}}$ )
9:      $\Gamma := \text{Simplify} (\Gamma \cup \{C_{\text{conflict}}\})$ 
10:  end while
11: end while
```

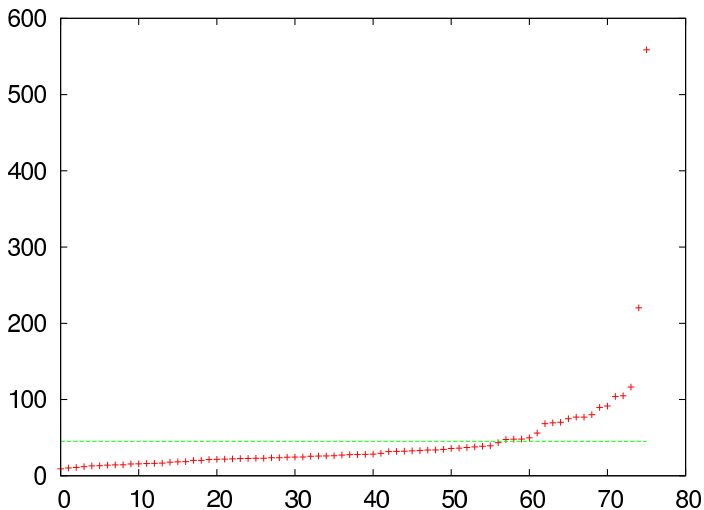









Average Learned Clause Length



Clause Learning

Data-structures

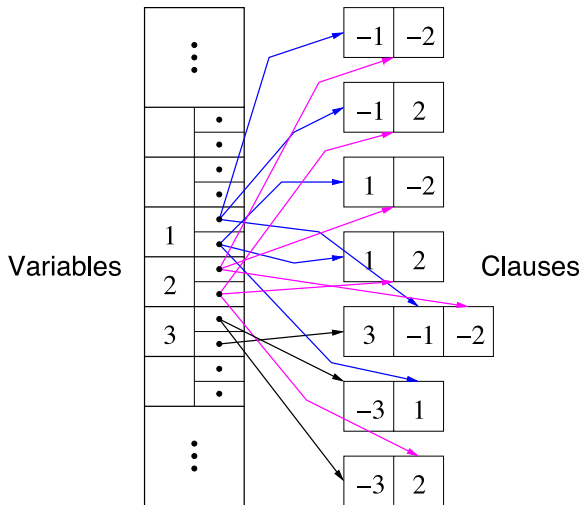
Heuristics

Clause Management

Conflict-Clause Minimization

Recent Advances and Conclusions

Simple data structure for unit propagation



Conflict-driven: Watch pointers (1) [MoskewiczMZZM'01]

$$\alpha = \{\chi_1 = *, \chi_2 = *, \chi_3 = *, \chi_4 = *, \chi_5 = *, \chi_6 = *\}$$



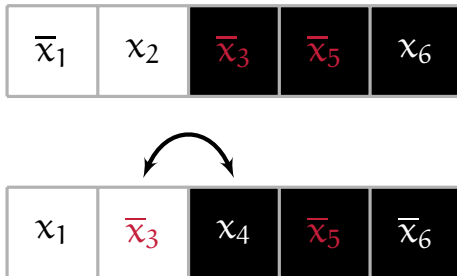
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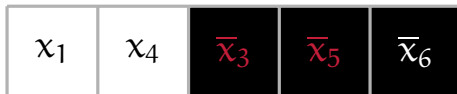
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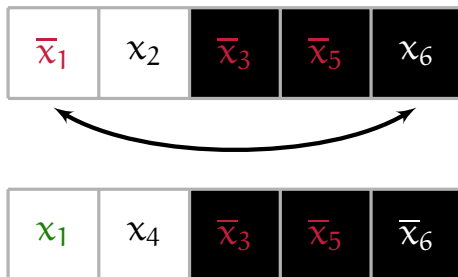
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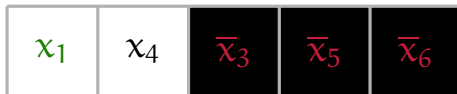
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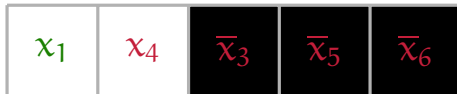
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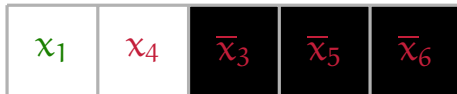
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Conflict-driven: Watch pointers (2) [MoskewiczMZZM'01]

Only examine (get in the cache) a clause when both

- a watch pointer gets falsified
- the other one is not satisfied

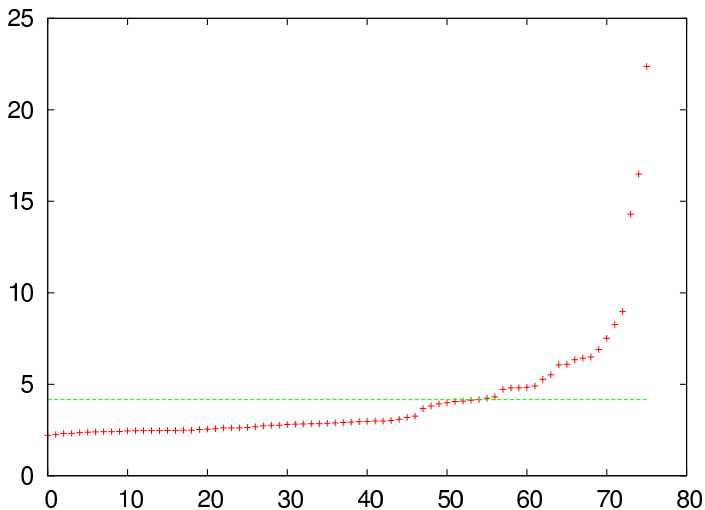
While backjumping, just unassign variables

Conflict clauses \rightarrow watch pointers

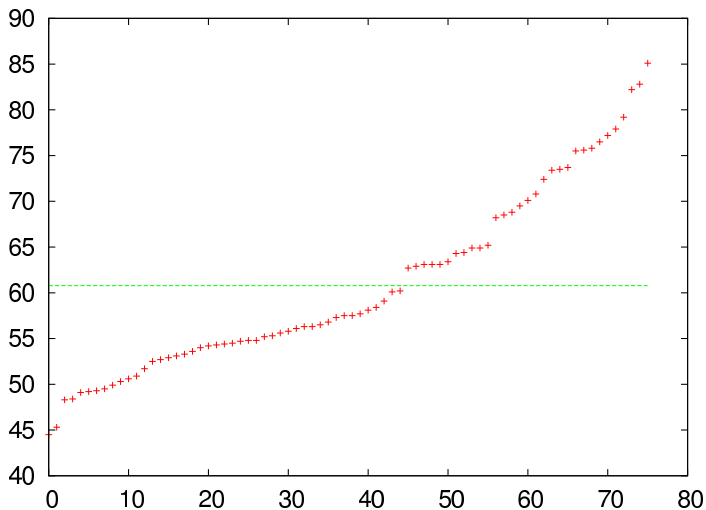
No detailed information available

Not used for binary clauses

Average Number Clauses Visited Per Propagation



Percentage visited clauses with other watched literal true



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- plus: could compensate a bad value selection

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Restart strategies

- aim: avoid heavy-tail behavior [GomesSelmanCrato'97]
- plus: focus search on recent conflicts when combined with dynamic heuristics

Variable selection heuristics

Based on the occurrences in the (reduced) formula

- examples: Jeroslow-Wang, Maximal Occurrence in clauses of Minimal Size (MOMS), look-aheads
- not practical for CDCL solver due to watch pointers

Variable selection heuristics

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Variable State Independent Decaying Sum (VSIDS)

- original idea (zChaff): for each conflict, increase the score of involved variables by 1, half all scores each 256 conflicts
[MoskewiczMZZM'01]
- improvement (MiniSAT): for each conflict, increase the score of involved variables by δ and increase $\delta := 1.05\delta$
[EenSörensson'03]

Visualization of VSIDS in PicoSAT

<http://www.youtube.com/watch?v=M0jhFywLre8>

Value selection heuristics

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- negative branching (early MiniSAT) [EenSörensson'03]

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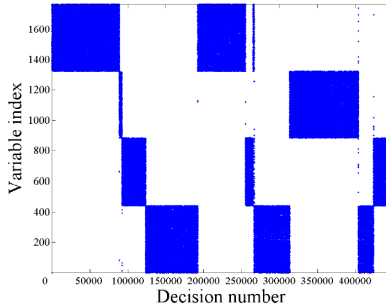
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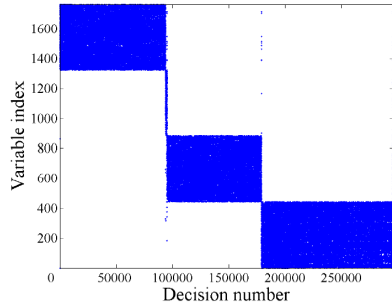
Based on the last implied value (phase-saving)

- introduced to CDCL [PipatsrisawatDarwiche'07]
- already used in local search [HirschKojevnikov'01]

Selecting the last implied value remembers solved components



negative branching



phase-saving

Restarts

Restarts in CDCL solvers:

- Counter heavy-tail behavior [GomesSelmanCrato'97]
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Restart strategies: [Walsh'99, LubySinclairZuckerman'93]

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- Luby sequence: e.g. 100, 100, 200, 100, 100, 200, 400, ...

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Rapid restarts by reusing trail: [vanderTakHeuleRamos'11]

- Partial restart same effect as full restart
- Optimal strategy Luby-1: 1, 1, 2, 1, 1, 2, 4, ...

Heuristics: SAT vs UNSAT [Oh'15]

The best heuristics choices depend on satisfiability: E.g.

- Restart **frequently** for UNSAT instances to get conflict early
- Restart **sporadically** for SAT instances to keep “progress”

Also, **keeping** learned clauses is **less important** on SAT instances and can actually slow down the search.

State-of-the-art CDCL solvers, such as CaDiCaL, have **separate modes** for SAT and UNSAT and they alternate between them.

Clause Learning

Data-structures

Heuristics

Clause Management

Conflict-Clause Minimization

Recent Advances and Conclusions

Clause deletion [EenSörensson'03, AudemardSimon'09]

Conflict clauses can significantly **slow down** CDCL solvers:

- Conflict clauses can quickly **outnumber** the original clauses
- Conflict clauses consists of **important variables**

Clause deletion is used to reduce the overhead:

- When the learned clause reach a limit, **remove half**
- **Increase limit** after every removal (completeness)

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Clause deletion heuristics:

- **length** of the clause
- **relevance** of the clause (when was it used in Analyze)
- the number of **involved decision levels**

Clause Learning

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Recent Advances and Conclusions

Self-Subsumption

Use self-subsumption to shorten conflict clauses

$$\frac{C \vee l \quad D \vee \bar{l}}{D} \quad C \subseteq D \quad \frac{(a \vee b \vee l) \quad (a \vee b \vee c \vee \bar{l})}{(a \vee b \vee c)}$$

Conflict clause minimization is an important optimization.

Self-Subsumption

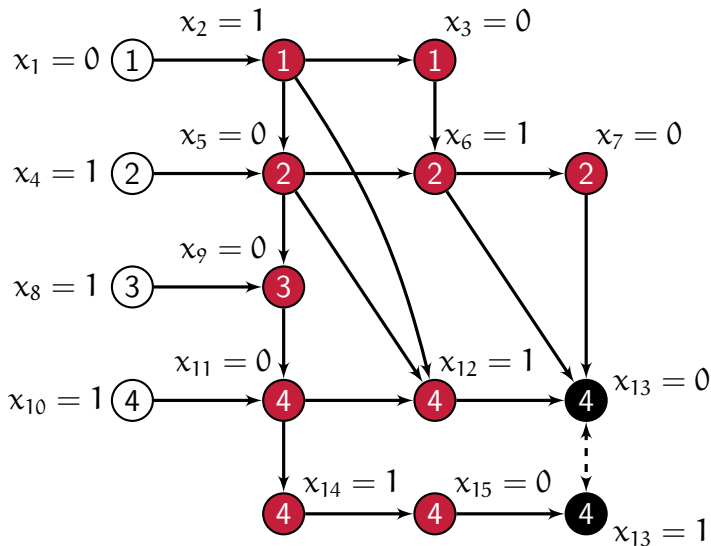
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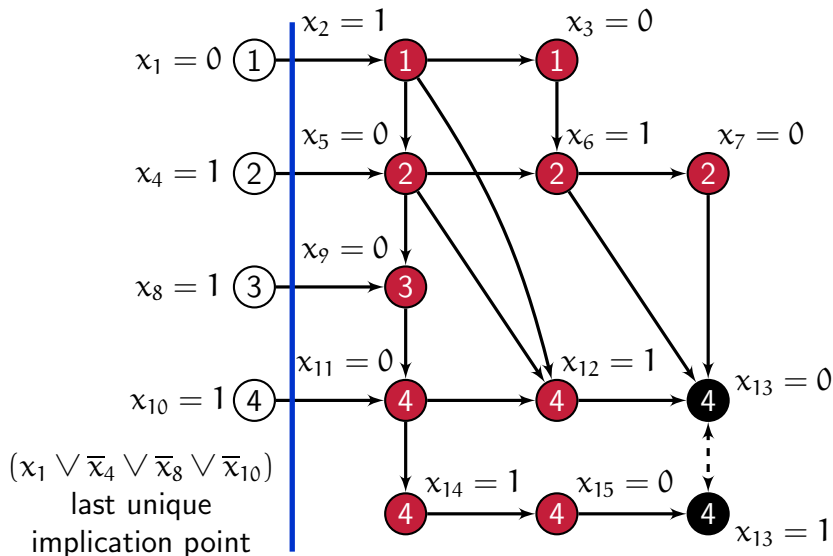
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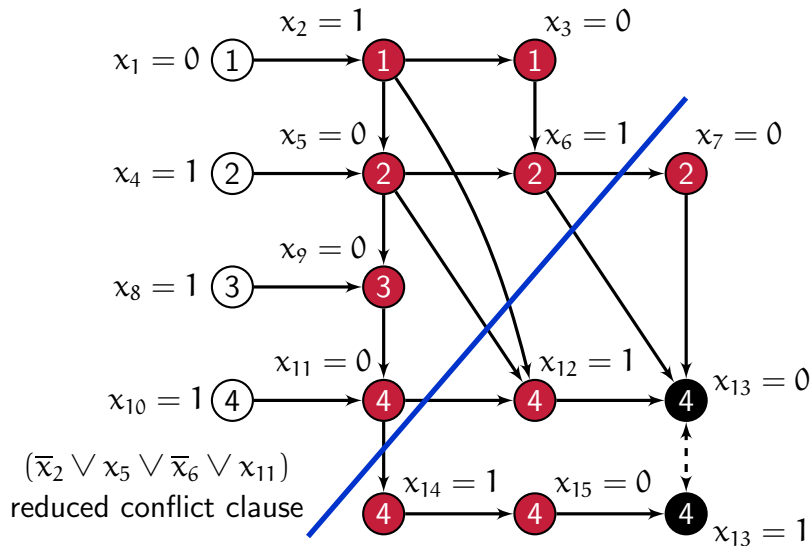
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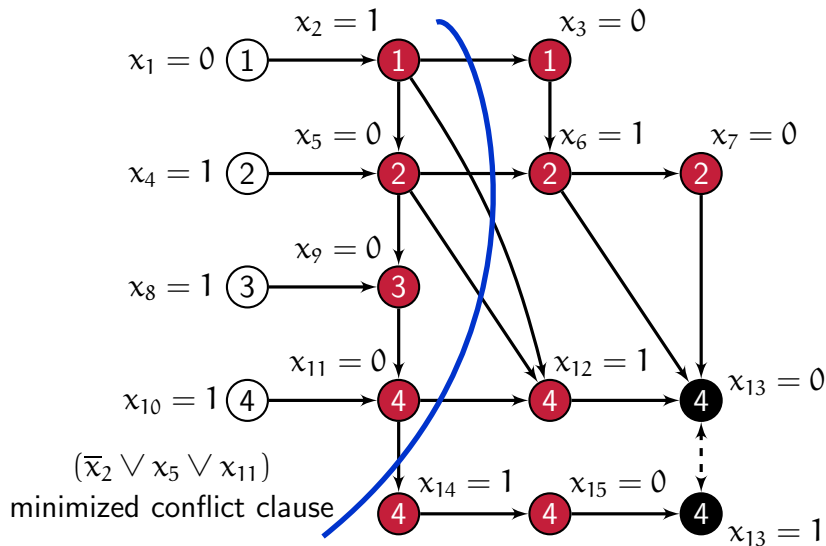
Use implication chains to further minimization:

$$\begin{aligned} & \dots (\bar{a} \vee b)(\bar{b} \vee c)(a \vee c \vee d) \dots \Rightarrow \\ & \dots (\bar{a} \vee b)(\bar{b} \vee c)(c \vee d) \dots \end{aligned}$$









Clause Learning

Data-structures

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Recent Advances and Conclusions

Recent Advances (I)

A new idea contributes to **winning the competition**.

Winner 2017: Clause vivification during search
[LuoLiXiaoManyáLü'17]

Winner 2018: Chronological backtracking
[NadelRyvchin'18]

Winner 2019: Multiple learnt clauses per conflict
[KochemazovZaikinKondratievSemenov'19]

Winner 2020: Back to C and “target phases”
[BiereFleury'20]

Recent Advances (II)

A new idea contributes to **winning the competition**.

Winner 2021: Selecting decision heuristics using RL
[CherifHabetTerrioux'21]

Winner 2022: Decision tree of local search strategies
[ZhengHeChenZhouLi'22]

Winner 2023: Structured Reencoding of Formulas
[HaberlandtGreen'23]

The 2023 winning solver was on a course project

Conclusions: state-of-the-art CDCL solver

Key contributions to CDCL solvers:

- concept of conflict clauses (grasp) [Marques-SilvaSakallah'96]
- restart strategies [GomesSC'97,LubySZ'93]
- 2-watch pointers and VSIDS (zChaff) [MoskewiczMZZM'01]
- efficient implementation (Minisat) [EenSörensson'03]
- phase-saving (Rsat) [PipatsrisawatDarwiche'07]
- conflict-clause minimization [SörenssonBiere'09]
- SAT vs UNSAT [Oh'15]

+ Pre- and in-processing techniques