Parallel Automated Reasoning

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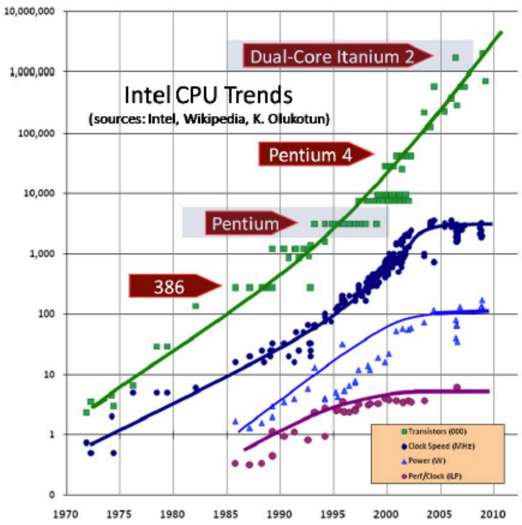
http://www.cs.cmu.edu/~mheule/15816-f19/

Automated Reasoning and Satisfiability, October 3, 2019

Why Parallelization? Power Wall



How to parallelize SAT?

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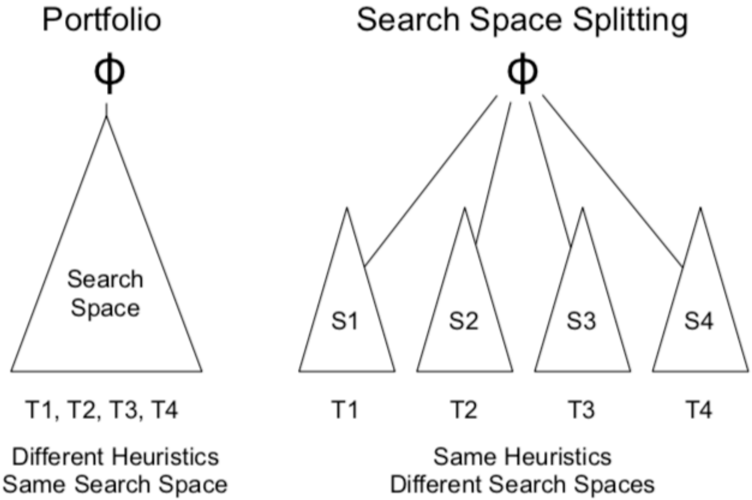

Portfolio Approach

Basic Idea:

- Run several solvers in parallel
- Stop when the first solver finds a solution or proves unsatisfiability

SAT Competition 2011:

- `ppfolio//` wins 11 medals, best solver in competition
- This solver is equivalent to type:
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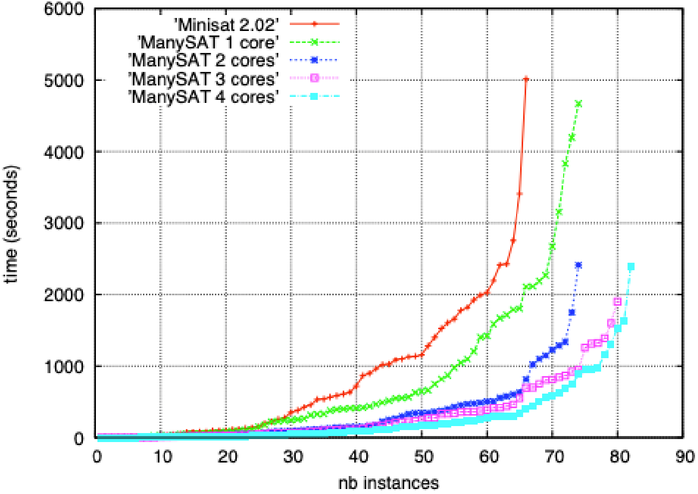
Can we do better?

- Exchange learned clauses
- Increase diversity between solvers

Portfolio Solvers: ManySAT

	Restart	Heuristic	Polarity	Learning
Core 0	Geometric $x_1 = 100$ $x_i = 1.5 \times x_{i-1}$	VSIDS (3% rand.)	if $\#occ(l) > \#occ(\neg l)$ $l = true$ else $l = false$	CDCL (extended)
Core 1	Dynamic (fast) $\alpha = 1200$ $x_1 = 100, x_2 = 100$ $x_i = f(y_{i-1}, y_i), i > 2$ if $y_{i-1} < y_i$ $f(y_{i-1,y_i}) =$ $\frac{\alpha}{y_i} \times \left\| cos(1 - \frac{y_{i-1}}{y_i}) \right\|$ else $f(y_{i-1,y_i}) =$ $\frac{\alpha}{y_i} \times \left\| cos(1 - \frac{y_i}{y_{i-1}}) \right\|$	VSIDS (2% rand.)	Progress saving	CDCL
Core 2	Arithmetic $x_1 = 16000$ $x_i = x_{i-1} + 16000$	VSIDS (2% rand.)	false	CDCL
Core 3	Luby 512	VSIDS (2% rand.)	Progress saving	CDCL (extended)

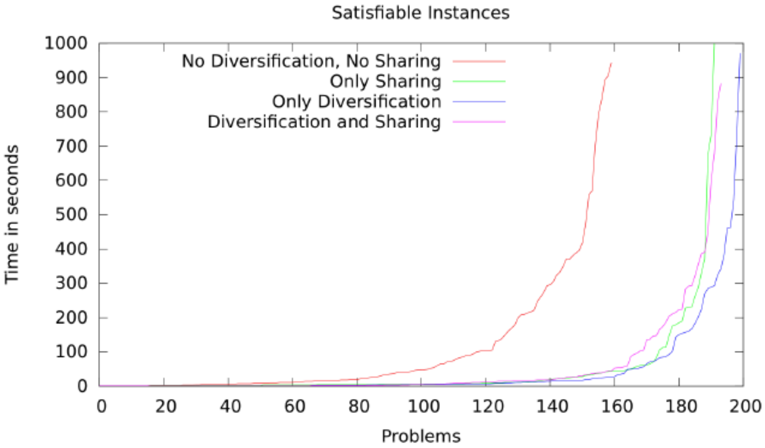
Portfolio Solvers: ManySAT



Portfolio Solvers: HordeSAT

What is the impact of diversification and clause exchange?

- 16 processes with 1 thread each
- Random 3-SAT, only satisfiable instances



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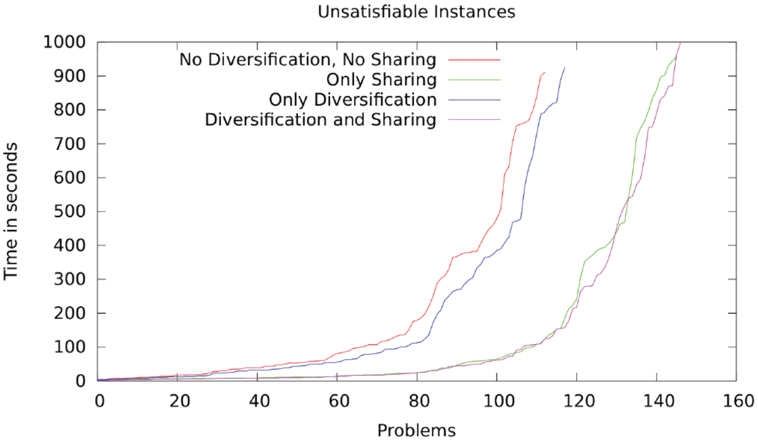

Portfolio Solvers: HordeSAT

How scalable are portfolio approaches?

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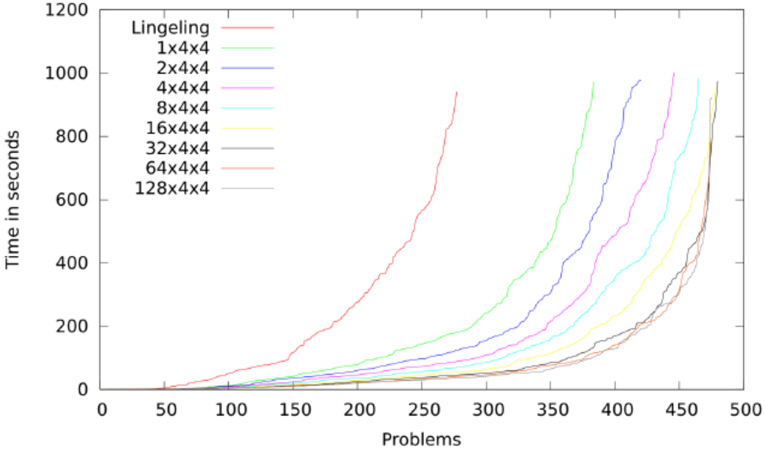
How scalable are portfolio approaches?

- (#nodes)x(#processes/node)x(#threads/process)



Search Space Splitting Approach

Basic idea:

- Split the search space into disjoint subspaces
- Each process searches in a disjoint subspace
- Load balancing mechanism to maintain all processes busy

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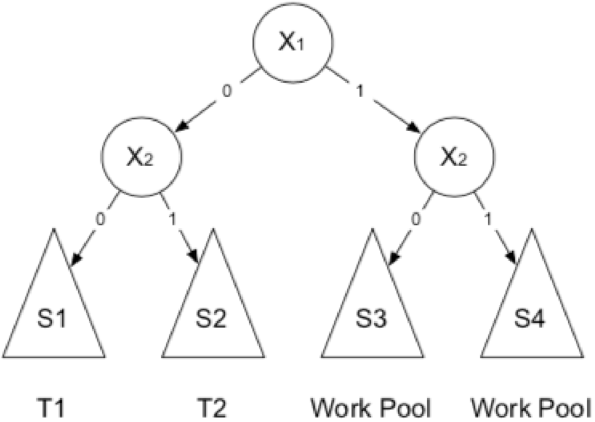
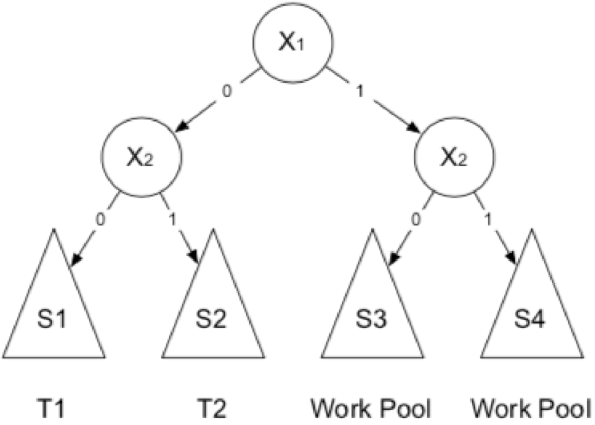
How to split the search space?

Search Space Splitting



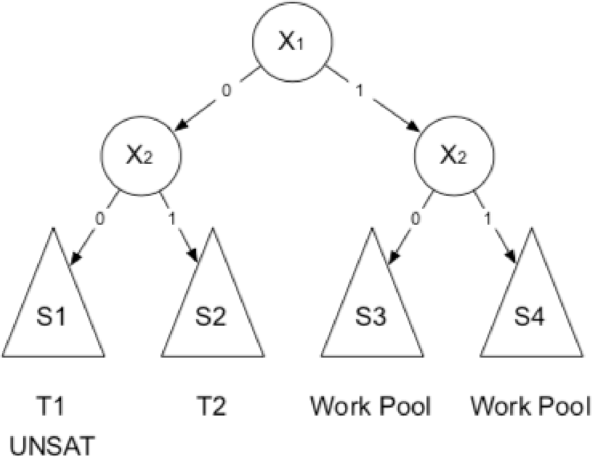
- Guiding path S1: $x_1 = 0, x_2 = 0$
- Restricts the search space of a given process

Search Space Splitting



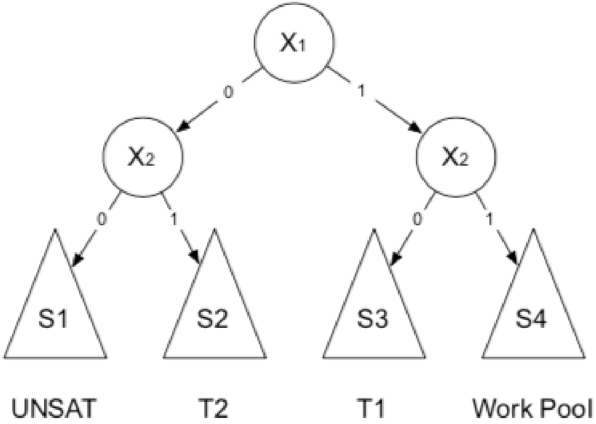
- Unused guiding paths are stored in the work queue

Search Space Splitting



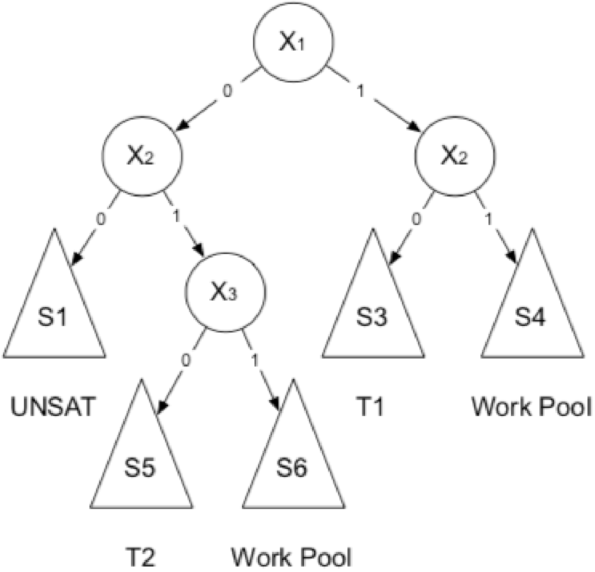
- If a subspace is unsatisfiable, then the process gets a new subspace

Search Space Splitting



- Dynamic work stealing procedure guarantees that all processes are always working

Search Space Splitting



Can we do a better split of the search space?

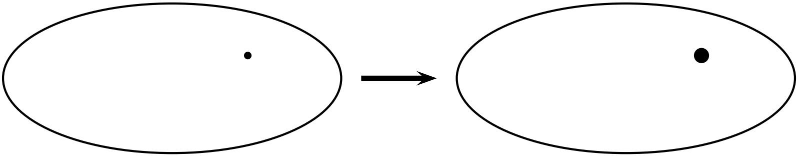
Conflict-Driven Clause Learning solvers

Highlights:

- goal: find small effective conflict clauses
- decisions: assign variables that occur in recent conflicts
- strength: powerful on "easy" problems

General CDCL situation:

- hit a conflict that can be generalized / analyzed to a large clause



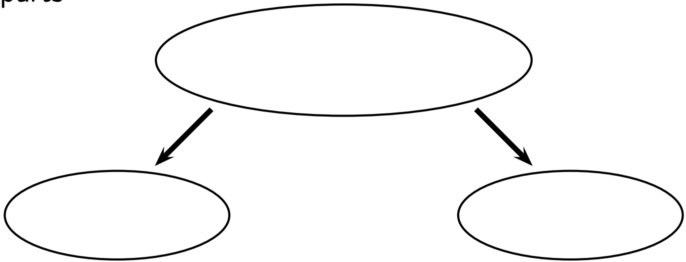
Lookahead solvers

Highlights:

- goal: construct a small binary search tree
- decisions: assign variables that cause a large reduction
- strength: powerful on small hard problems

Ideal lookahead situation:

- split the search space into two equally large but smaller parts



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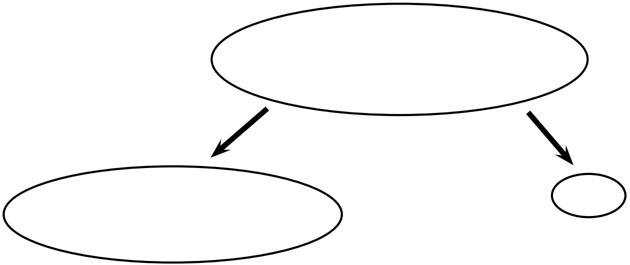
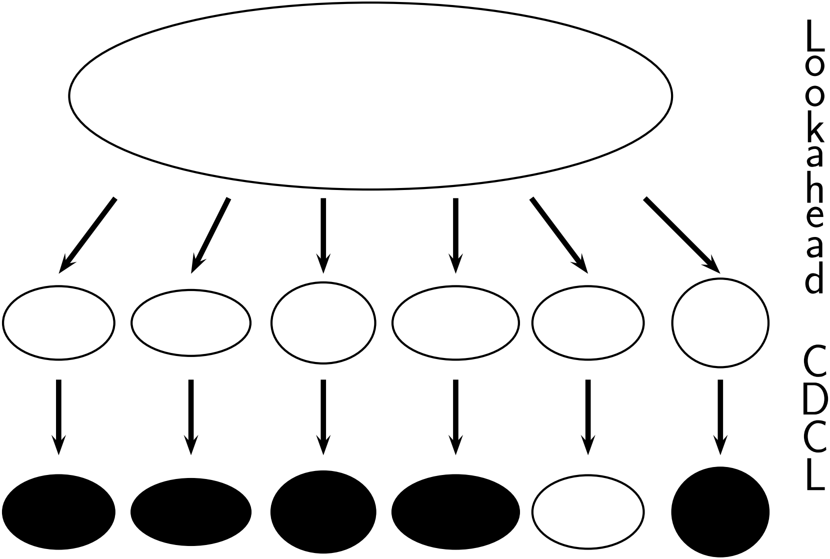
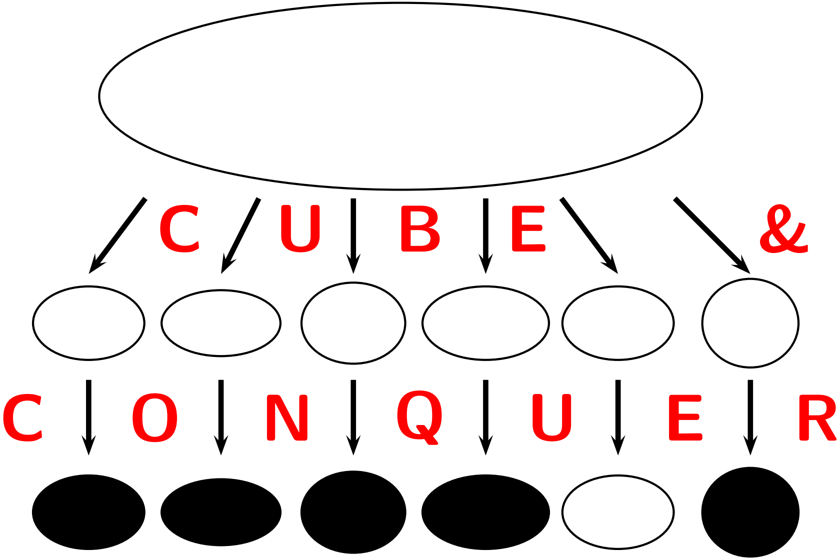


Best of both worlds: Combining Lookahead and CDCL



Best of both worlds: Cube and Conquer



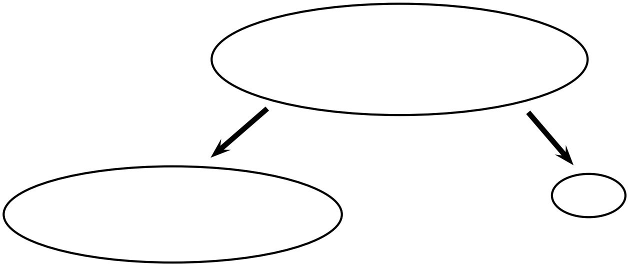
Cube: key observation

Split until the problem becomes easy

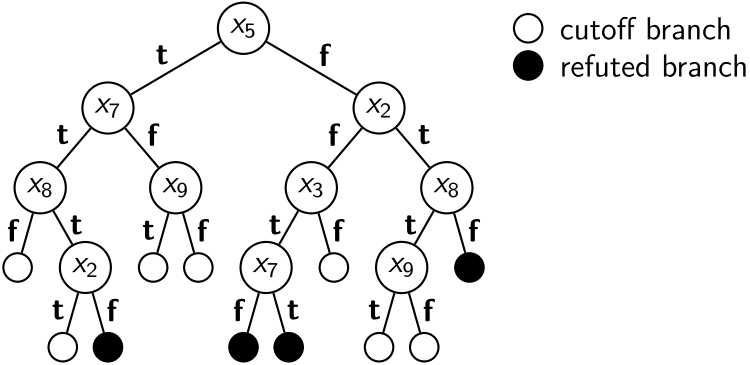
- do not have a fixed cut off depth
- determine hardness by number of assigned variables
- create many thousands or even millions of cubes

General lookahead situation:

- the search space is split into a large and a small part



Cube: example



$$F_1 := F \wedge (x_5 \wedge x_7 \wedge \neg x_8)$$
$$F_2 := F \wedge (x_5 \wedge x_7 \wedge x_8 \wedge x_2)$$
$$F_3 := F \wedge (x_5 \wedge \neg x_7 \wedge x_9)$$
$$F_4 := F \wedge (x_5 \wedge \neg x_7 \wedge \neg x_9)$$
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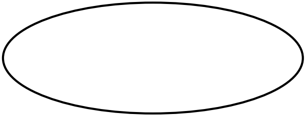
$$F_6 := F \wedge (\neg x_5 \wedge x_2 \wedge x_8 \wedge x_9)$$
$$F_7 := F \wedge (\neg x_5 \wedge x_2 \wedge x_8 \wedge \neg x_9)$$

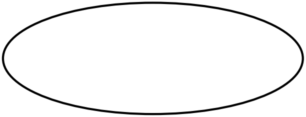
Conquer: describing cubes

How much information to send to the CDCL solver?

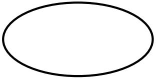
- Only the decisions

 $\wedge\ \varphi_{\mathrm{dec}}$

- The full assignment (including failed literals)

 $\wedge\ \varphi_{\mathrm{dec}}\ \wedge\ \varphi_{\mathrm{imp}}$

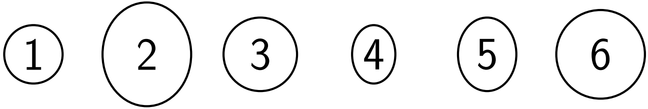
- The simplified formula (including local learnt clauses)



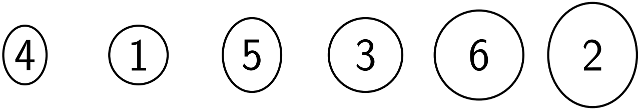
Conquer: ordering cubes

What is the optimal order to solve the cubes?

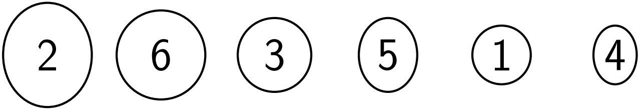
- Depth-first search (in lookahead order)



- Solves cubes with increasing (approximated) search space



- Solves cubes with decreasing (approximated) search space



Conquer: parallel solving

Strategies to solve cubes in parallel:

1. cores solve different cubes in parallel
2. cores solve the same cube in parallel
3. start with (1) till no new cubes are available, continue with (2)

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What to share between cores?

- nothing, so hardly communication required (only ask / receive cubes)
- sharing the learned clauses (maybe only to master)
- sharing the short conflict clauses, units (maybe also binaries)

Results: two experiments

1^{st} experiment: single core on Van der Waerden numbers

- hard combinatorial problem in Ramsey Theory
- comparison with the best solver for each instance
- cube solver: `OKsolver`
- conquer solver: `minisat`

2^{nd} experiment: multi core on challenging applications

- unsolved application instances from the SAT09 benchmarks
- comparison with the best parallel solvers
- cube solver: `march`
- conquer solver: `lingeling`

Results: palindromic Van der Waerden numbers

- k_1 : arithmetic progression of first set;
- k_2 : arithmetic progression of second set;
- n : number of elements to partition;
- best solver : time of fastest sequential solver;
- D : cut off depth.

k_1	k_2	n	#cls	?	best solver	D	#cubes	C&C
3	25	586	45779	S	$\sim$ 13 days	45	9120	6.5 hours
3	25	607	49427	U	$\sim$ 13 days	45	13462	2 days
4	12	387	15544	S	$>$ 14 days	30	132131	2 days
4	12	394	15889	U	$>$ 14 days	34	147237	8 hours
5	8	312	9121	S	3.5 days	20	2248	5 hours
5	8	313	9973	U	53 days	20	87667	40 hours

Results: parallel SAT solving

Portfolio solvers:

- run multiple versions of the same solver (different seeds)
- share short conflict clauses such as units
- solver `pLingeling` (`pLing`), on a 12-core machine

Grid based SAT solving approach:

- run solvers with different cubes on a grid
- grid constraints: limited communication, possible delay and timeout
- solver `PartitionTree` (`PTree`) on a grid, up to 60 jobs in parallel

Results: hard application benchmarks

Benchmark	?	#cubes	I total	II total	II 12-core	pLing 12-core
9dlx_vliw_at_b_iq8	U	121	150	—	—	**3256**
9dlx_vliw_at_b_iq9	U	100	179	—	—	**5164**
AProVE07-25	U	84247	89	100340	**8690**	—
dated-5-19-u	U	57716	418	3214	**1451**	4465
eq.atree.braun.12	U	86541	85	3261	**273**	—
eq.atree.braun.13	U	81313	77	18165	**1517**	—
gss-24-s100	S	18237	48	4975	**415**	2930
gss-26-s100	S	19455	57	37259	**3108**	18173
gus-md5-14	U	60102	961	—	—	—
ndhf_xits_09_UNS	U	37358	82	71096	12041	—
rbcl_xits_09_UNK	U	54669	132	94911	11542	—
rpoc_xits_09_UNS	U	30681	114	48028	**8366**	—
sortnet-8-ipc5-h19	S	724	153	48668	4067	**2700**
total-10-17-u	U	9192	288	5638	4517	**3672**
total-5-15-u	U	14914	215	—	—	—

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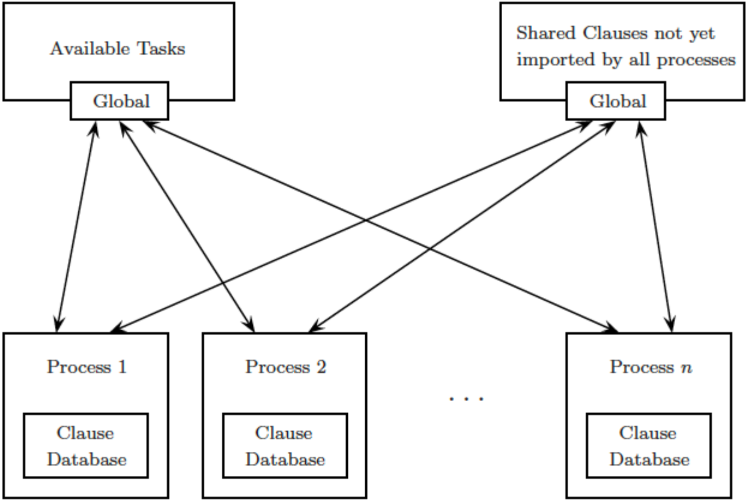
Architecture of parallel SAT solvers



- Most parallel SAT solvers has its own clause database!

Other forms of parallelization

Parallel unit propagation:

- More than 90% of the SAT solver is spent doing unit propagations

Number of new implied literals	Ratio
2	13%
4	4%

Can we find these implied literals in parallel?

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Does not scale beyond 2 cores!

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In the worst case unit propagation is inherently sequential:

$$\varphi = (\neg x_1) \wedge (x_1 \vee x_2) \wedge (x_1 \vee \neg x_2 \vee x_3) \wedge (\neg x_2 \vee \neg x_3 \vee x_4) \dots$$

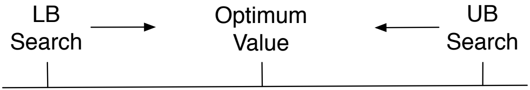
Applying unit propagation to φ results in the following chain of successive (sequential) and unique implications:

- $x_1 = 0 \rightarrow x_2 = 1 \rightarrow x_3 = 1 \rightarrow x_4 = 1 \dots$

What about parallel MaxSAT?

Parallel MaxSAT solver (2 threads)

- Search in the lower and upper bound values of the optimal solution:



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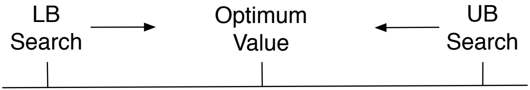

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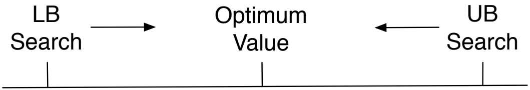

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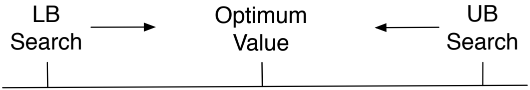

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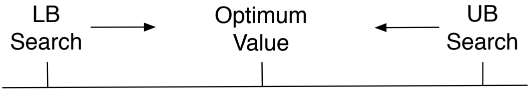

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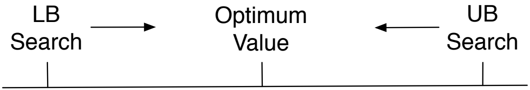

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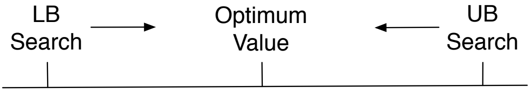


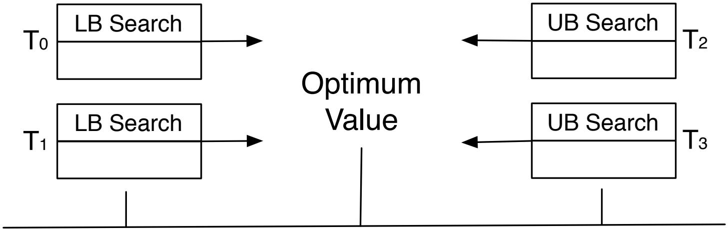
- The optimum value is found when:
  - LB or UB search terminates with a solution;
  - or when LB value = UB value.

		T_0; T_1										
		LB; UB										
0	1	2	3	4	5	6	7	8	9	10	11	12

LB value = UB value, optimal value has been found

Parallel MaxSAT solver (n threads)

- Search in the lower and upper bound values of the optimal solution:



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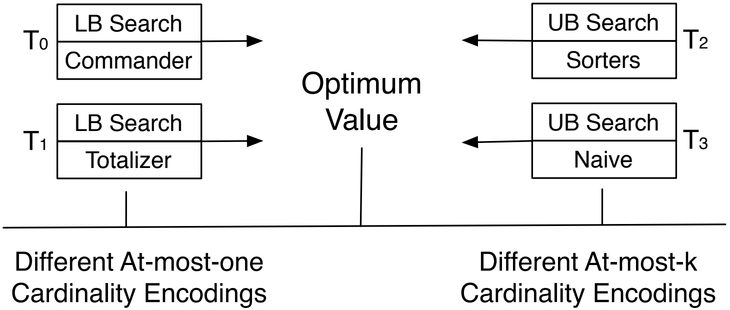

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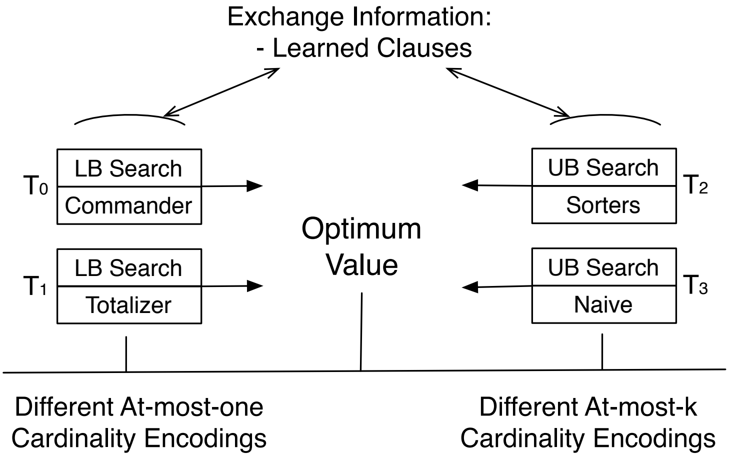
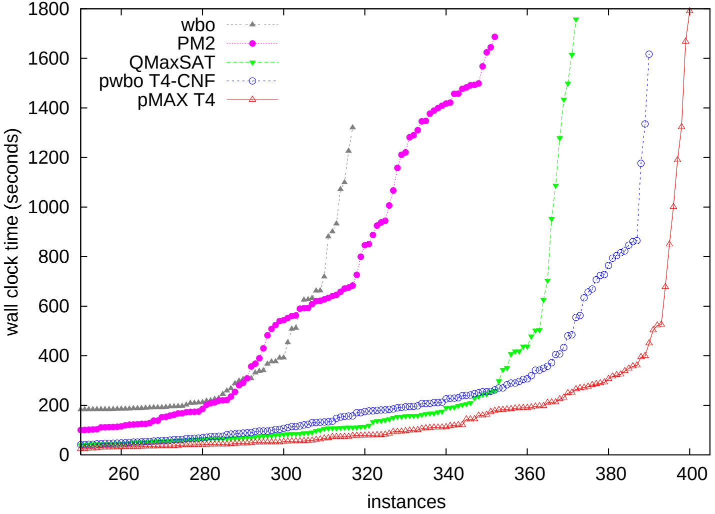
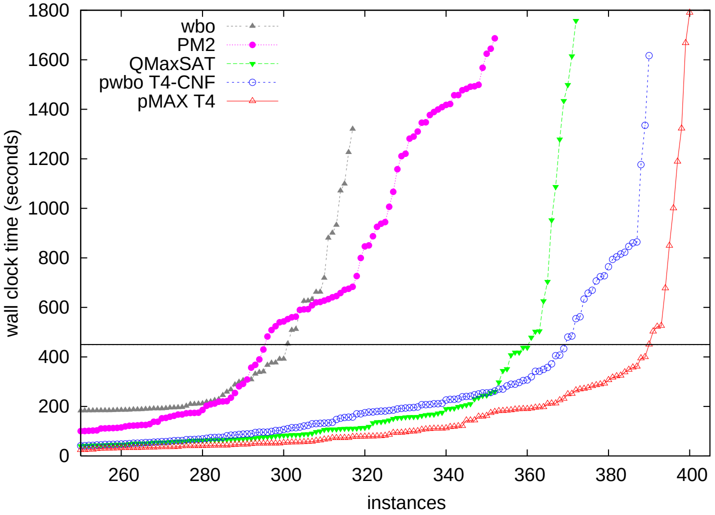


Experimental Results

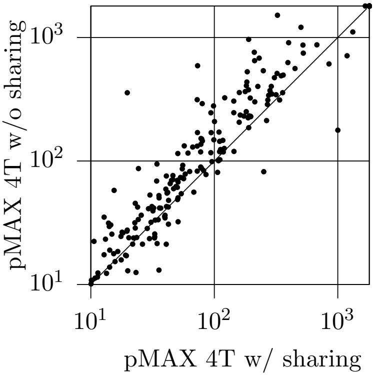


Experimental Results



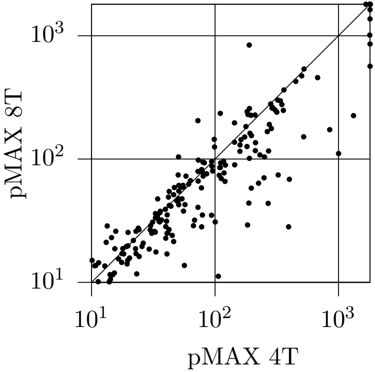
Experimental Results

- Impact of sharing learned clauses (seconds):



Experimental Results

- Scalability of pMAX (seconds):



Experimental Results

- Speedup on instances solved by all solvers:

Solver	Time (s)	Speedup
wbo	67,947.41	1.00
pwbo 4T-CNF	18,015.69	3.77
pMAX 4T	11,382.91	5.97
pMAX 8T	7,990.10	8.50

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- Does not scale beyond 8 cores!

Conclusions

Parallelization of SAT algorithms is hard!

Success stories:

- Cube and Conquer on thousands of nodes to solve hard combinatorial problems:
  - Pythagorean Triples
  - Schur Number Five
- Variable Elimination using GPUs
  - $66\times$ speedup
  - NVIDIA Titan Xp GPU (30 SMs with 128 cores each)

Still many open research directions for scalability in parallel SAT solving!

Parallel Automated Reasoning

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Automated Reasoning and Satisfiability, October 3, 2019