Assignment 2

due Thursday, September 9, 2021

Remember that homework is due at 6pm on the due date.

Problem 1 (3 points)

Express the following using summation notation, and prove it by induction:

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \cdots + \frac{1}{n \cdot (n+1)} = \frac{n}{n+1}.$$

(You can use notation like $\sum_{i \leq n}$ or $\sum_{i=1}^{n}$, as you prefer.)

Problem 2 (3 points)

In class, we described an algorithm to solve the tower of Hanoi problem and proved that n disks can be moved from one peg to another with $2^n - 1$ steps. Prove, as clearly as you can, that this is optimal: it is impossible to move n disks from one peg to another with a smaller number of steps.

Problem 3 (3 points)

Prove that for $n \geq 3$ a convex n-gon has $n(n-3)/2$ diagonals.

Problem 4 (3 points)

Recall that the Fibonacci numbers are defined recursively as follows:

$$\begin{aligned} F_0 &= 0 \\ F_1 &= 1 \\ F_{n+2} &= F_{n+1} + F_n. \end{aligned}$$

Show $\sum_{i<n} F_i = F_{n+1} - 1$.

Problem 5 (3 points)

Let α and β be the two roots of $x^2 = x + 1$. Show that for every n, $F_n = (\alpha^n - \beta^n)/\sqrt{5}$.

Problem 6 (3 points)

Remember the recursive definition of the greatest common divisor function:

$$\gcd(x, y) = \begin{cases} x & \text{if } y = 0 \\ \gcd(y, \text{mod}(x, y)) & \text{otherwise} \end{cases}$$

Notice that the easiest way to show that the recursion is well founded is to notice that the second argument decreases with each recursive call.

Show that for every nonnegative x and y, there are integers a and b such that $\gcd(x, y) = ax + by$.