



10-601 Introduction to Machine Learning

Machine Learning Department School of Computer Science Carnegie Mellon University

Midterm Exam Review + Multinomial Logistic Reg. + Feature Engineering

+ Regularization

Matt Gormley Lecture 10 Feb. 18, 2019

Reminders

- Homework 4: Logistic Regression
 - Out: Fri, Feb 15
 - Due: Fri, Mar 1 at 11:59pm
- Midterm Exam 1
 - Thu, Feb 21, 6:30pm 8:00pm
- Today's In-Class Poll
 - http://p10.mlcourse.org
- Reading on Probabilistic Learning is reused later in the course for MLE/MAP

Outline

- Midterm Exam Logistics
- Sample Questions
- Classification and Regression:
 The Big Picture
- Q&A

MIDTERM EXAM LOGISTICS

Midterm Exam

Time / Location

- Time: Evening ExamThu, Feb. 21 at 6:30pm 8:00pm
- Room: We will contact each student individually with your room assignment. The rooms are not based on section.
- Seats: There will be assigned seats. Please arrive early.
- Please watch Piazza carefully for announcements regarding room / seat assignments.

Logistics

- Covered material: Lecture 1 Lecture 8
- Format of questions:
 - Multiple choice
 - True / False (with justification)
 - Derivations
 - Short answers
 - Interpreting figures
 - Implementing algorithms on paper
- No electronic devices
- You are allowed to bring one 8½ x 11 sheet of notes (front and back)

Midterm Exam

How to Prepare

- Attend the midterm review lecture (right now!)
- Review prior year's exam and solutions (we'll post them)
- Review this year's homework problems
- Consider whether you have achieved the "learning objectives" for each lecture / section

Midterm Exam

Advice (for during the exam)

- Solve the easy problems first
 (e.g. multiple choice before derivations)
 - if a problem seems extremely complicated you're likely missing something
- Don't leave any answer blank!
- If you make an assumption, write it down
- If you look at a question and don't know the answer:
 - we probably haven't told you the answer
 - but we've told you enough to work it out
 - imagine arguing for some answer and see if you like it

Topics for Midterm

- Foundations
 - Probability, Linear
 Algebra, Geometry,
 Calculus
 - Optimization
- Important Concepts
 - Overfitting
 - Experimental Design

- Classification
 - Decision Tree
 - KNN
 - Perceptron
- Regression
 - Linear Regression

SAMPLE QUESTIONS

1.4 Probability

Assume we have a sample space Ω . Answer each question with **T** or **F**.

(a) [1 pts.] **T** or **F**: If events A, B, and C are disjoint then they are independent.

(b) [1 pts.] **T** or **F**:
$$P(A|B) \propto \frac{P(A)P(B|A)}{P(A|B)}$$
. (The sign ' \propto ' means 'is proportional to')

5.2 Constructing decision trees

Consider the problem of predicting whether the university will be closed on a particular day. We will assume that the factors which decide this are whether there is a snowstorm, whether it is a weekend or an official holiday. Suppose we have the training examples described in the Table 5.2.

Soovstorm	Holiday	Weekend	Closed
T	T	F	F
T	T	F	T
F	T	F	F
T	T	F	F
F	F	F	F
F	F	F	T
T	F	F	T
F	F	F	T

Table 1: Training examples for decision tree

- [2 points] What would be the effect of the Weekend attribute on the decision tree if it were made the root? Explain in terms of information gain.
- [8 points] If we cannot make Weekend the root node, which attribute should be made the root node of the decision tree? Explain your reasoning and show your calculations. (You may use $\log_2 0.75 = -0.4$ and $\log_2 0.25 = -2$)

4 K-NN [12 pts]

Now we will apply K-Nearest Neighbors using Euclidean distance to a binary classification task. We assign the class of the test point to be the class of the majority of the k nearest neighbors. A point can be its own neighbor.

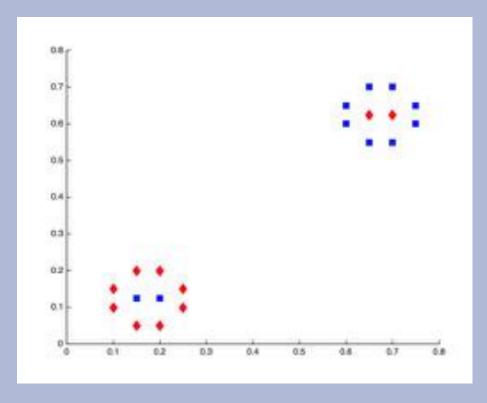


Figure 5

3. [2 pts] What value of k minimizes leave-one-out cross-validation error for the dataset shown in Figure 5? What is the resulting error?

4.1 True or False

Answer each of the following questions with **T** or **F** and **provide a one line justification**.

(a) [2 pts.] Consider two datasets $D^{(1)}$ and $D^{(2)}$ where $D^{(1)} = \{(x_1^{(1)}, y_1^{(1)}), ..., (x_n^{(1)}, y_n^{(1)})\}$ and $D^{(2)} = \{(x_1^{(2)}, y_1^{(2)}), ..., (x_m^{(2)}, y_m^{(2)})\}$ such that $x_i^{(1)} \in \mathbb{R}^{d_1}, x_i^{(2)} \in \mathbb{R}^{d_2}$. Suppose $d_1 > d_2$ and n > m. Then the maximum number of mistakes a perceptron algorithm will make is higher on dataset $D^{(1)}$ than on dataset $D^{(2)}$.

3.1 Linear regression

Consider the dataset S plotted in Fig. 1 along with its associated regression line. For each of the altered data sets S^{new} plotted in Fig. 3, indicate which regression line (relative to the original one) in Fig. 2 corresponds to the regression line for the new data set. Write your answers in the table below.

Dataset	(a)	(b)	(c)	(d)	(e)
Regression line					

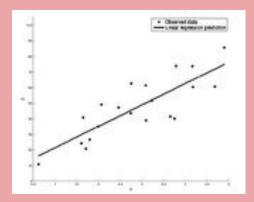


Figure 1: An observed data set and its associated regression line.

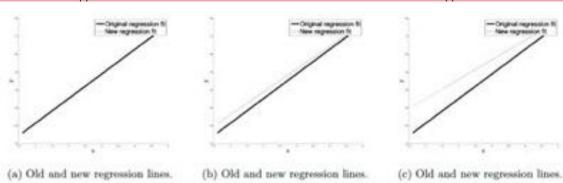
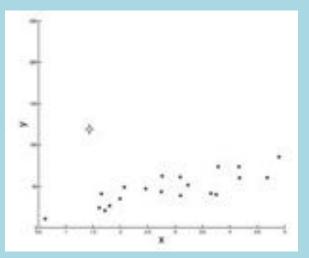


Figure 2: New regression lines for altered data sets S^{new} .



(a) Adding one outlier to the original data set.

3.1 Linear regression

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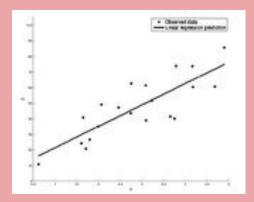


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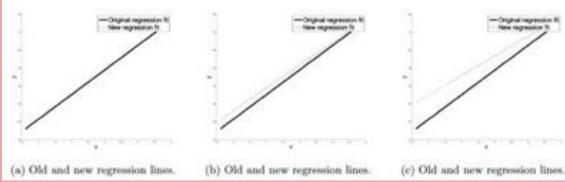
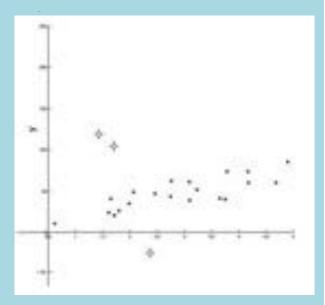


Figure 2: New regression lines for altered data sets S^{new} .



(c) Adding three outliers to the original data set. Two on one side and one on the other side.

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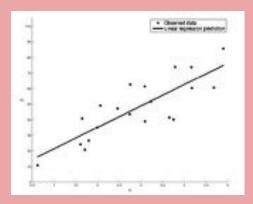


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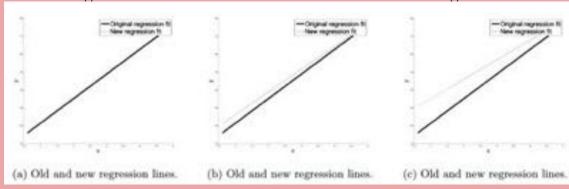
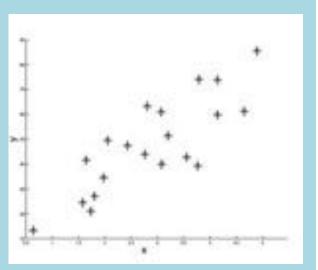


Figure 2: New regression lines for altered data sets S^{new} .



(d) Duplicating the original data set.

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Consider the dataset S plotted in Fig. 1 along with its associated regression line. For each of the altered data sets S^{new} plotted in Fig. 3, indicate which regression line (relative to the original one) in Fig. 2 corresponds to the regression line for the new data set. Write your answers in the table below.

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Regression line					

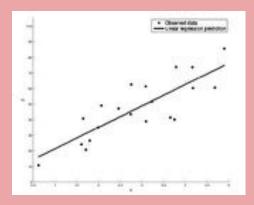


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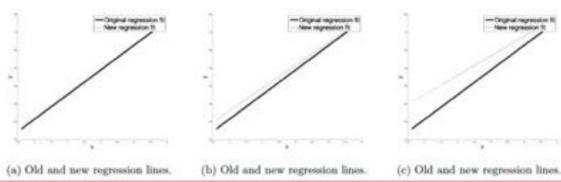
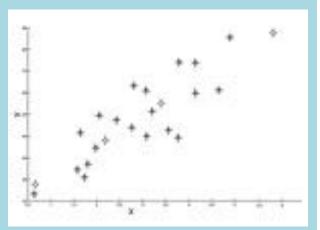


Figure 2: New regression lines for altered data sets S^{new} .



(e) Duplicating the original data set and adding four points that lie on the trajectory of the original regression line.

Matching Game

Goal: Match the Algorithm to its Update Rule

1. SGD for Logistic Regression

$$h_{\boldsymbol{\theta}}(\mathbf{x}) = p(y|x)$$

2. Least Mean Squares

$$h_{\boldsymbol{\theta}}(\mathbf{x}) = \boldsymbol{\theta}^T \mathbf{x}$$

3. Perceptron

$$h_{\boldsymbol{\theta}}(\mathbf{x}) = \operatorname{sign}(\boldsymbol{\theta}^T \mathbf{x})$$

$$\theta_k \leftarrow \theta_k + (h_{\theta}(\mathbf{x}^{(i)}) - y^{(i)})$$

$$\theta_k \leftarrow \theta_k + \frac{1}{1 + \exp \lambda (h_{\theta}(\mathbf{x}^{(i)}) - y^{(i)})}$$

$$\theta_k \leftarrow \theta_k + \lambda (h_{\theta}(\mathbf{x}^{(i)}) - y^{(i)}) x_k^{(i)}$$

Q&A

MULTINOMIAL LOGISTIC REGRESSION



Multinomial Logistic Regression

Chalkboard

- Background: Multinomial distribution
- Definition: Multi-class classification
- Geometric intuitions
- Multinomial logistic regression model
- Generative story
- Reduction to binary logistic regression
- Partial derivatives and gradients
- Applying Gradient Descent and SGD
- Implementation w/ sparse features

Debug that Program!

In-Class Exercise: Think-Pair-Share

Debug the following program which is (incorrectly) attempting to run SGD for multinomial logistic regression

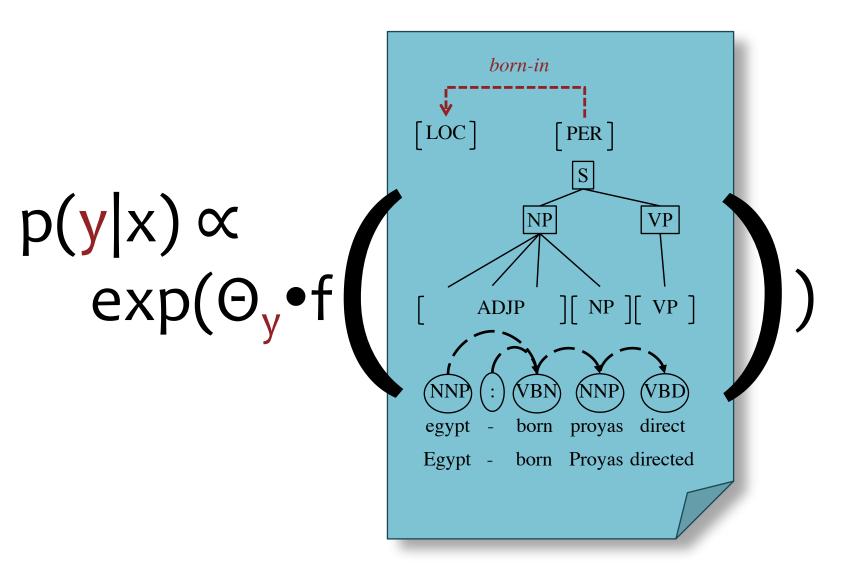
Buggy Program:

```
while not converged:
   for i in shuffle([1,...,N]):
      for k in [1,...,K]:
        theta[k] = theta[k] - lambda * grad(x[i], y[i], theta, k)
```

Assume: grad(x[i], y[i], theta, k) returns the gradient of the negative log-likelihood of the training example (x[i],y[i]) with respect to vector theta[k]. lambda is the learning rate. N = # of examples. K = # of output classes. M = # of features. theta is a K by M matrix.

FEATURE ENGINEERING

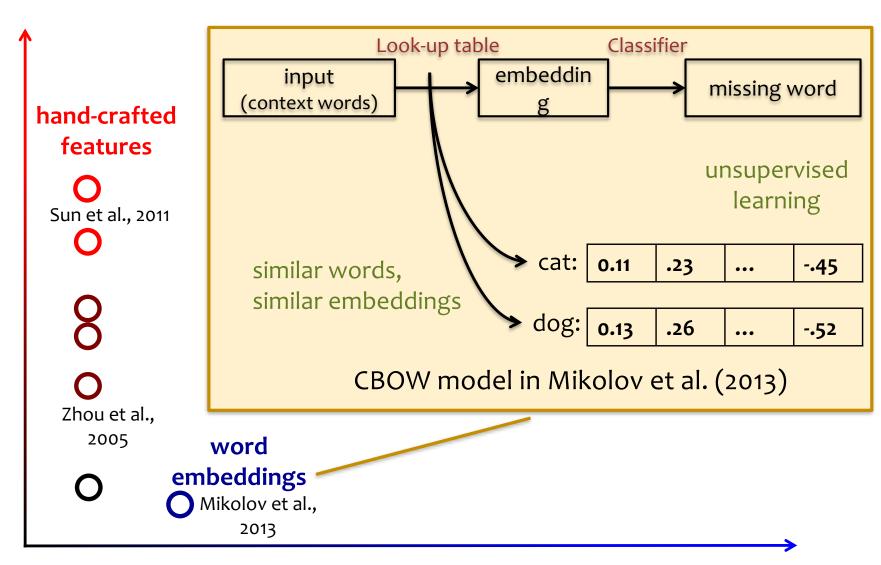
Handcrafted Features



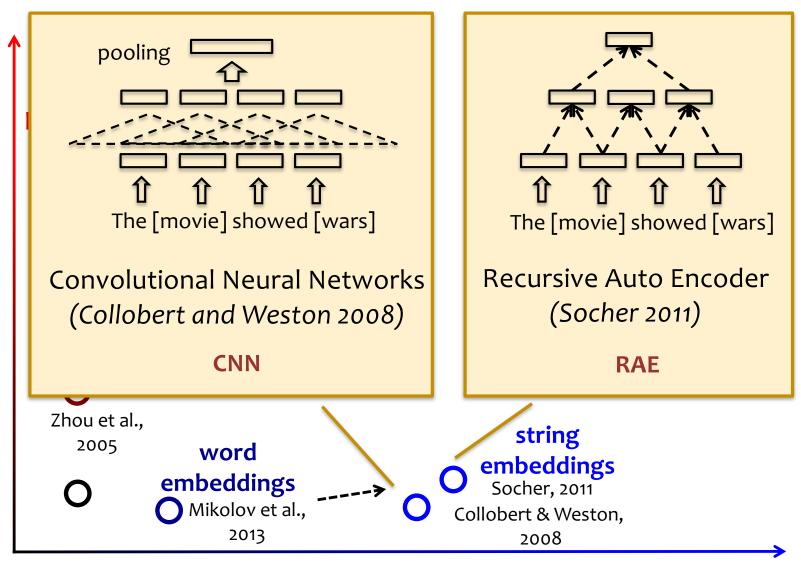
First word before M1 Second word before M1 hand-crafted Bag-of-words in M1 features Head word of M1 Other word in between First word after M2 Sun et al., 2011 Second word after M2 Bag-of-words in M2 Head word of M2 Bigrams in between Words on dependency path Country name list Personal relative triggers Personal title list Zhou et al., WordNet Tags 2005 Heads of chunks in between Path of phrase labels Combination of entity types

Feature Learning

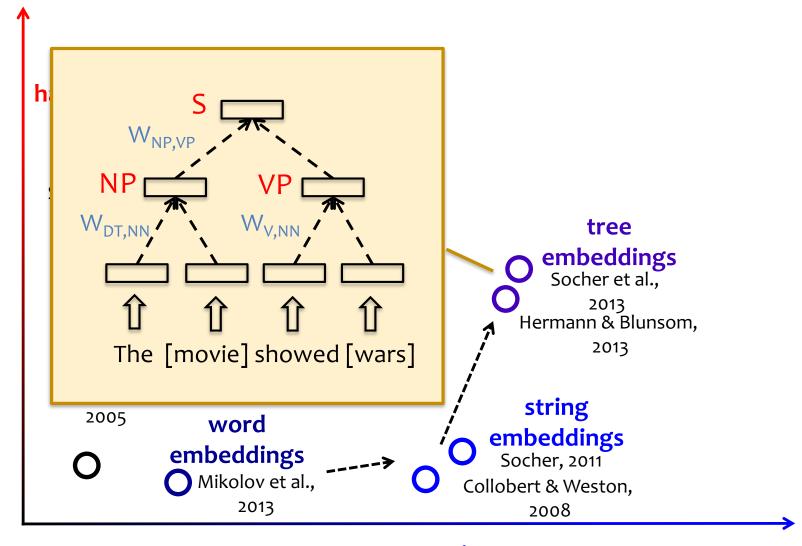
Feature Engineering



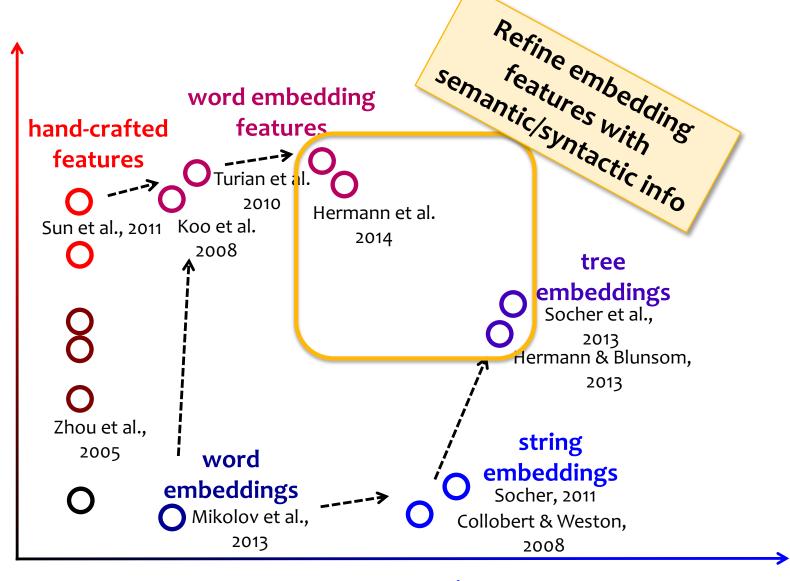
Feature Learning



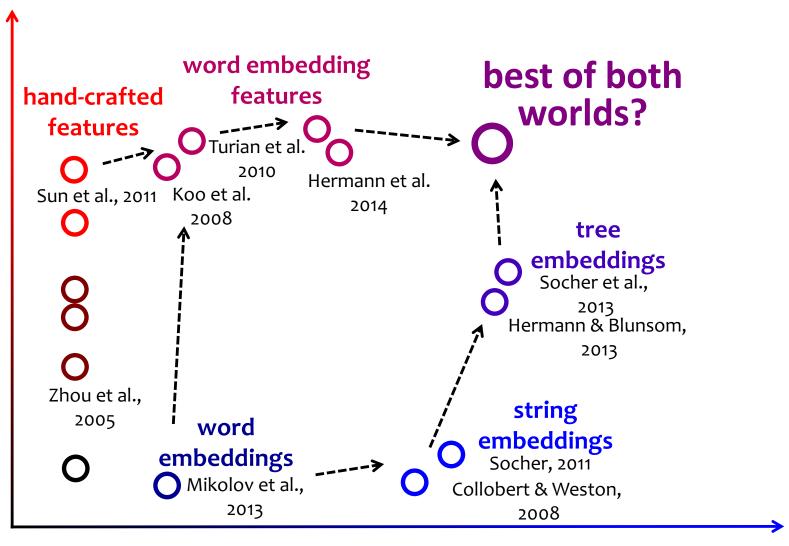
Feature Learning



Feature Learning



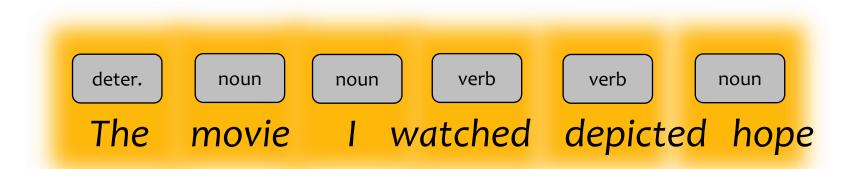
Feature Learning



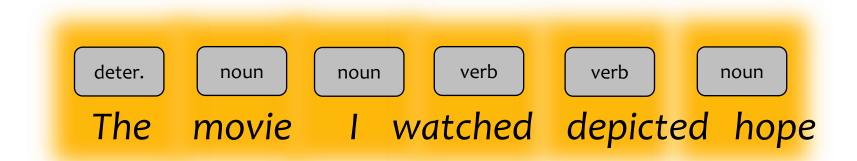
Feature Learning

Suppose you build a logistic regression model to predict a part-of-speech (POS) tag for each word in a sentence.

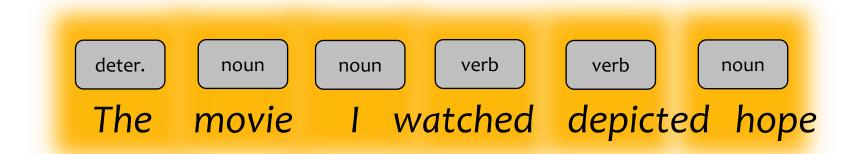
What features should you use?



Per-word Features:



Context Features:



Context Features:

 $w_{i} == "I"$ $w_{i+1} == "I"$ $w_{i-1} == "I"$ $w_{i+2} == "I"$ $w_{i-2} == "I"$

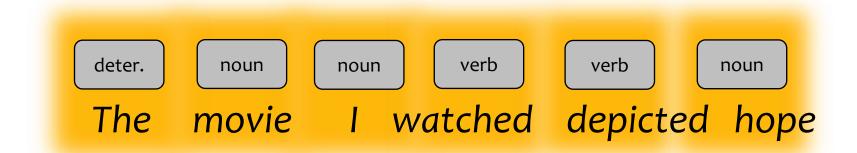
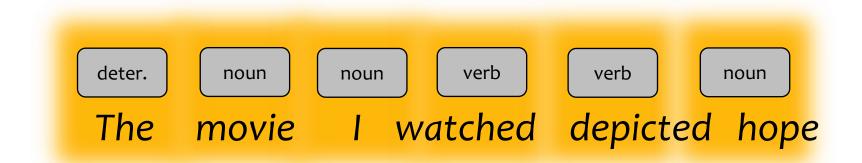


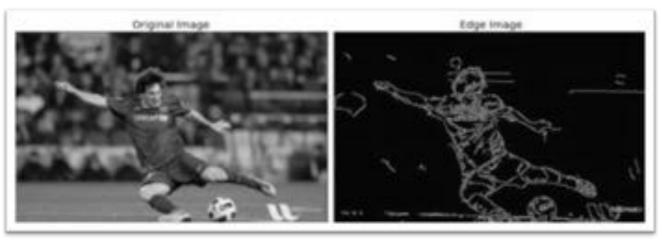
Table 3. Tagging accuracies with different feature templates and other changes on the WSJ 19-21 development set.

Model	Feature Templates	#	Sent.	Token	Unk.
		Feats	Acc.	Acc.	Acc.
3GRAMMEMM	See text	248,798	52.07%	96.92%	88.99%
NAACL 2003	See text and [1]	$460,\!552$	55.31%	97.15%	88.61%
Replication	See text and [1]	$460,\!551$	55.62%	97.18%	88.92%
Replication'	+rareFeatureThresh $= 5$	$482,\!364$	55.67%	97.19%	88.96%
$5 \mathrm{W}$	$+\langle t_0, w_{-2}\rangle, \langle t_0, w_2\rangle$	$730,\!178$	56.23%	97.20%	89.03%
5wShapes	$+\langle t_0, s_{-1}\rangle, \langle t_0, s_0\rangle, \langle t_0, s_{+1}\rangle$	731,661	56.52%	97.25%	89.81%
5wShapesDS	+ distributional similarity	737,955	56.79%	97.28%	90.46%

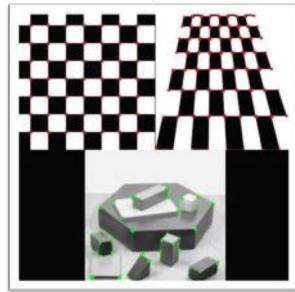


Feature Engineering for CV

Edge detection (Canny)



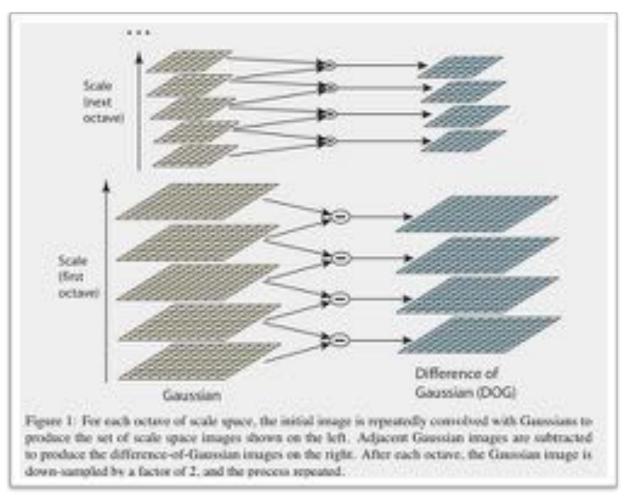
Corner Detection (Harris)



Feature Engineering for CV

Scale Invariant Feature Transform (SIFT)





NON-LINEAR FEATURES

Nonlinear Features

- aka. "nonlinear basis functions"
- So far, input was always $\mathbf{x} = [x_1, \dots, x_M]$
- **Key Idea:** let input be some function of **x**
 - $\begin{array}{ll} & \text{original input:} & \mathbf{x} \in \mathbb{R}^M \\ & \text{new input:} & \mathbf{x}' \in \mathbb{R}^{M'} \end{array} \text{ where } M' > M \text{ (usually)}$

 - define $\mathbf{x}' = b(\mathbf{x}) = [b_1(\mathbf{x}), b_2(\mathbf{x}), \dots, b_{M'}(\mathbf{x})]$ where $b_i: \mathbb{R}^M \to \mathbb{R}$ is any function
- Examples: (M = 1)

polynomial
$$b_j(x) = x^j \quad \forall j \in \{1,\dots,J\}$$
 radial basis function $b_j(x) = \exp\left(\frac{-(x-\mu_j)^2}{2\sigma_j^2}\right)$

sigmoid
$$b_j(x) = \frac{1}{1 + \exp(-\omega_j x)}$$

$$\log b_j(x) = \log(x)$$

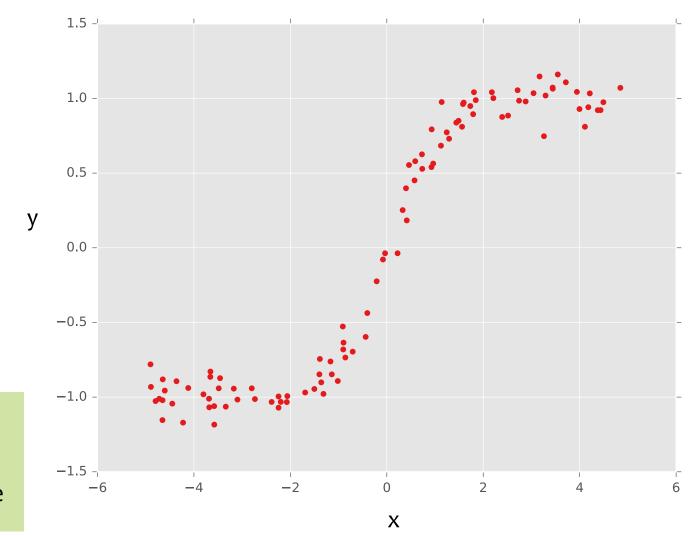
For a linear model: still a linear function of b(x) even though a nonlinear function of X

Examples:

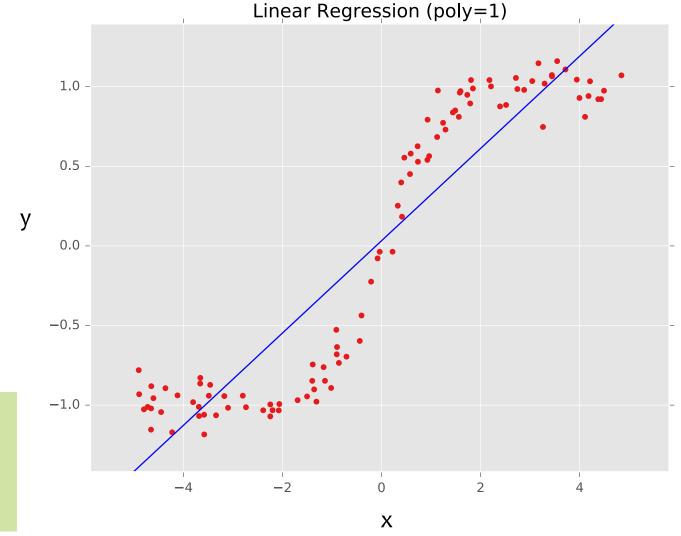
- Perceptron
- Linear regression
- Logistic regression

Goal: Learn $y = \mathbf{w}^T f(\mathbf{x}) + b$ where f(.) is a polynomial

basis function

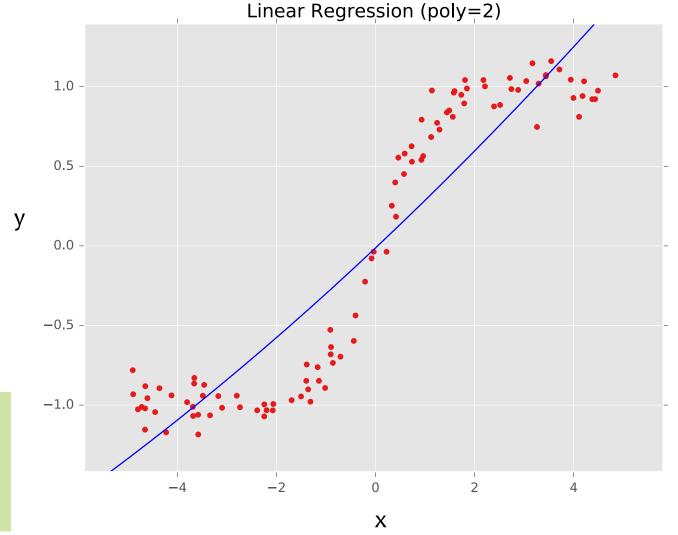


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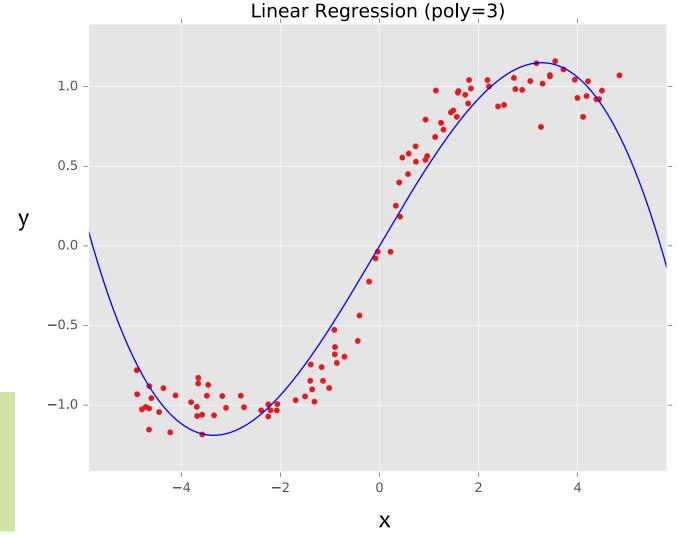
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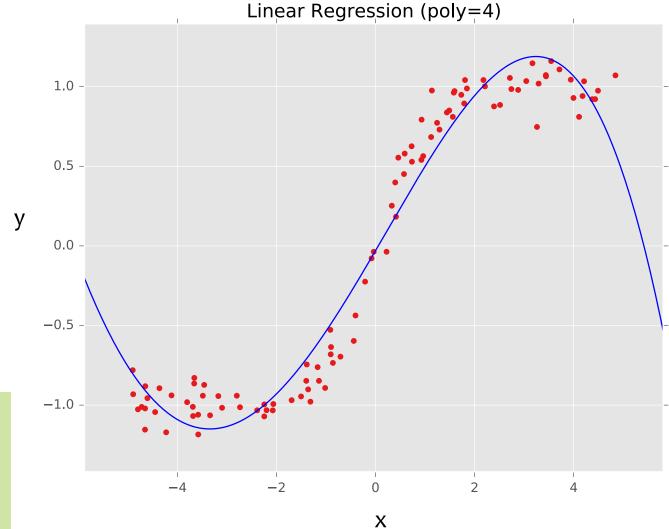
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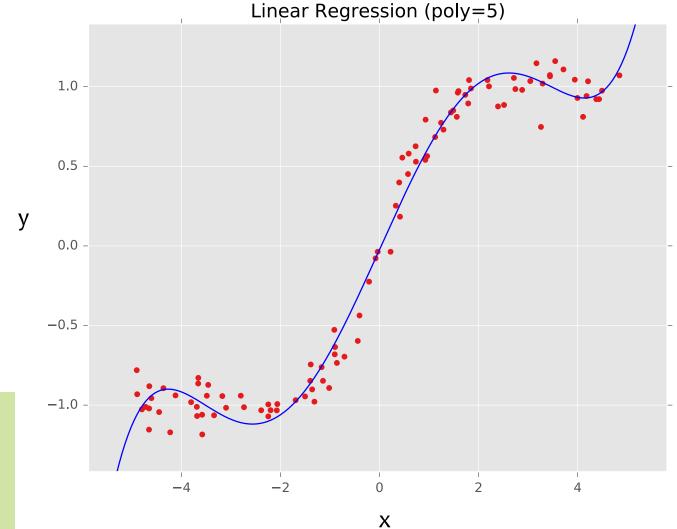


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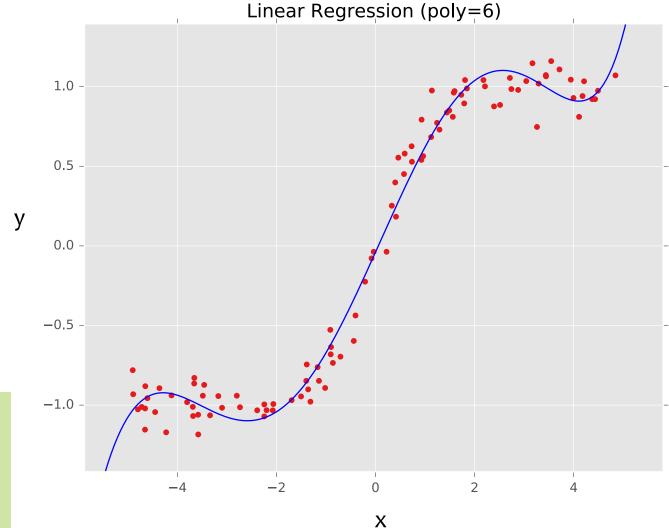




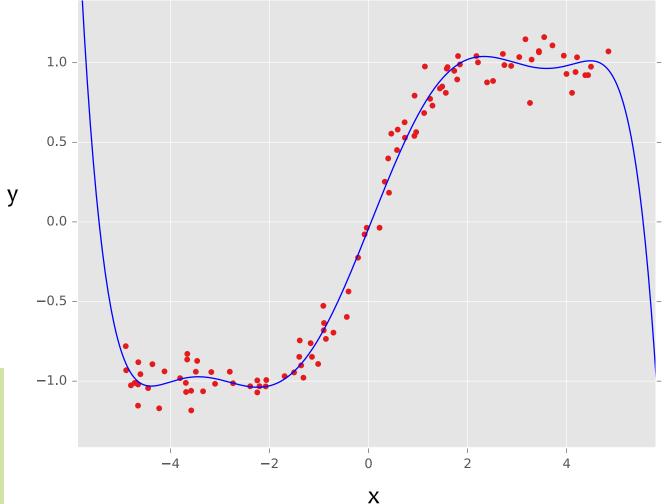
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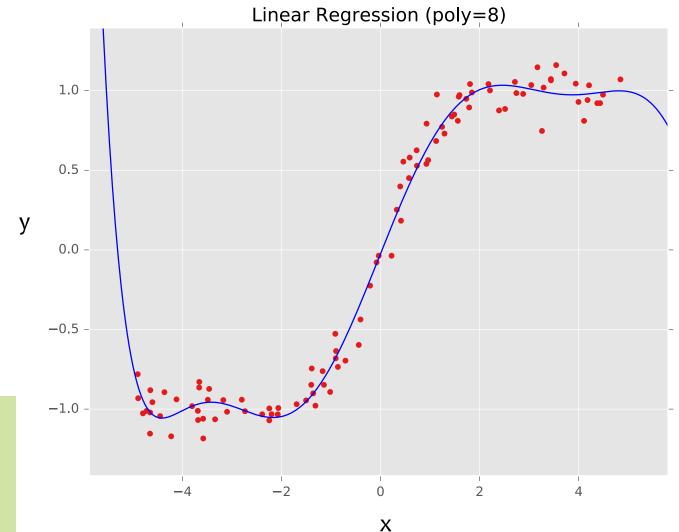


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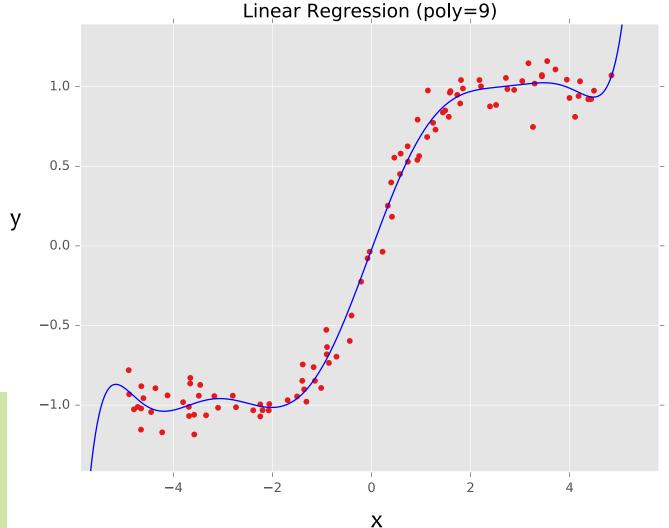


Linear Regression (poly=7)

Goal: Learn $y = \mathbf{w}^T f(\mathbf{x}) + b$ where f(.) is a polynomial basis function

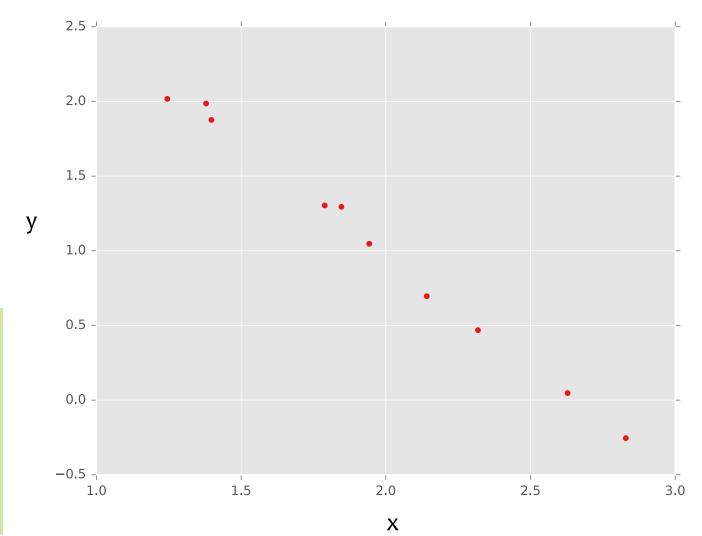


Goal: Learn $y = \mathbf{w}^T f(\mathbf{x}) + b$ where f(.) is a polynomial basis function



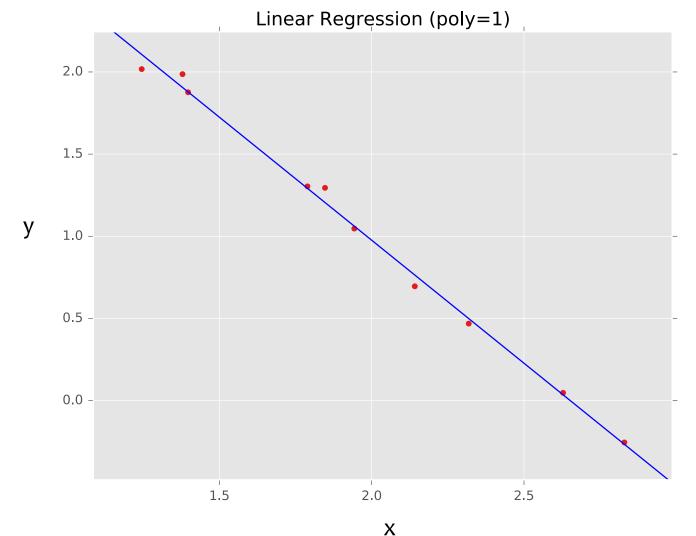
Goal: Learn $y = \mathbf{w}^T f(\mathbf{x}) + \mathbf{b}$ where f(.) is a polynomial

basis function



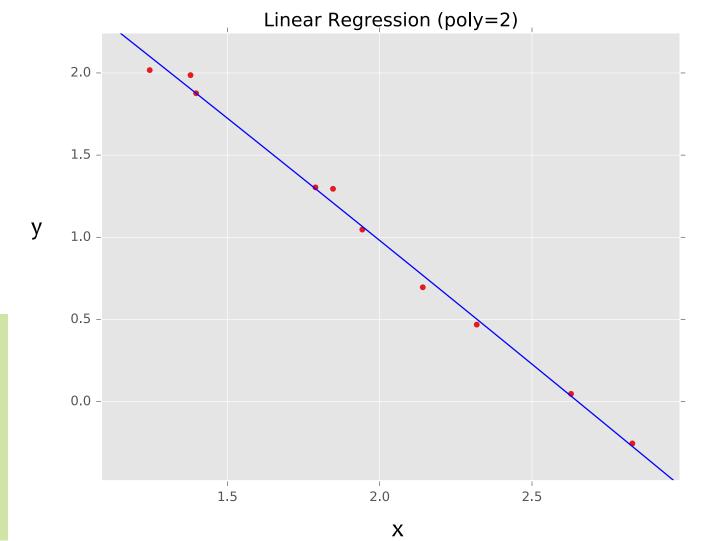
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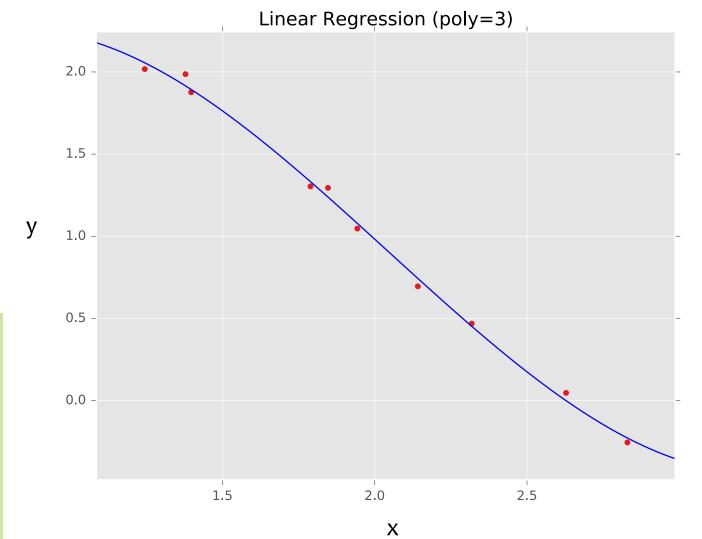
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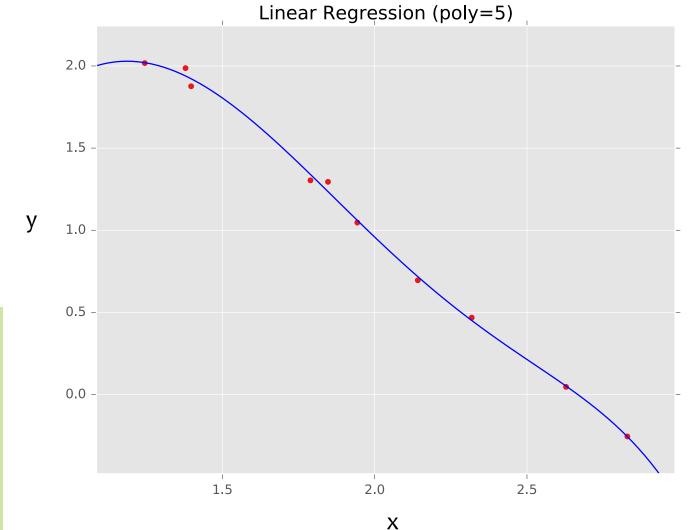


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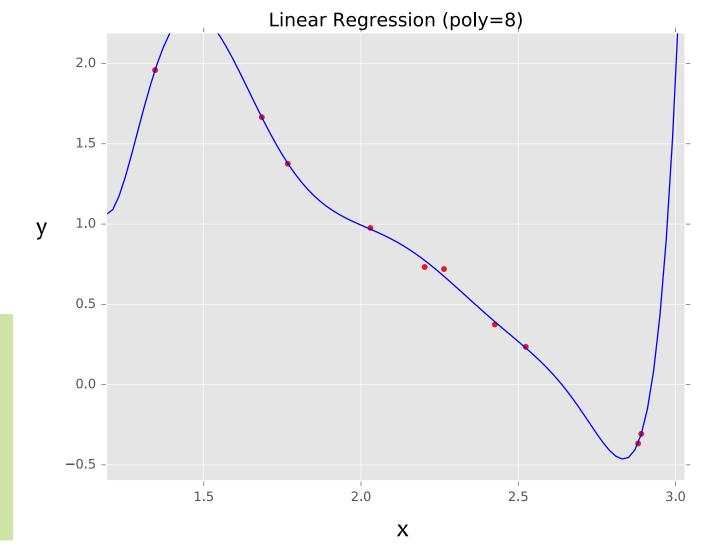


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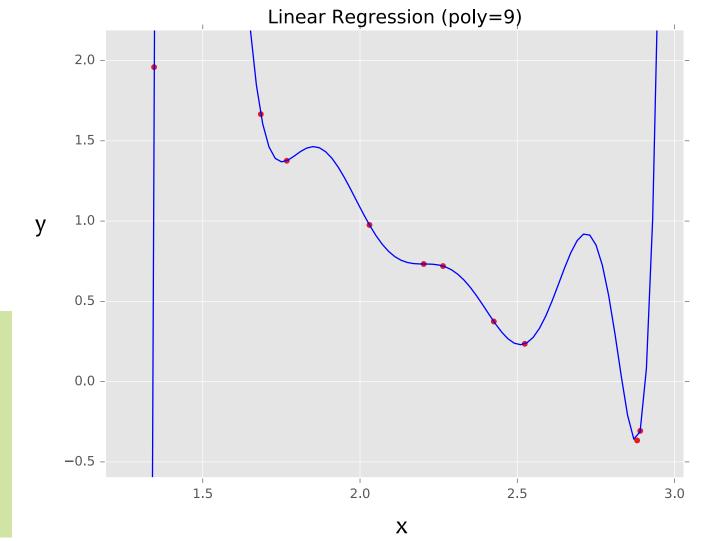
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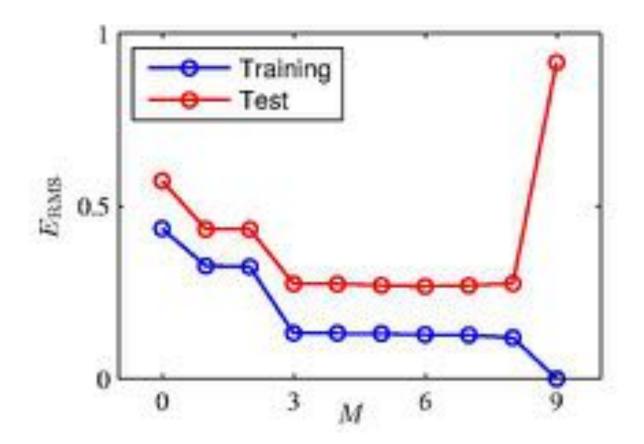


Goal: Learn $y = \mathbf{w}^T f(\mathbf{x}) + b$ where f(.) is a polynomial

basis function



Over-fitting



Root-Mean-Square (RMS) Error:

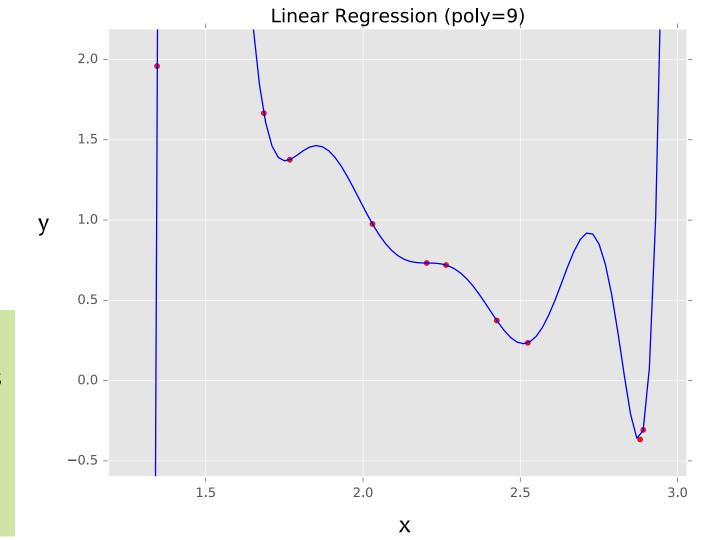
$$E_{\rm RMS} = \sqrt{2E(\mathbf{w}^{\star})/N}$$

Polynomial Coefficients

	M = 0	M = 1	M = 3	M = 9
$\overline{\theta_0}$	0.19	0.82	0.31	0.35
$ heta_1$		-1.27	7.99	232.37
$ heta_2$			-25.43	-5321.83
$ heta_3$			17.37	48568.31
$ heta_4$				-231639.30
$ heta_5$				640042.26
$ heta_6$				-1061800.52
$ heta_7$				1042400.18
$ heta_8$				-557682.99
$ heta_9$				125201.43

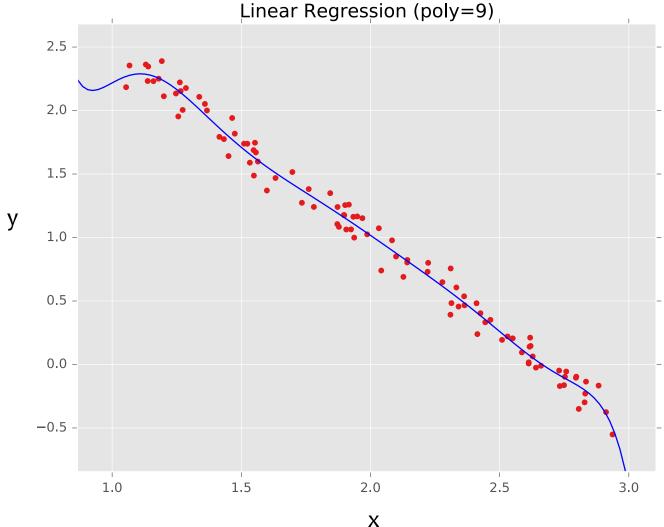
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basis function



Goal: Learn $y = \mathbf{w}^T f(\mathbf{x}) + b$ where f(.) is a polynomial basis function

Same as before, but now with N = 100 points



REGULARIZATION

Overfitting

Definition: The problem of **overfitting** is when the model captures the noise in the training data instead of the underlying structure

Overfitting can occur in all the models we've seen so far:

- Decision Trees (e.g. when tree is too deep)
- KNN (e.g. when k is small)
- Perceptron (e.g. when sample isn't representative)
- Linear Regression (e.g. with nonlinear features)
- Logistic Regression (e.g. with many rare features)

Motivation: Regularization

Example: Stock Prices

- Suppose we wish to predict Google's stock price at time t+1
- What features should we use? (putting all computational concerns aside)
 - Stock prices of all other stocks at times t, t-1, t-2, ..., t - k
 - Mentions of Google with positive / negative sentiment words in all newspapers and social media outlets



 Do we believe that all of these features are going to be useful?

Motivation: Regularization

 Occam's Razor: prefer the simplest hypothesis

- What does it mean for a hypothesis (or model) to be simple?
 - 1. small number of features (model selection)
 - small number of "important" features (shrinkage)

Regularization

Chalkboard

- L2, L1, Lo Regularization
- Example: Linear Regression

Regularization

Don't Regularize the Bias (Intercept) Parameter!

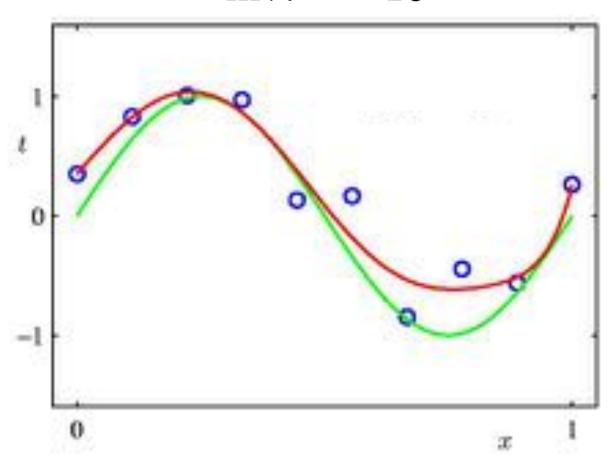
- In our models so far, the bias / intercept parameter is usually denoted by θ_0 -- that is, the parameter for which we fixed $x_0=1$
- Regularizers always avoid penalizing this bias / intercept parameter
- Why? Because otherwise the learning algorithms wouldn't be invariant to a shift in the y-values

Whitening Data

- It's common to whiten each feature by subtracting its mean and dividing by its variance
- For regularization, this helps all the features be penalized in the same units (e.g. convert both centimeters and kilometers to z-scores)

Regularization:

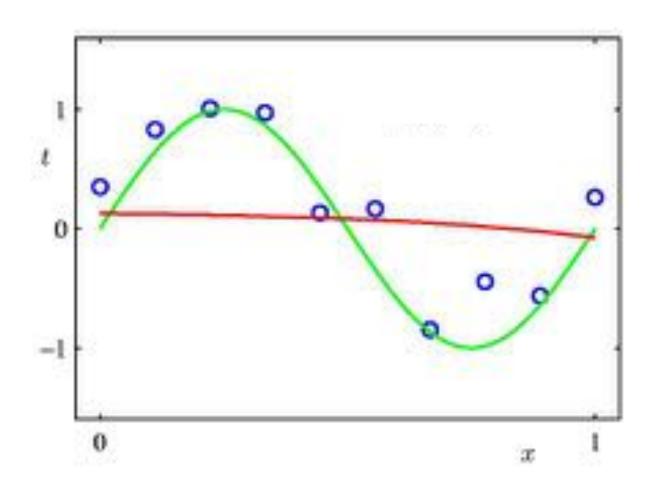
$$\ln \lambda = +18$$



Polynomial Coefficients

none		exp(18)	huge
w_0^\star	0.35	0.35	0.13
w_1^{\star}	232.37	4.74	-0.05
w_2^{\star}	-5321.83	-0.77	-0.06
w_3^{\star}	48568.31	-31.97	-0.05
w_4^{\star}	-231639.30	-3.89	-0.03
w_5^{\star}	640042.26	55.28	-0.02
w_6^{\star}	-1061800.52	41.32	-0.01
w_7^{\star}	1042400.18	-45.95	-0.00
w_8^*	-557682.99	-91.53	0.00
w_9^{\star}	125201.43	72.68	0.01

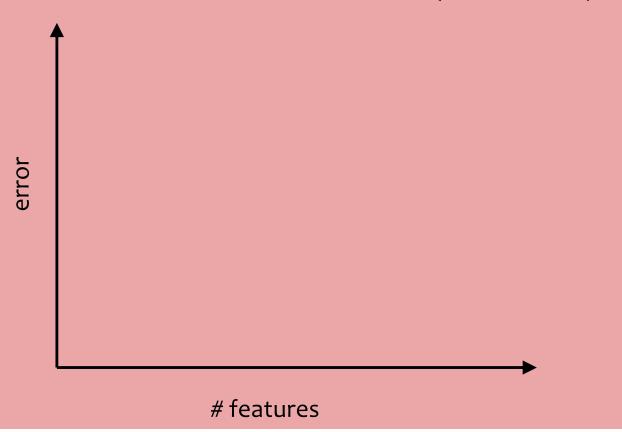
Over Regularization:



Regularization Exercise

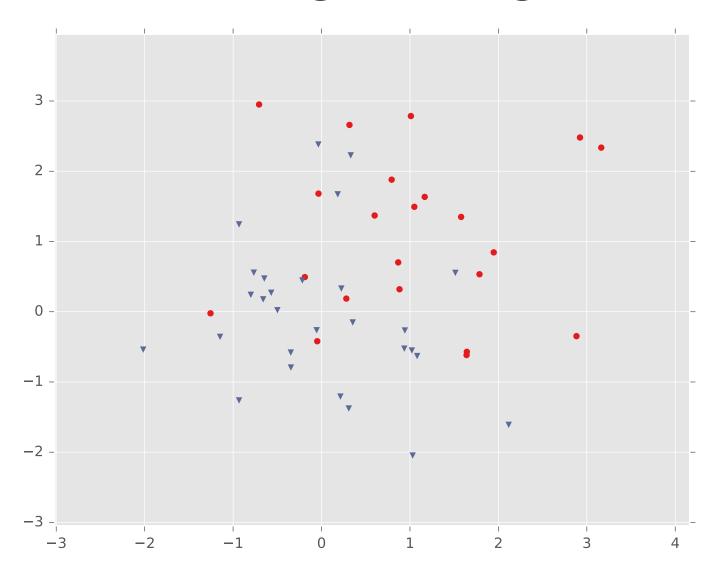
In-class Exercise

- 1. Plot train error vs. # features (cartoon)
- 2. Plot test error vs. # features (cartoon)

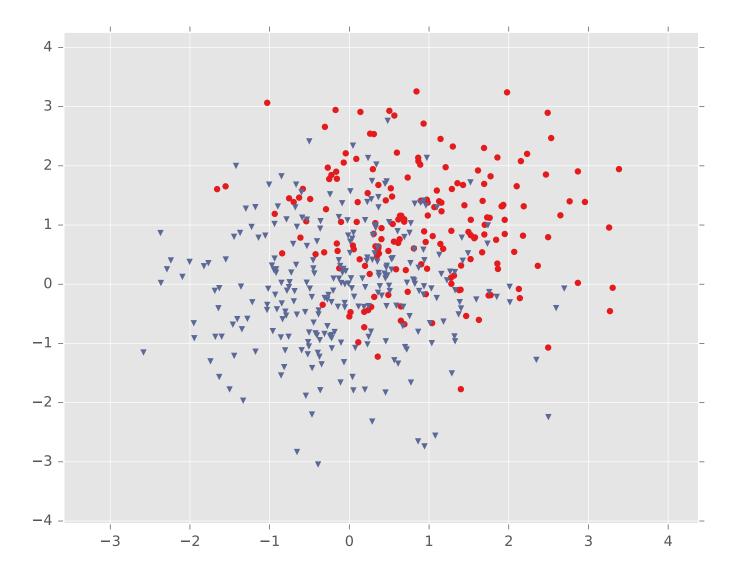


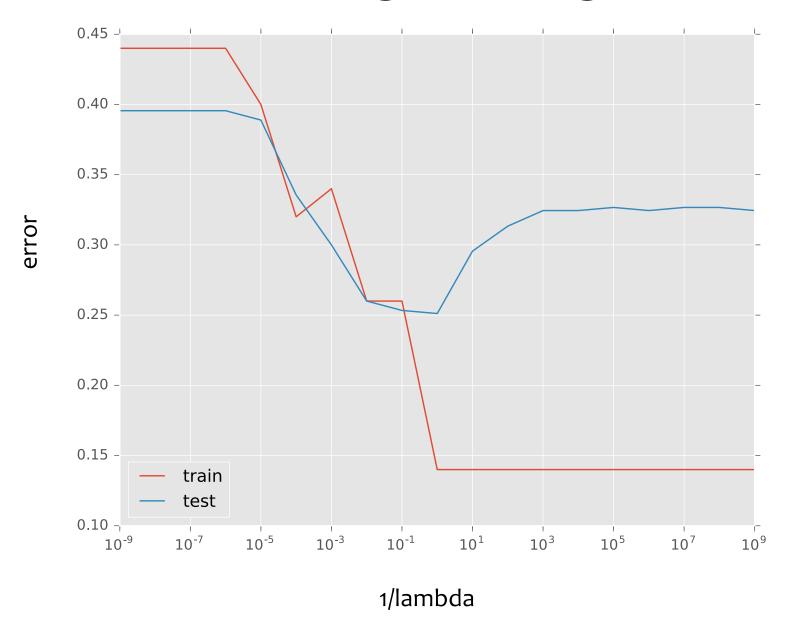
Example: Logistic Regression

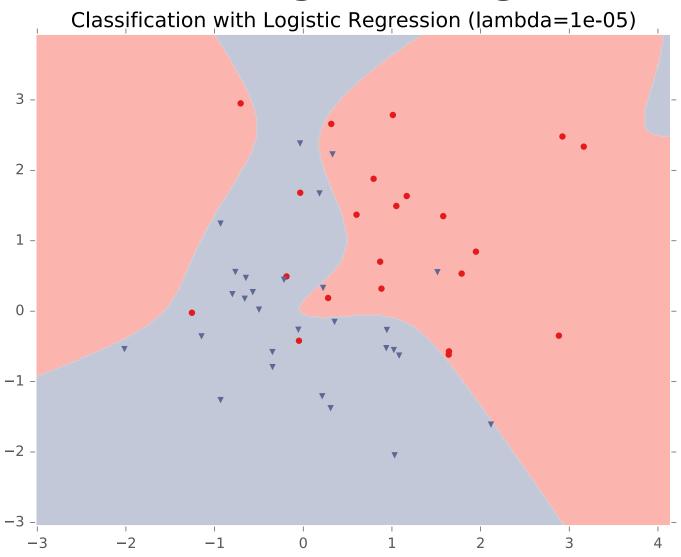
Training Data

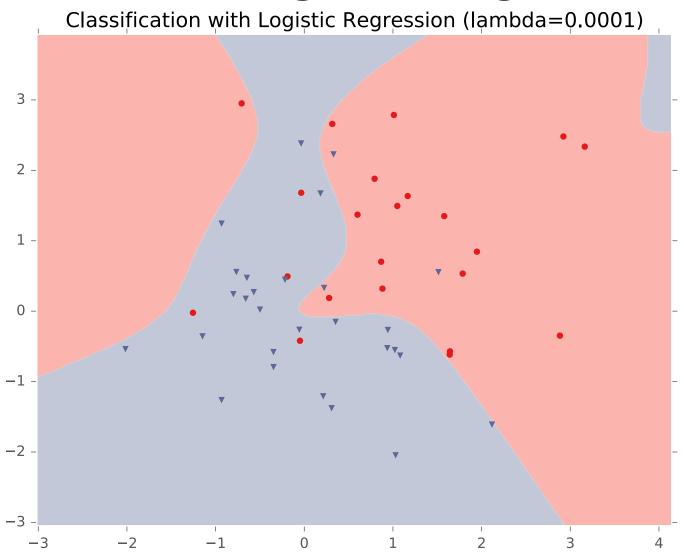


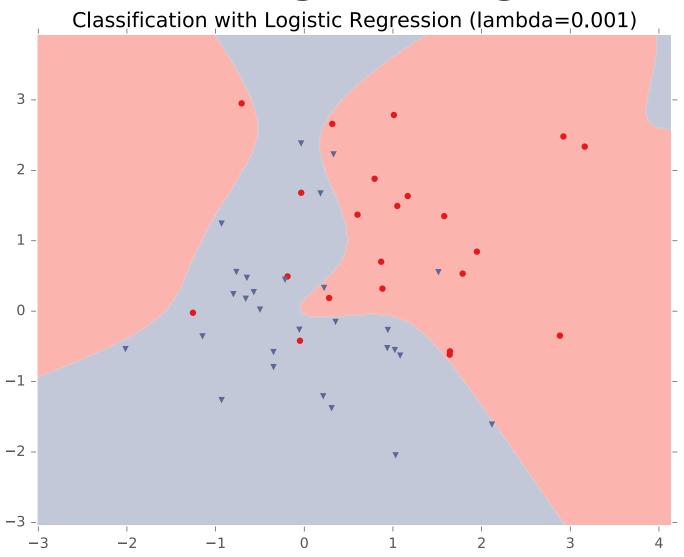


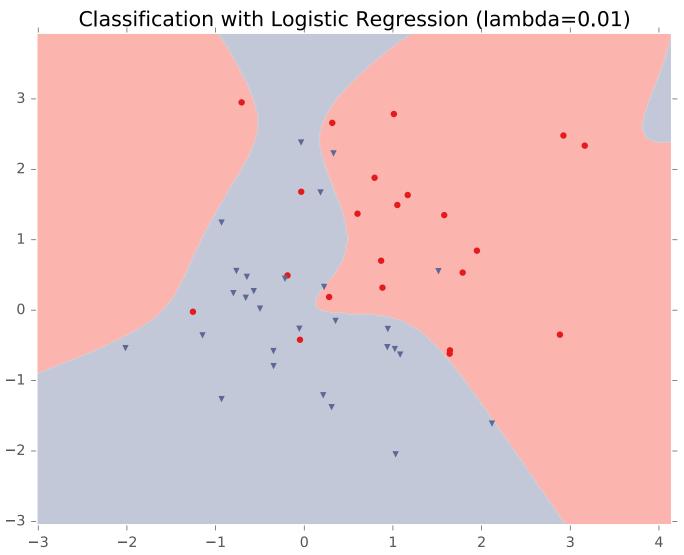


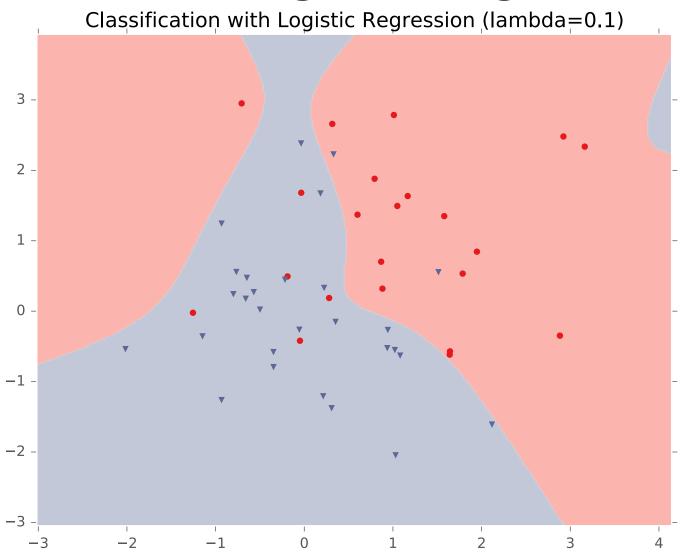


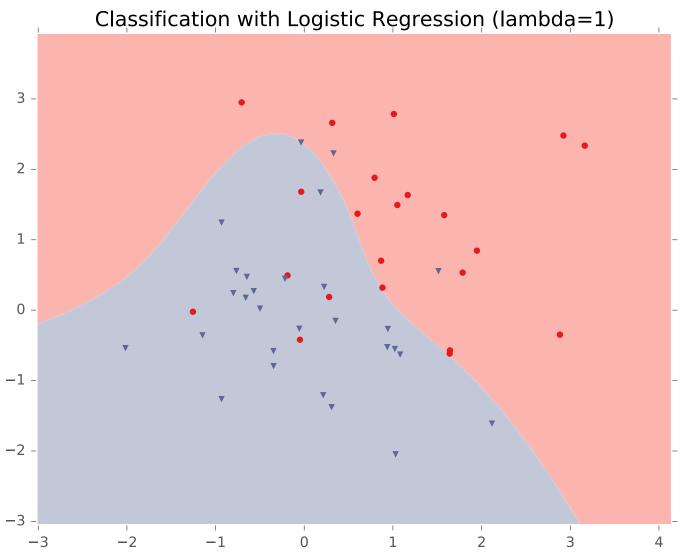


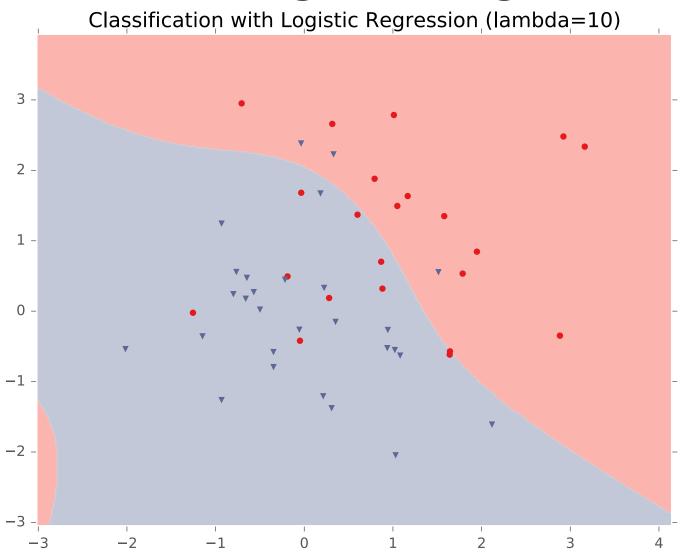


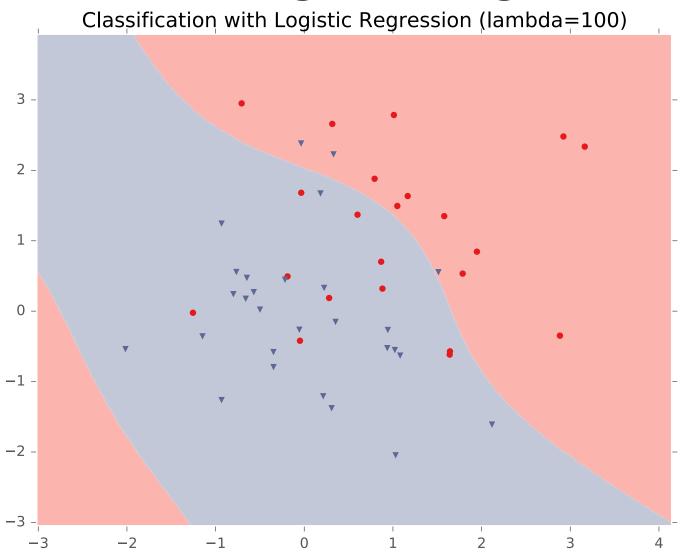


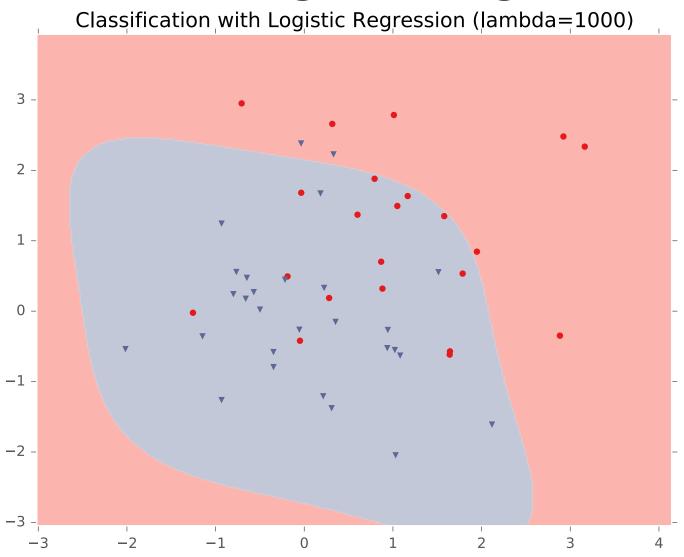


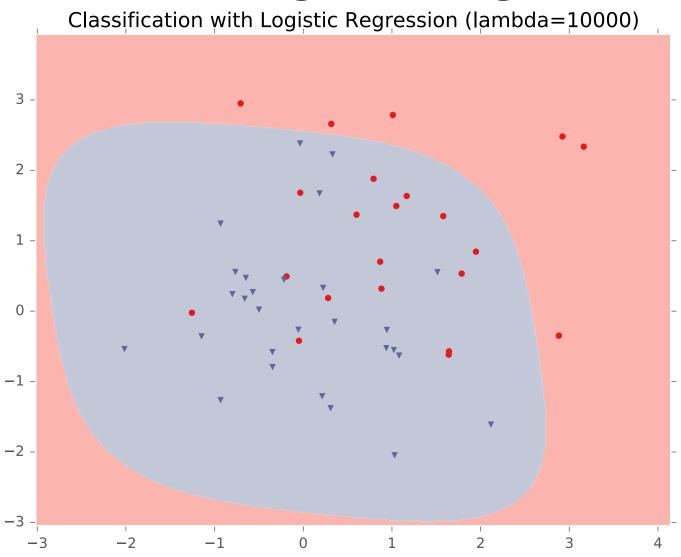


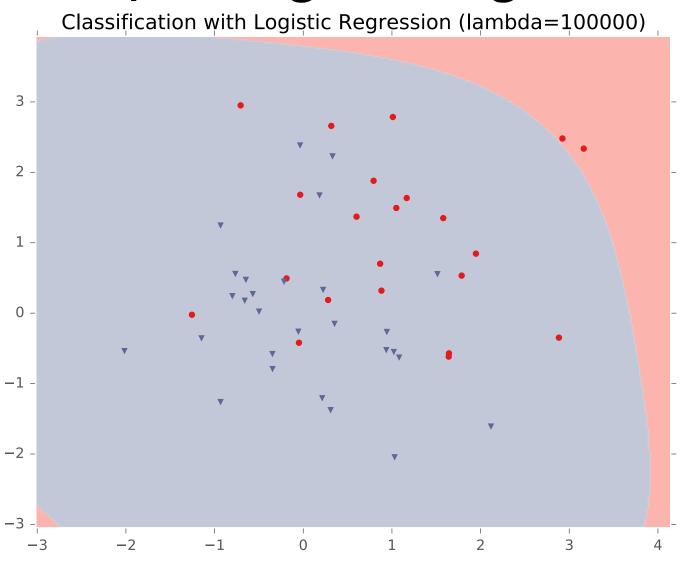


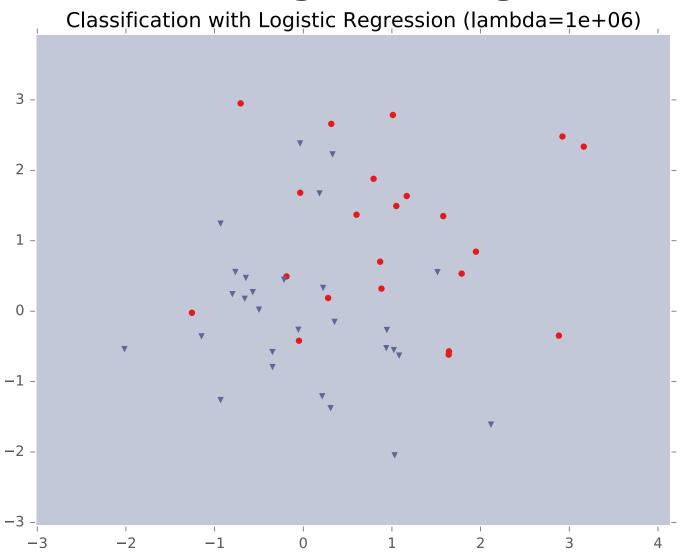


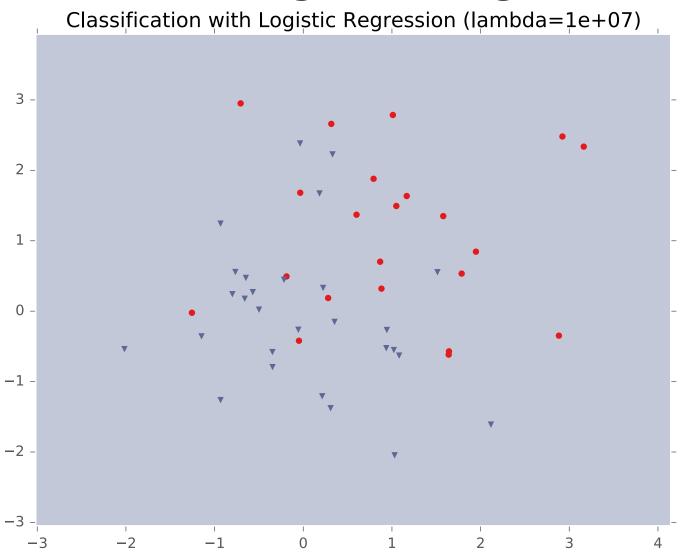


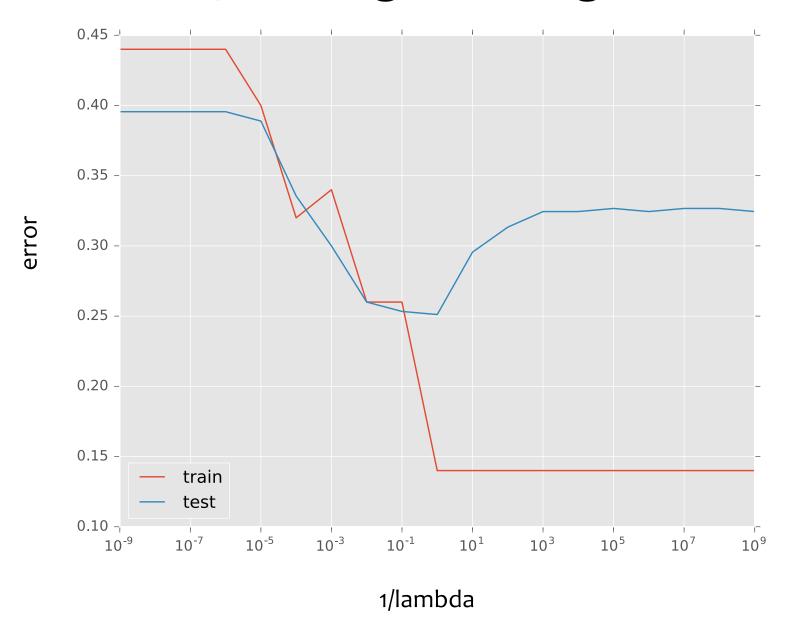












Regularization as MAP

- L1 and L2 regularization can be interpreted as maximum a-posteriori (MAP) estimation of the parameters
- To be discussed later in the course...

Takeaways

- 1. Nonlinear basis functions allow linear models (e.g. Linear Regression, Logistic Regression) to capture nonlinear aspects of the original input
- Nonlinear features are require no changes to the model (i.e. just preprocessing)
- 3. Regularization helps to avoid overfitting
- **4. Regularization** and **MAP estimation** are equivalent for appropriately chosen priors

Feature Engineering / Regularization Objectives

You should be able to...

- Engineer appropriate features for a new task
- Use feature selection techniques to identify and remove irrelevant features
- Identify when a model is overfitting
- Add a regularizer to an existing objective in order to combat overfitting
- Explain why we should not regularize the bias term
- Convert linearly inseparable dataset to a linearly separable dataset in higher dimensions
- Describe feature engineering in common application areas