Machine Learning Department
School of Computer Science
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Learning Large Language Models and Decoding

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Lecture 3
Jan. 22, 2025

Slide credit: Matt Gormley and Henry Chai

v2

A Recipe for Machine Learning

Recall…

1. Given training data:

$$\{\boldsymbol{x}_i, \boldsymbol{y}_i\}_{i=1}^N$$

2. Choose each of these:
   - Decision function

$$\hat{\boldsymbol{y}} = f_{\boldsymbol{\theta}}(\boldsymbol{x}_i)$$

   - Loss function

$$\ell(\hat{\boldsymbol{y}}, \boldsymbol{y}_i) \in \mathbb{R}$$

3. Define goal:

$$\boldsymbol{\theta}^* = \arg\min_{\boldsymbol{\theta}} \sum_{i=1}^N \ell(f_{\boldsymbol{\theta}}(\boldsymbol{x}_i), \boldsymbol{y}_i)$$

4. Train with SGD:

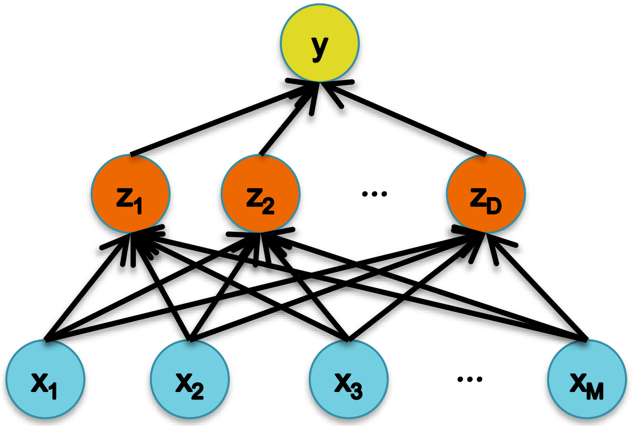
(take small steps opposite the gradient)

$$\boldsymbol{\theta}^{(t+1)} = \boldsymbol{\theta}^{(t)} - \eta_t \nabla \ell(f_{\boldsymbol{\theta}}(\boldsymbol{x}_i), \boldsymbol{y}_i)$$

Recall...

Backpropagation

Example:
Neural Network



	Forward	Backward
Loss	$J = y^* \log y + (1 - y^*) \log(1 - y)$	$g_y = \frac{y^*}{y} + \frac{(1 - y^*)}{y - 1}$
Sigmoid	$y = \frac{1}{1 + \exp(-b)}$	$g_b = g_y \frac{\partial y}{\partial b}, \frac{\partial y}{\partial b} = y(1 - y)$
Linear	$b = \sum_{j=0}^{D} \beta_j z_j$	$g_{\beta_j} = g_b \frac{\partial b}{\partial \beta_j}, \frac{\partial b}{\partial \beta_j} = z_j$ $g_{z_j} = g_b \frac{\partial b}{\partial z_j}, \frac{\partial b}{\partial z_j} = \beta_j$
Sigmoid	$z_j = \frac{1}{1 + \exp(-a_j)}$	$g_{a_j} = g_{z_j} \frac{\partial z_j}{\partial a_j}, \frac{\partial z_j}{\partial a_j} = z_j(1 - z_j)$
Linear	$a_j = \sum_{i=0}^{M} \alpha_{ji} x_i$	$g_{\alpha_{ji}} = g_{a_j} \frac{\partial a_j}{\partial \alpha_{ji}}, \frac{\partial a_j}{\partial \alpha_{ji}} = x_i$ $g_{x_i} = \sum_{j=0}^{D} g_{a_j} \frac{\partial a_j}{\partial x_i}, \frac{\partial a_j}{\partial x_i} = \alpha_{ji}$

Backpropagation

Recall…

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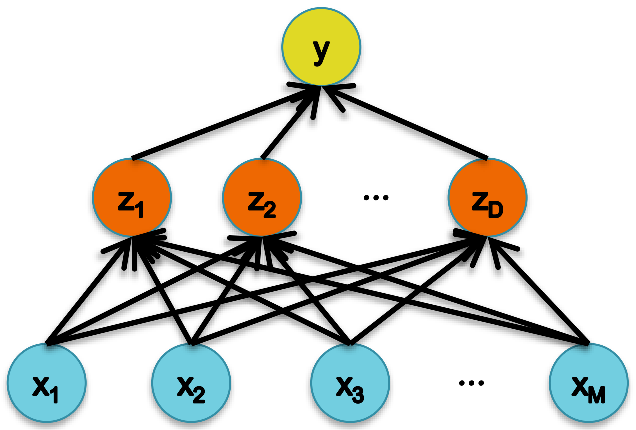

This whole "Backward" columns is now computed for us automatically by AutoDiff

		Backward
Loss		$g_y = \frac{y^*}{y} + \frac{(1-y^*)}{y-1}$
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LEARNING A TRANSFORMER LM

Language Models

1. Given training data:

$$\{\boldsymbol{x}_i, \boldsymbol{y}_i\}_{i=1}^N$$

2. Choose Decision function

$$\hat{\boldsymbol{y}} = f_{\boldsymbol{\theta}}(\boldsymbol{x}_i)$$

But what is the task?

Data

I am Sam .
I am Sam .
Sam I am .
That Sam-I-am .
That Sam-I-am !
I do not like that Sam-I-am
Do you like green eggs and ham
I do not like them , Sam-I-am .
I do not like green eggs and ham .
Would you like them here or there ?
I would not like them here or there .
I would not like them anywhere .
I do not like green eggs and ham .
I do not like them , Sam-I-am .
Would you like them in a house ?
Would you like them with a mouse ?
I do not like them in a house .
I do not like them with a mouse .

Language Models

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$$\{\boldsymbol{x}_i, \boldsymbol{y}_i\}_{i=1}^N$$

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Language Models

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Language Models

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EFFICIENT TRANSFORMERS

Why does efficiency matter?

Case Study: GPT-3

- # of training tokens = 500 billion
- # of parameters = 175 billion
- # of cycles = 50 petaflop/s-days (each of which are 8.64e+19 flops)

Dataset	Quantity (tokens)	Weight in training mix	Epochs elapsed when training for 300B tokens
Common Crawl (filtered)	410 billion	60%	0.44
WebText2	19 billion	22%	2.9
Books1	12 billion	8%	1.9
Books2	55 billion	8%	0.43
Wikipedia	3 billion	3%	3.4

Table 2.2: Datasets used to train GPT-3. "Weight in training mix" refers to the fraction of examples during training that are drawn from a given dataset, which we intentionally do not make proportional to the size of the dataset. As a result, when we train for 300 billion tokens, some datasets are seen up to 3.4 times during training while other datasets are seen less than once.

Model Name	n_{params}	n_{layers}	d_{model}	n_{heads}	d_{head}	Batch Size	Learning Rate
GPT-3 Small	125M	12	768	12	64	0.5M	6.0×10^{-4}
GPT-3 Medium	350M	24	1024	16	64	0.5M	3.0×10^{-4}
GPT-3 Large	760M	24	1536	16	96	0.5M	2.5×10^{-4}
GPT-3 XL	1.3B	24	2048	24	128	1M	2.0×10^{-4}
GPT-3 2.7B	2.7B	32	2560	32	80	1M	1.6×10^{-4}
GPT-3 6.7B	6.7B	32	4096	32	128	2M	1.2×10^{-4}
GPT-3 13B	13.0B	40	5140	40	128	2M	1.0×10^{-4}
GPT-3 175B or "GPT-3"	175.0B	96	12288	96	128	3.2M	0.6×10^{-4}

Table 2.1: Sizes, architectures, and learning hyper-parameters (batch size in tokens and learning rate) of the models which we trained. All models were trained for a total of 300 billion tokens.

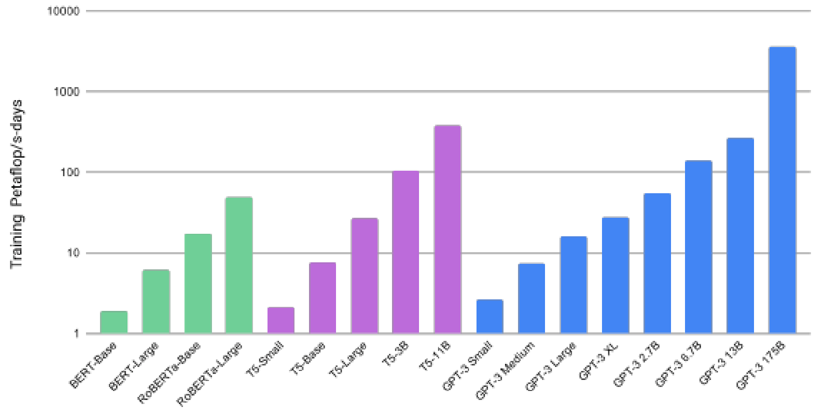

Figure 2.2: Total compute used during training. Based on the analysis in Scaling Laws For Neural Language Models [KMH+20] we train much larger models on many fewer tokens than is typical. As a consequence, although GPT-3 3B is almost 10x larger than RoBERTa-Large (355M params), both models took roughly 50 petaflop/s-days of compute during pre-training. Methodology for these calculations can be found in Appendix D.

Figure from https://arxiv.org/pdf/2005.14165.

Efficient Parallelism for Transformers

Transformers can be trained very efficiently!

(This is arguably one of the key reasons they have been so successful.)

- **Batching**: Rather than processing one sentence at a time, Transformers take in a batch of B sentences at a time. The computation is identical for each batch and is trivially parallelized.
- **Scaled Dot-product Attention**: can be easily parallelized because the attention scores of one timestep do not depend on other timesteps.
- **Multi-headed Attention**: computes each head independently, which permits yet more parallelism.
- **Matrix multiplication:** The core computation in attention is matrix multiplication, and specialized hardware (GPUs and TPUs) makes this very fast.
- **Model parallelism:** For huge models, we can divide the model over multiple GPUs/machines.
- **Key-value caching**: The keys and values are re-used over many timesteps, but we do not need to cache the queries, similarity scores, and attention weights.

Batching: Padding and Truncation

- Suppose we have 8 training sentences
- We set our block size (maximum sequence length) to 10
- Before collecting them into a batch, we:
  1. truncate those sentences that are too long
  2. pad the sentences that are too short
  3. convert each token to an integer via a lookup table (vocabulary)
  4. convert each token to an embedding vector of fixed length

i	w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8	w_9	w_{10}	w_{11}	w_{12}
1	In	the	hole	in	the	ground	there	lived	a	hobbit		
2	It	is	our	choices	that	show	what	we	truly	are		
3	It	was	the	best	of	times	it	was	the	worst	of	times
4	Even	miracles	take	a	little	time						
5	The	more	that	you	read	the	more	things	you	will	know	
6	We'll	always	have	each	other	no	matter	what	happens			
7	The	sun	did	not	shine	it	was	too	wet	to	play	
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3	It	was	the	best	of	times	it	was	the	worst	of	times
4	Even	miracles	take	a	little	time	<PAD>	<PAD>	<PAD>	<PAD>		
5	The	more	that	you	read	the	more	things	you	will	know	
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2	3	20	32	10	40	36	53	51	49	8
3	3	50	41	9	30	46	21	50	41	55
4	1	25	39	6	22	45	0	0	0	0
5	4	26	40	56	34	41	26	44	56	54
6	5	7	15	12	31	28	24	53	14	0
7	4	38	11	29	35	21	50	48	52	47
8	4	18	43	20	47	27	37	33	0	0

Vocabulary:

```
{
    '<PAD>': 0,
    'Even': 1,
    'In': 2,
    'It': 3,
    'The': 4,
    "We'll": 5,
    'a': 6,
    'always': 7,
    'are': 8,
    'best': 9,
     …
    'what': 53,
    'will': 54,
    'worst': 55,
    'you': 56
}
```

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1										
2										
3										
4										
5										
6										
7										
8										

Embeddings:

```
{
    0 :
    1 :
    2 :
    3 :
    4 :
    5 :
    6 :
    7 :
    ...
    55 :
    56 :
}
```

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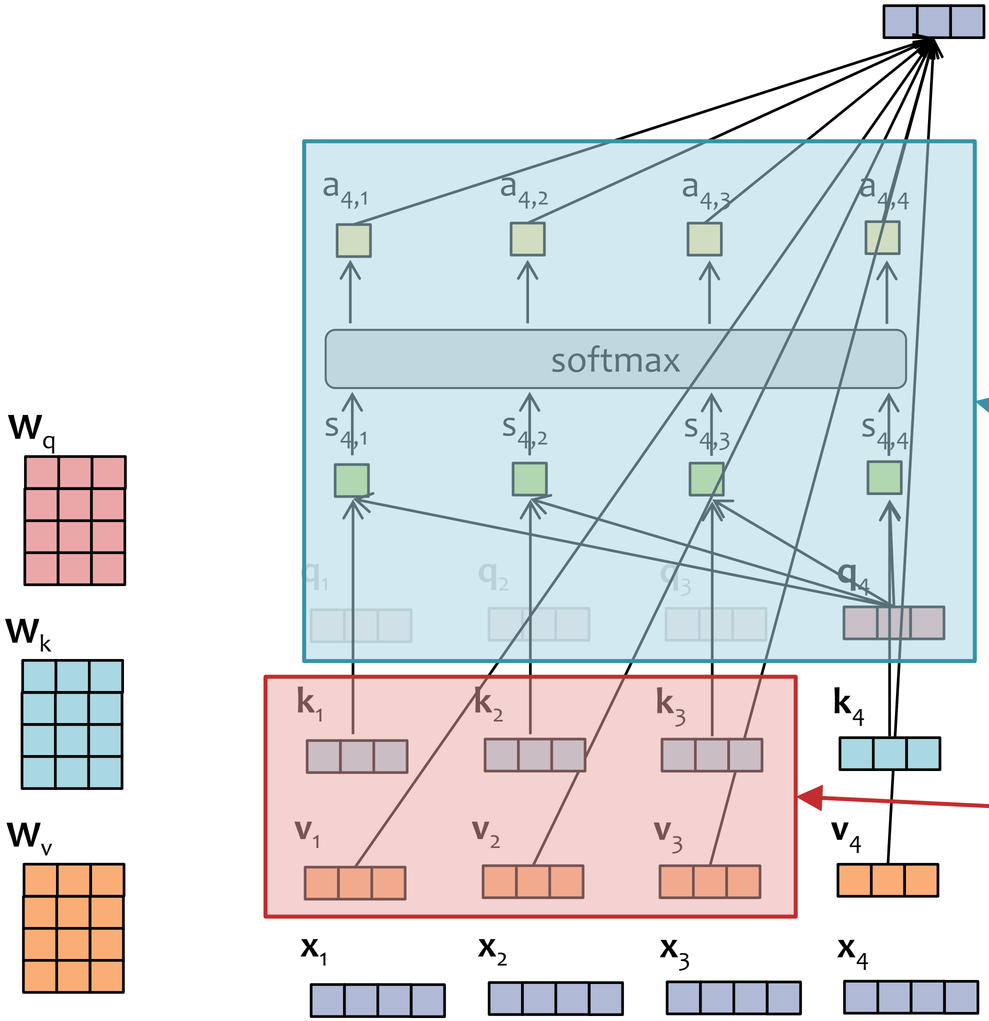
Key-Value Cache



$$\mathbf{x}'_4 = \sum_{j=1}^{4} a_{4,j} \mathbf{v}_j$$

$$\mathbf{a}_4 = \text{softmax}(\mathbf{s}_4)$$

$$s_{4,j} = \mathbf{k}_j^T \mathbf{q}_4 / \sqrt{d_k}$$

$$\mathbf{q}_j = \mathbf{W}_q^T \mathbf{x}_j$$

$$\mathbf{k}_j = \mathbf{W}_k^T \mathbf{x}_j$$

$$\mathbf{v}_j = \mathbf{W}_v^T \mathbf{x}_j$$

- At each timestep, we reuse all previous keys and values (i.e. we need to cache them)
- But we can get rid of the queries, similarity scores, and attention weights (i.e. we can let them fall out of the cache)

TOKENIZATION

Tokenization

Word-based Tokenizer:

Input: “Henry is giving a lecture on transformers”

Output: [“henry”, “is”, “giving”, “a”, “lecture”, “on”, “transformers”]

Pros/Cons:

- Can have difficulty trading off between vocabulary size and computational tractability
- Similar words e.g., “transformers” and “transformer” can get mapped to completely disparate representations
- Typos will typically be out-of-vocabulary (OOV)

Slide adapted from Henry Chai

Tokenization

Word-based Tokenizer:

Input: “Henry is givin’ a lectrue on transformers”

Output: [“henry”, “is”, <OOV>, “a”, <OOV>, “on”, “transformers”]

Pros/Cons:

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Tokenization

Character-based Tokenizer:

Input: “Henry is givin’ a lectrue on transformers”

Output: [“h”, “e”, “n”, “r”, “y”, “i”, “s”, “g”, “i”, “v”, “i”, “n”, “ ’ ”, …]

Pros/Cons:

- Much smaller vocabularies but a lot of semantic meaning is lost…
- Sequences will be much longer than word-based tokenization, potentially causing computational issues
- Can do well on logographic languages e.g., Kanji 漢字

Tokenization

Subword-based Tokenizer:

Input: “Henry is givin’ a lectrue on transformers”

Output: [“henry”, “is”, “giv”, “##in”, “ ‘ ”, “a”, “lec” “##true”, “on”, “transform”, “##ers”]

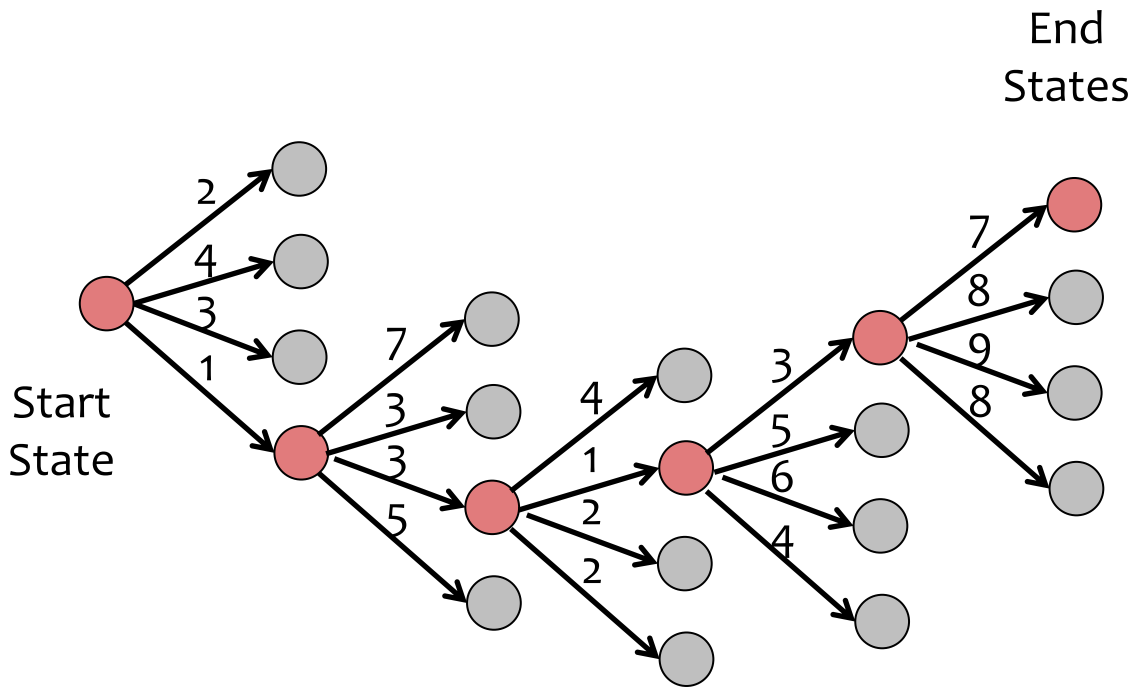
Pros/Cons:

- Split long or rare words into smaller, semantically meaningful components or subwords
- No out-of-vocabulary words – any non-subword token can be constructed from other subwords (always includ all characters as subwords)
- Examples algorithms for learning a subword tokenization:
  - Byte-Pair-Encoding (BPE), WordPiece, SentencePiece

Slide adapted from Henry Chai

GREEDY DECODING FOR A LANGUAGE MODEL

Background: Greedy Search



Goal:

- Search space consists of nodes and weighted edges
- Goal is to find the lowest (total) weight path from root to a leaf

Greedy Search:

- At each node, selects the edge with lowest (immediate) weight
- **Heuristic** method of search (i.e. does *not* necessarily find the best path)
- Computation time: **linear** in max path length

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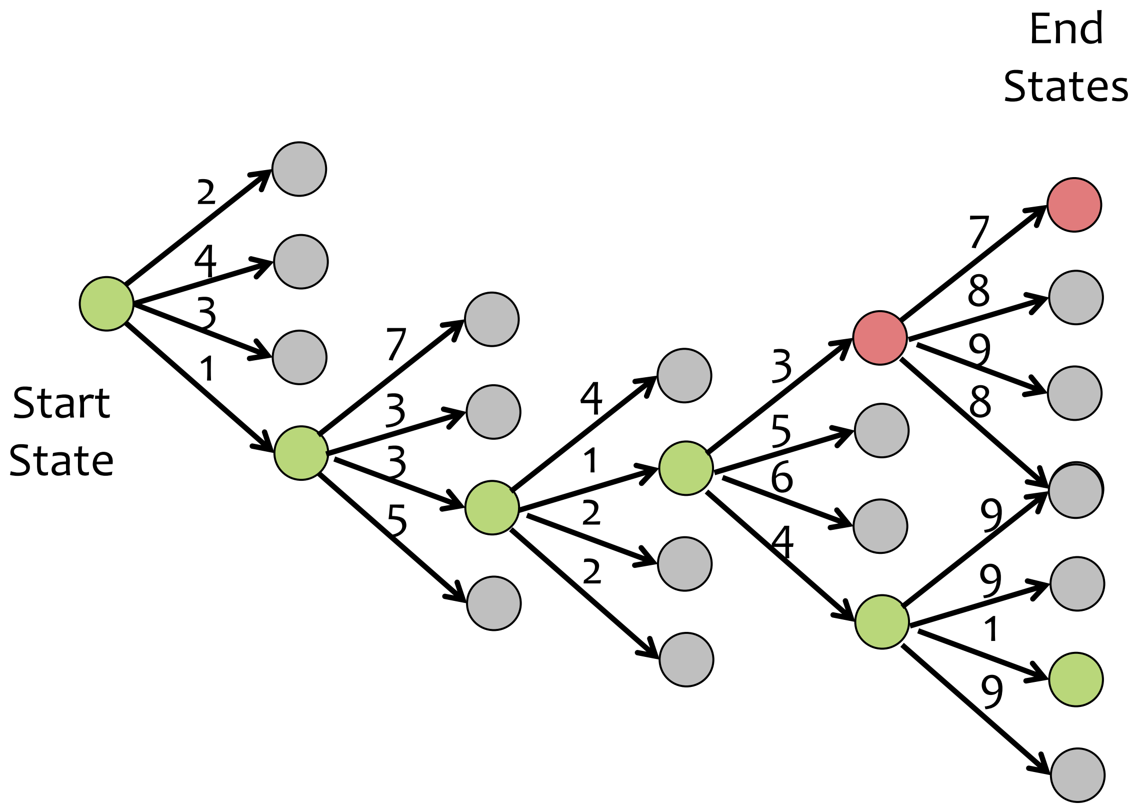

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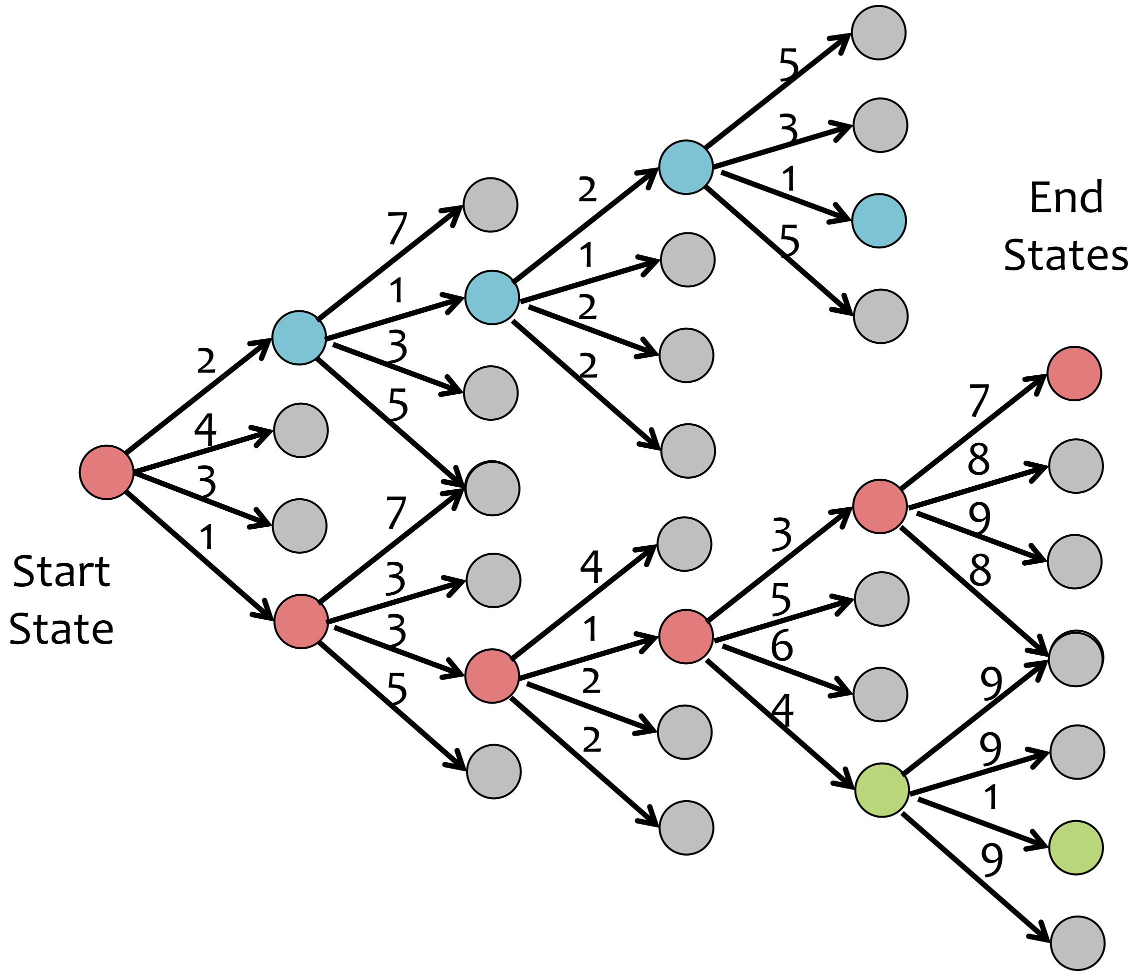

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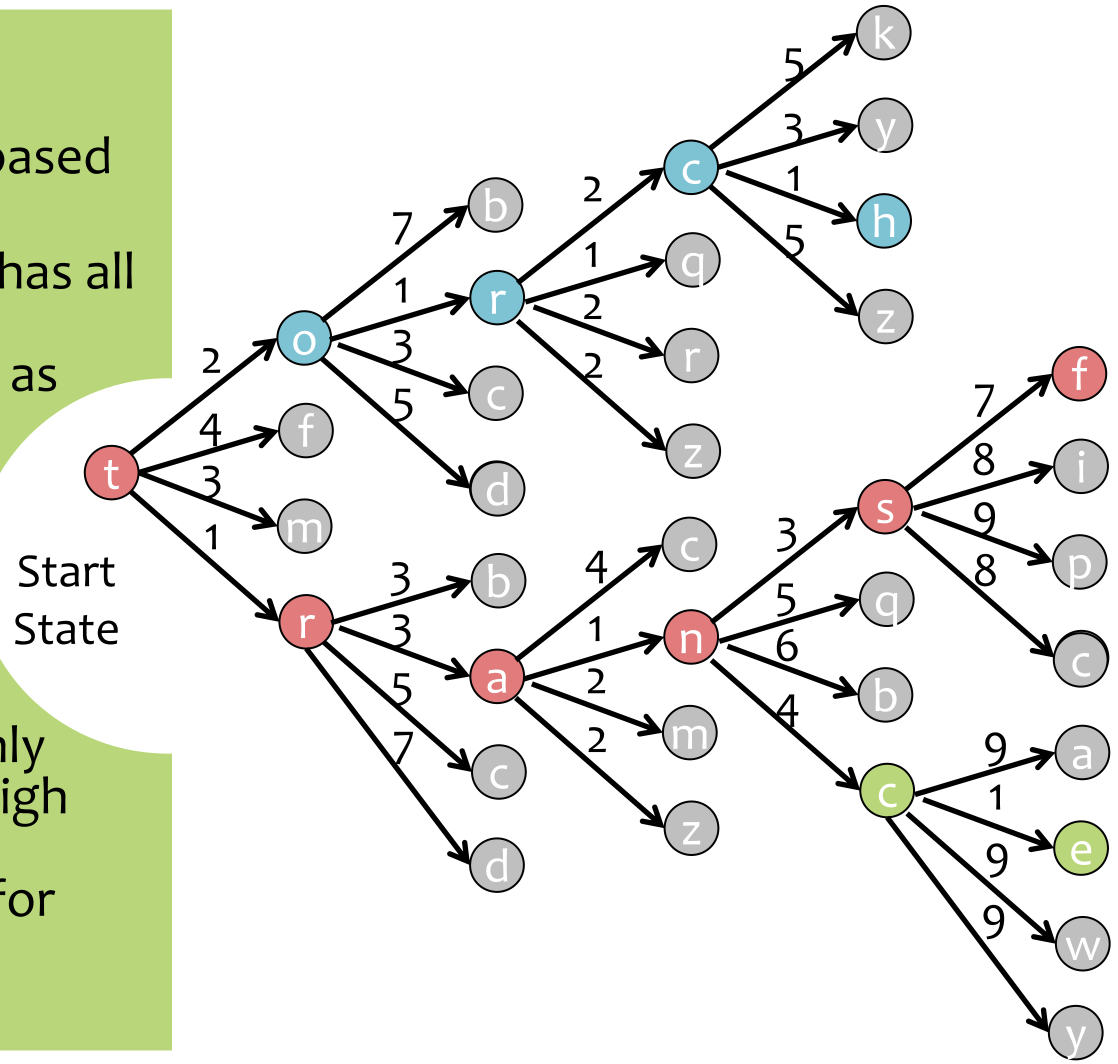
Greedy Decoding for a Language Model

Setup:

- Assume a character-based tokenizer
- Each node has all characters {a,b,c,…,z} as neighbors
- Here we only show the high probability neighbors for space



Goal:

- Search space consists of nodes (partial sentences) and weighted by negative log probability
- Goal is to find the highest probably (lowest negative log probability) path from root to a leaf

Greedy Search:

- At each node, selects the edge with lowest negative log probability
- **Heuristic** method of search (i.e. does *not* necessarily find the best path)
- Computation time: **linear** in max path length

Sampling from a Language Model

Setup:

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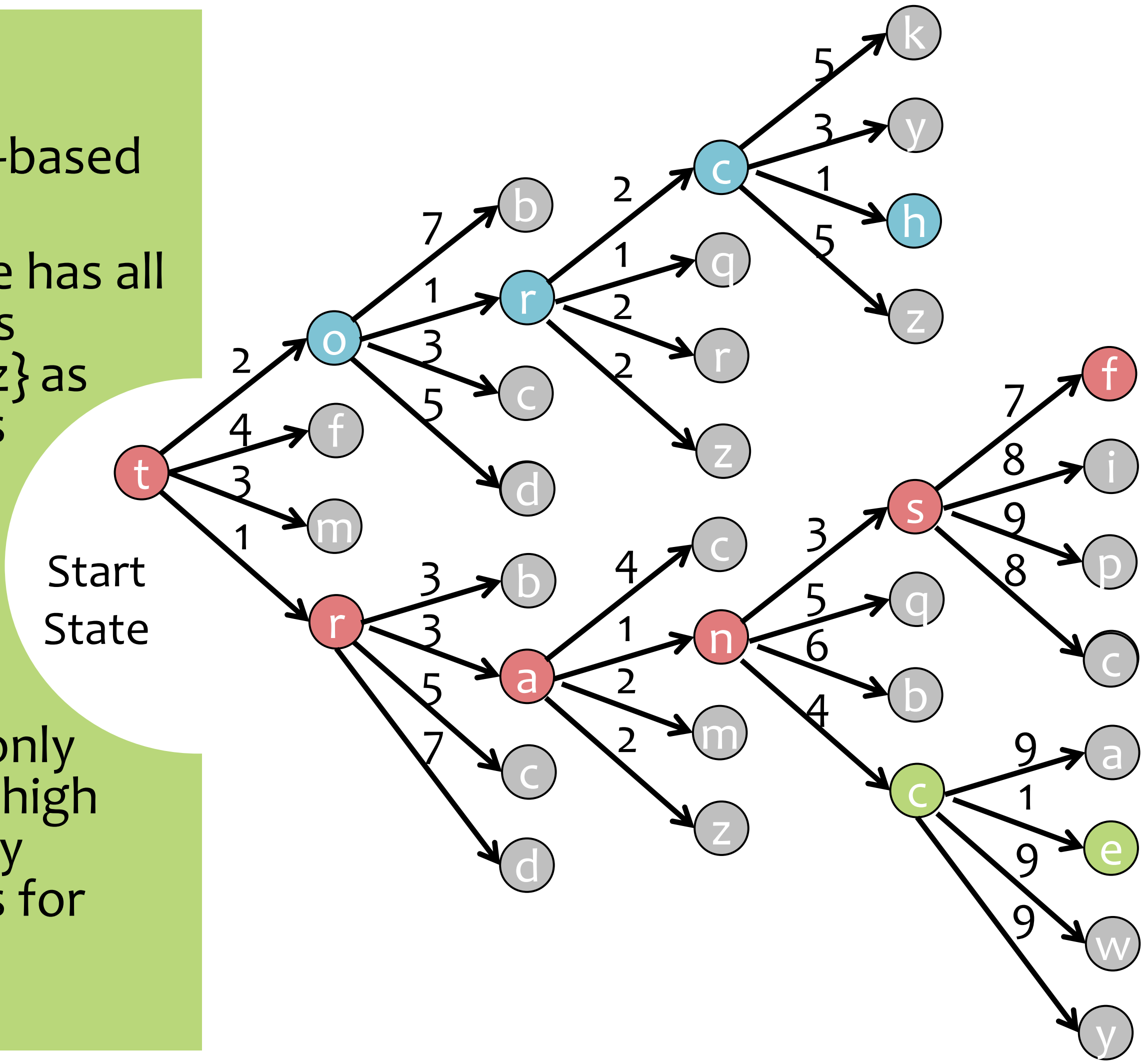

Goal:

- Search space consists of nodes (partial sentences) and weighted by negative log probability
- Goal is to sample a path from root to a leaf with probability according to the probability of that path

Ancestral Sampling:

- At each node, randomly pick an edge with probability (converting from negative log probability)
- **Exact** method of sampling, assuming a locally normalized distribution (i.e. does *not* necessarily find the best path)
- Computation time: **linear** in max path length