Machine Learning Department
School of Computer Science
Carnegie Mellon University

Efficient Attention (FlashAttention)

Matt Gormley & Pat Virtue
Lecture 18
Mar. 24, 2025

Reminders

- Homework 4: Visual Language Models
  - Out: Thu, Mar 13
  - Due: Mon, Mar 24 at 11:59pm
- Exam
  - Date: In-class, Monday, Mar 31
  - Time: 75 minutes, taking up the whole class time
  - Covered Material: Lectures 1 – 15 (same as Quiz 1 – Quiz 4)
  - You may bring one sheet of notes (front and back)
  - Format of questions: Unlike the Quiz questions, which were all multiple choice, Exam questions will include open-ended questions as well
  - Check Piazza for seat assignment

Why do we care about FlashAttention?

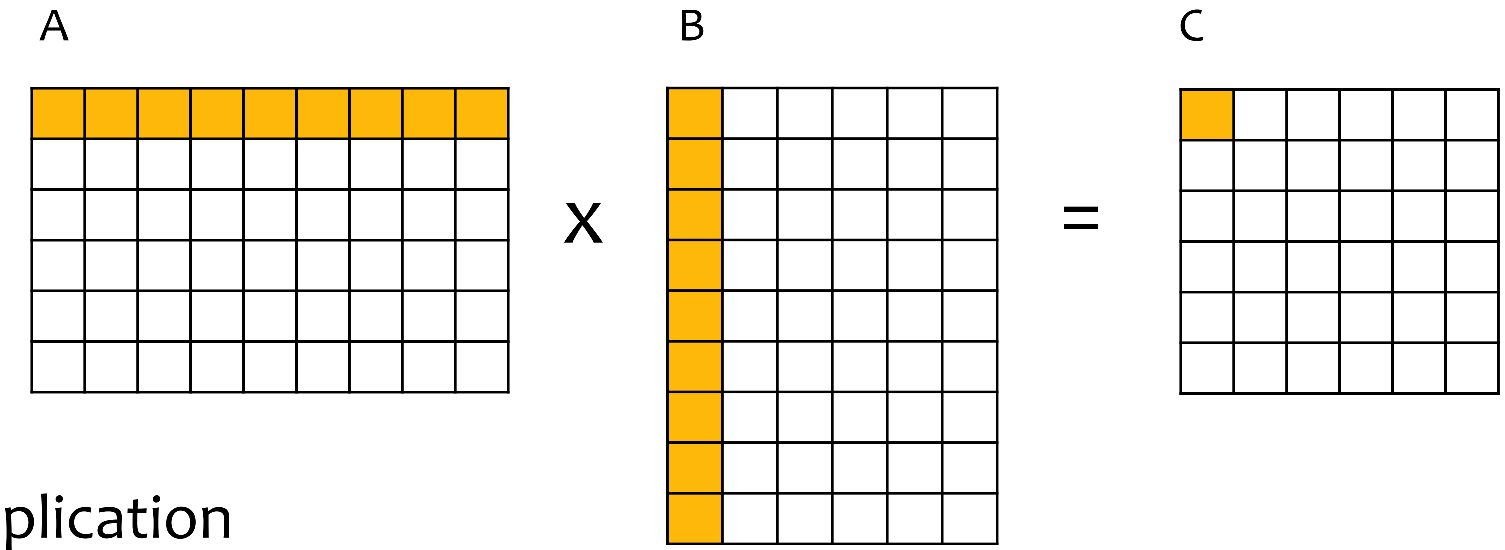
- The algorithm is performing exact attention, so we see no reduction in perplexity or quality of the model
- The key metric is runtime

Model implementations	OpenWebText (ppl)	Training time (speedup)
GPT-2 small - Huggingface [87]	18.2	9.5 days (1.0×)
GPT-2 small - Megatron-LM [77]	18.2	4.7 days (2.0×)
GPT-2 small - FlashAttention	18.2	**2.7 days (3.5×)**
GPT-2 medium - Huggingface [87]	14.2	21.0 days (1.0×)
GPT-2 medium - Megatron-LM [77]	14.3	11.5 days (1.8×)
GPT-2 medium - FlashAttention	14.3	**6.9 days (3.0×)**

Background

TILING FOR MATRIX MULTIPLICATION

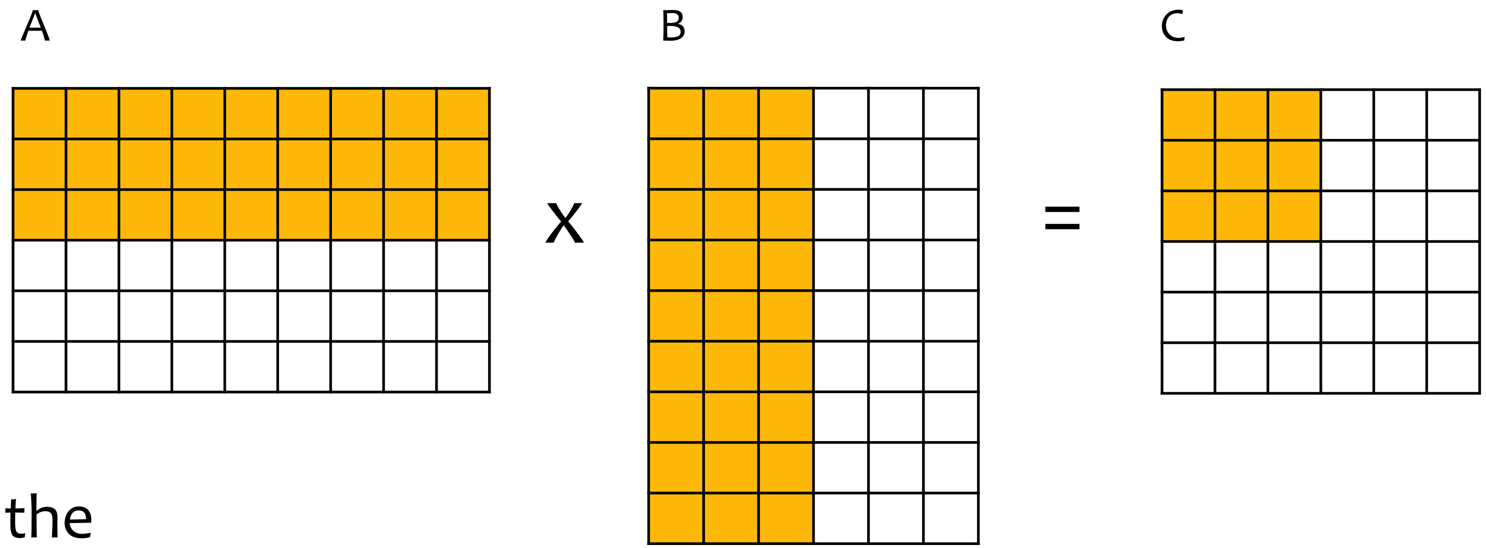
Tiling for Matrix Multiplication



- Matrix multiplication computes each output value as a dot-product of a row/column pair from the input matrices

$$C_{ij} = \sum_{m=1}^{M} \sum_{n=1}^{N} A_{im} B_{nj}$$

Tiling for Matrix Multiplication

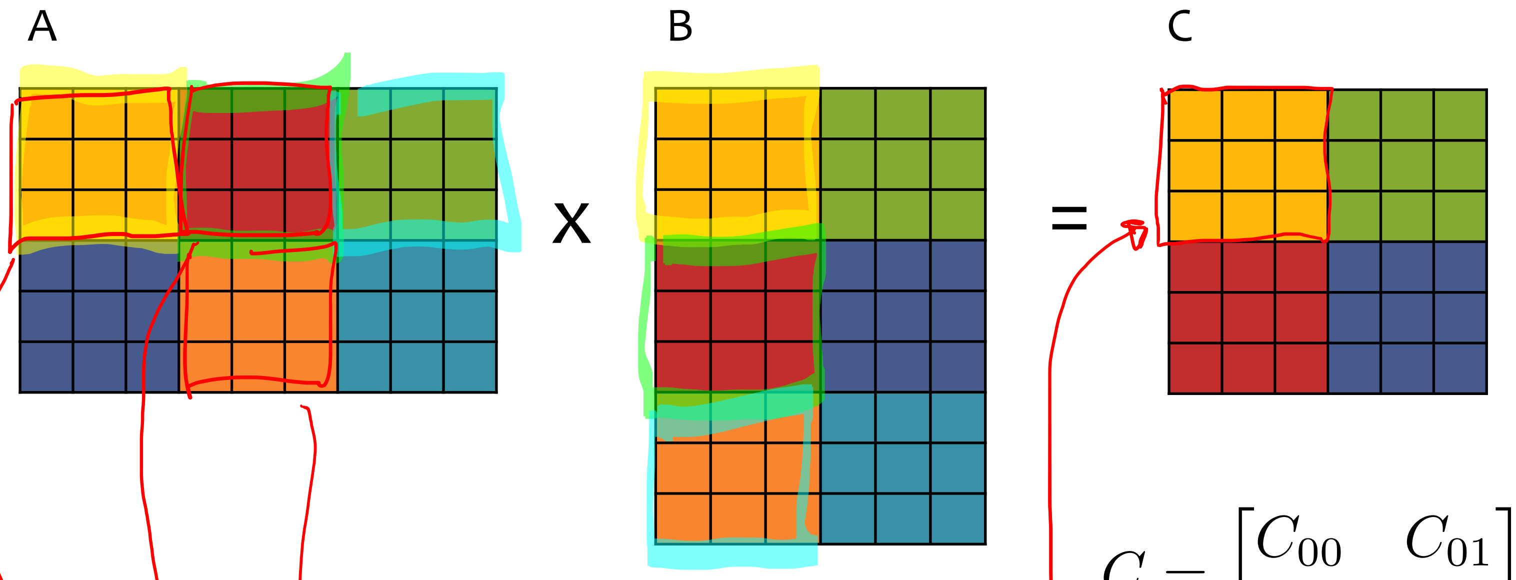


- We can view the computation as decomposing if we consider subsets of rows/columns

$$C_{(1,1):(3,3)} = A_{(1,1):(3,9)} \times B_{(1,1):(9,3)}$$

Tiling for Matrix Multiplication

- Tiling capitalizes on this decomposition
- Each output tile is computed by multiplying a pair of input tiles and adding it to the appropriate output tile



$$A = \begin{bmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \end{bmatrix}$$

with each $A_{ij} \in \mathbb{R}^{3\times 3}$

$$B = \begin{bmatrix} B_{00} & B_{01} \\ B_{10} & B_{11} \\ B_{20} & B_{21} \end{bmatrix}$$

with each $B_{ij} \in \mathbb{R}^{3\times 3}$

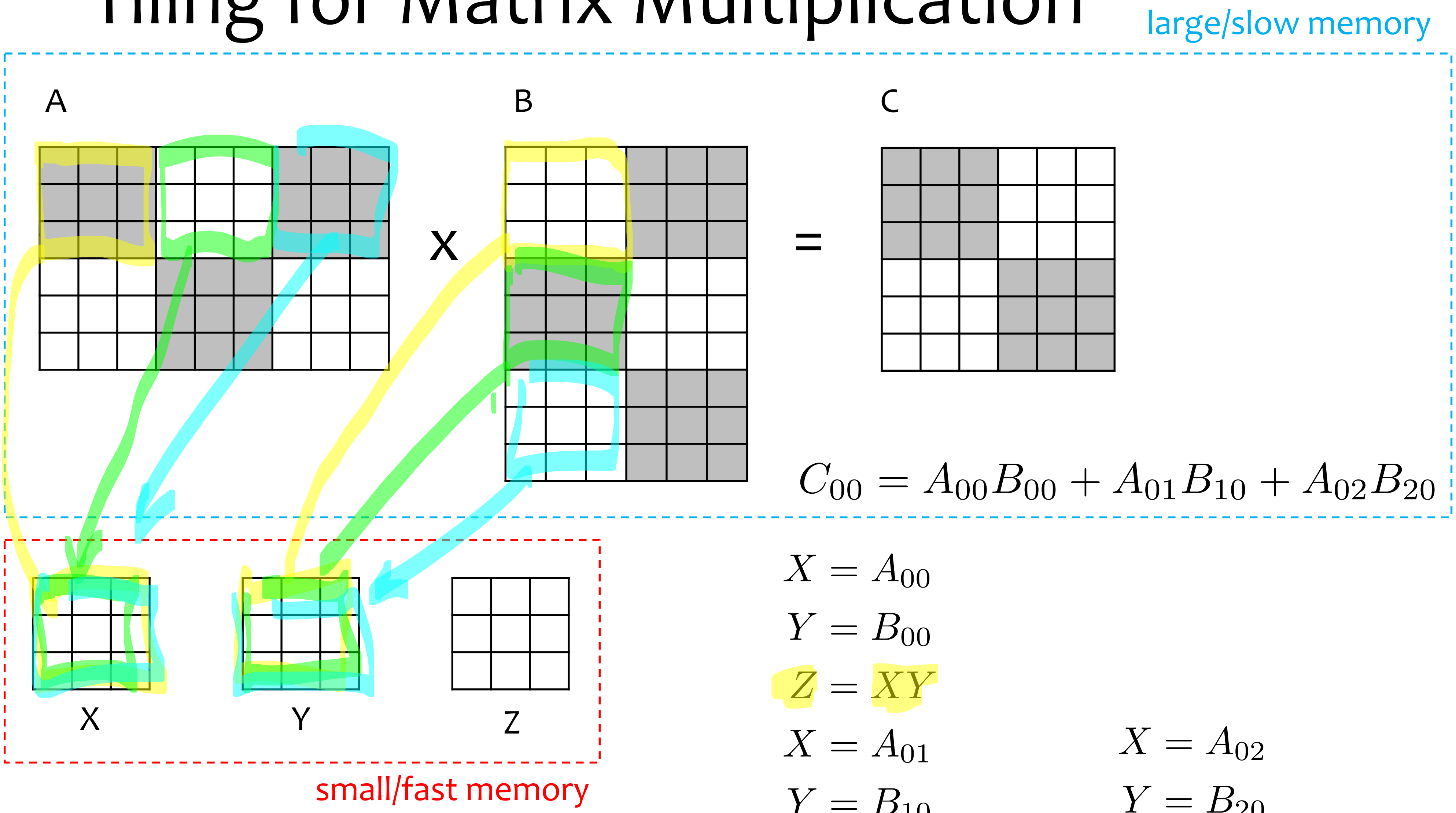
$$C = \begin{bmatrix} C_{00} & C_{01} \\ C_{10} & C_{11} \end{bmatrix}$$

with each $C_{ij} \in \mathbb{R}^{3\times 3}$

$$C_{00} = A_{00}B_{00} + A_{01}B_{10} + A_{02}B_{20}$$
$$C_{01} = A_{00}B_{01} + A_{01}B_{11} + A_{02}B_{21}$$
$$C_{10} = A_{10}B_{00} + A_{11}B_{10} + A_{12}B_{20}$$
$$C_{11} = A_{10}B_{01} + A_{11}B_{11} + A_{12}B_{21}$$

Tiling for Matrix Multiplication

- Tiling enables matrix multiplication of two **very** large matrices to capitalize on the small amount of fast memory on a device (e.g. GPU)
- Start by putting the input matrices and storage for the output matrix into large/slow memory
- Do the primary computation in slow/fast memory



$$C_{00} = A_{00}B_{00} + A_{01}B_{10} + A_{02}B_{20}$$

$$X = A_{00}$$
$$Y = B_{00}$$
$$Z = XY$$

$$X = A_{01}$$
$$Y = B_{10}$$
$$Z = Z + XY$$

$$X = A_{02}$$
$$Y = B_{20}$$
$$Z = Z + XY$$

$$C_{00} = Z$$

Tiling for Self-Attention?

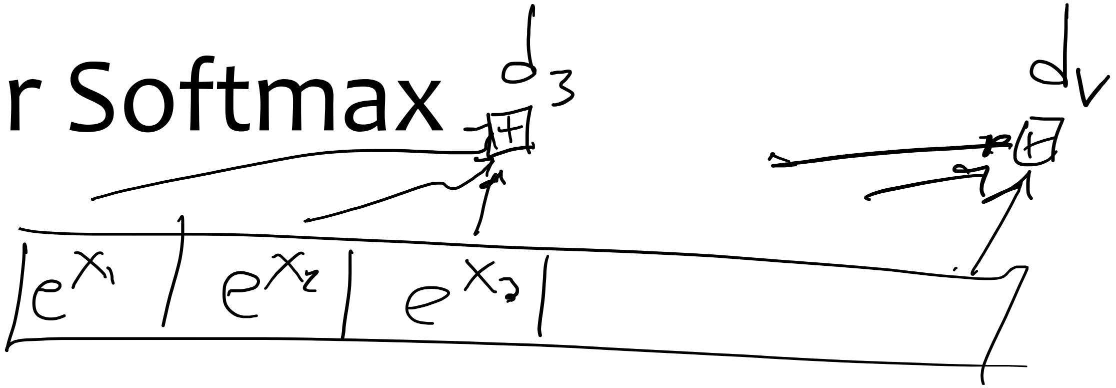
- It would be great if we could directly use tiling for self-attention
- Unfortunately, whereas the addition in matrix multiplication is associative, the softmax in self-attention is not!

$$\mathbf{X}' = \text{softmax}(\mathbf{Q}\mathbf{K}^T / \sqrt{d_k})\mathbf{V}$$

Background

ONLINE SOFTMAX

Regular Softmax



- The standard softmax computation is used heavily throughout deep learning
- Yet, often we need to compute softmax on very large logits
- To avoid issues of overflow when raising e to some large power, we can use the safe softmax instead
- Every deep learning library implements this

$$y_i = \frac{e^{x_i}}{\sum_{j=1}^{V} e^{x_j}}$$

Algorithm 1 Naive softmax

$d_0 \leftarrow 0$
for $j \leftarrow 1, V$ **do**
    $d_j \leftarrow d_{j-1} + e^{x_j}$
end for
for $i \leftarrow 1, V$ **do**
    $y_i \leftarrow \frac{e^{x_i}}{d_V}$
end for

Figure from https://arxiv.org/pdf/1805.02867

Safe Softmax

- The standard softmax computation is used heavily throughout deep learning
- Yet, often we need to compute softmax on very large logits
- To avoid issues of overflow when raising e to some large power, we can use the safe softmax instead
- Every deep learning library implements this

$$y_i = \frac{e^{x_i - \max_{k=1}^{V} x_k}}{\sum_{j=1}^{V} e^{x_j - \max_{k=1}^{V} x_k}}$$

$$= \left(\frac{e^{x_i}}{\sum_{j=1}^{V} e^{x_j}} \right) \cdot \left(\frac{e^{-m_V}}{e^{-m_V}} \right)$$

Algorithm 2 Safe softmax

$m_0 \leftarrow -\infty$
for $k \leftarrow 1, V$ **do**
    $m_k \leftarrow \max(m_{k-1}, x_k)$
end for
$d_0 \leftarrow 0$
for $j \leftarrow 1, V$ **do**
    $d_j \leftarrow d_{j-1} + e^{x_j - m_V}$
end for
for $i \leftarrow 1, V$ **do**
    $y_i \leftarrow \frac{e^{x_i - m_V}}{d_V}$
end for

$$m_V = \max_{k=1}^{V} x_k$$

Figure from https://arxiv.org/pdf/1805.02867

Online Softmax

- The problem with the usual safe softmax is that it requires three iterations, with each one accessing memory
- Online softmax reduces this to only two iterations through the data!
- This results in not only a 1.33x apparent speedup, but also a 1.3x speedup in practice because of reduced memory bandwidth requirements

Algorithm 3 Safe softmax with online normalizer calculation

$m_0 \leftarrow -\infty$
$d_0 \leftarrow 0$
for $j \leftarrow 1, V$ **do**
    $m_j \leftarrow \max(m_{j-1}, x_j)$
    $d_j \leftarrow d_{j-1} \times e^{m_{j-1}-m_j} + e^{x_j - m_j}$
end for
for $i \leftarrow 1, V$ **do**
    $y_i \leftarrow \frac{e^{x_i - m_V}}{d_V}$
end for

$d_j = \sum_{k=1}^{j} e^{x_k - m_j}$

Figure from https://arxiv.org/pdf/1805.02867

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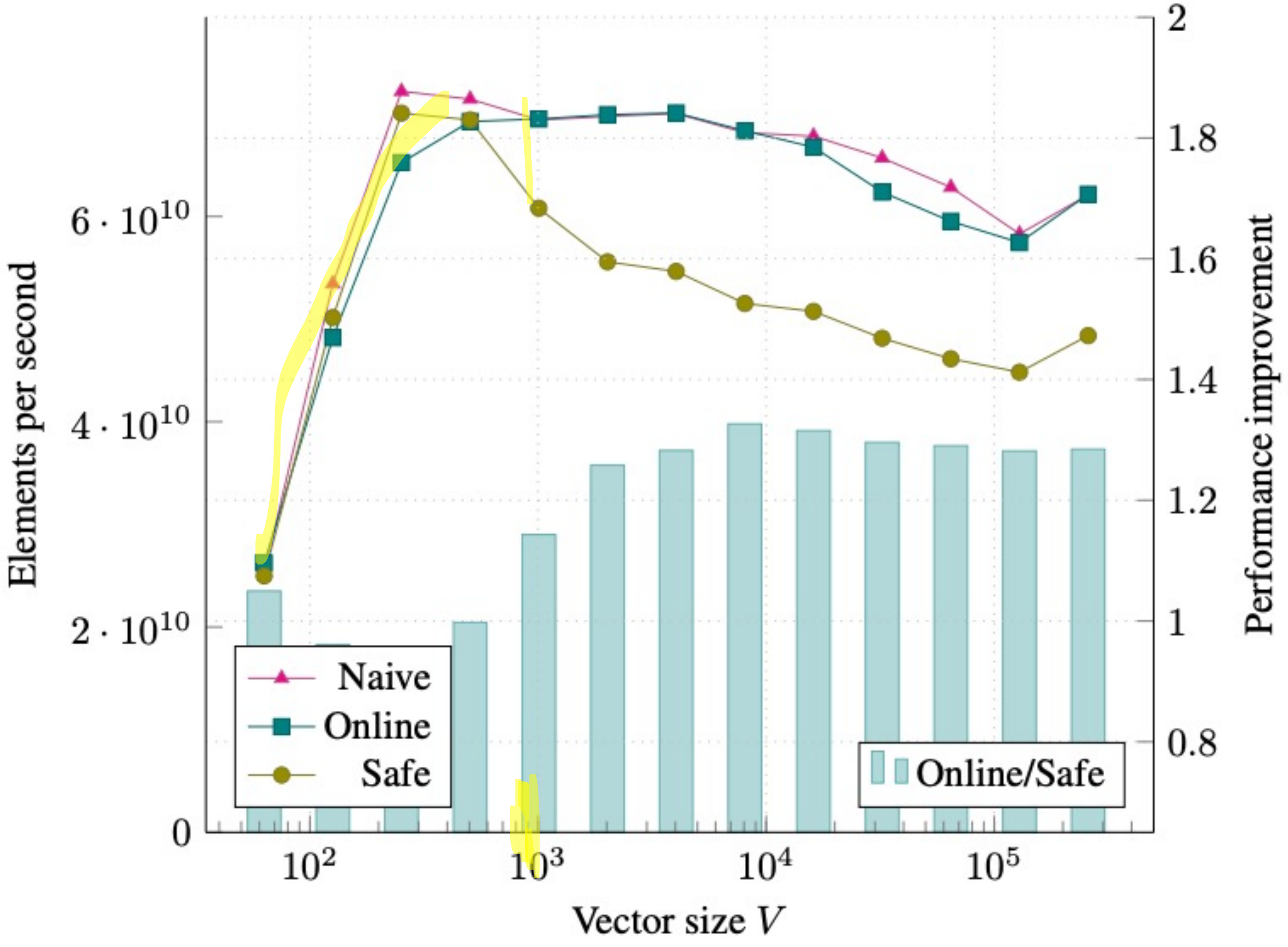

Figure 1: Benchmarking softmax, Tesla V100, fp32, batch size 4000 vectors

Figure from https://arxiv.org/pdf/1805.02867

Online Softmax

Theorem 1. *The lines 1-6 of the algorithm 3 compute* $m_V = \max_{k=1}^{V} x_k$ *and* $d_V = \sum_{j=1}^{V} e^{x_j - m_V}$

Proof. We will use a proof by induction.

◊ *Base case*: $V = 1$

$$m_1 \leftarrow x_1 \qquad \text{by line 4 of the algorithm 3}$$

$$= \max_{k=1}^{1} x_k$$

$$d_1 \leftarrow e^{x_1 - m_1} \qquad \text{by line 5 of the algorithm 3}$$

$$= \sum_{j=1}^{1} e^{x_j - m_1}$$

The theorem holds for $V = 1$.

Figure from https://arxiv.org/pdf/1805.02867

Online Softmax

Theorem 1. *The lines 1-6 of the algorithm 3 compute* $m_V = \max_{k=1}^{V} x_k$ *and* $d_V = \sum_{j=1}^{V} e^{x_j - m_V}$

Proof. We will use a proof by induction.

◊ *Inductive step*: We assume the theorem statement holds for $V = S - 1$, that is the lines 1-6 of the algorithm 3 compute $m_{S-1} = \max_{k=1}^{S-1} x_k$ and $d_{S-1} = \sum_{j=1}^{S-1} e^{x_j - m_{S-1}}$. Let's see what the algorithm computes for $V = S$

$$m_S \leftarrow \max(m_{S-1}, x_S) \quad \text{by line 4 of the algorithm 3}$$

$$= \max(\max_{k=1}^{S-1} x_k, x_S) \quad \text{by the inductive hypothesis}$$

$$= \max_{k=1}^{S} x_k$$

$$d_S \leftarrow d_{S-1} \times e^{m_{S-1} - m_S} + e^{x_S - m_S} \quad \text{by line 5 of the algorithm 3}$$

$$= \left(\sum_{j=1}^{S-1} e^{x_j - m_{S-1}}\right) \times e^{m_{S-1} - m_S} + e^{x_S - m_S} \quad \text{by the inductive hypothesis}$$

$$= \sum_{j=1}^{S-1} e^{x_j - m_S} + e^{x_S - m_S}$$

$$= \sum_{j=1}^{S} e^{x_j - m_S}$$

The inductive step holds as well. □

Figure from https://arxiv.org/pdf/1805.02867

FLASHATTENTION

Figure from https://slideslive.com/38988230/flashattention-fast-and-memoryefficient-exact-attention-with-ioawareness

FlashAttention

- One of the most impactful ideas in ML recently
- Even though many people probably don’t even know they are using it!
- Introduced at HAET Workshop @ ICML July 2022
- Published @ NeurIPS Dec 2022

Figure from https://slideslive.com/38988230/flashattention-fast-and-memoryefficient-exact-attention-with-ioawareness

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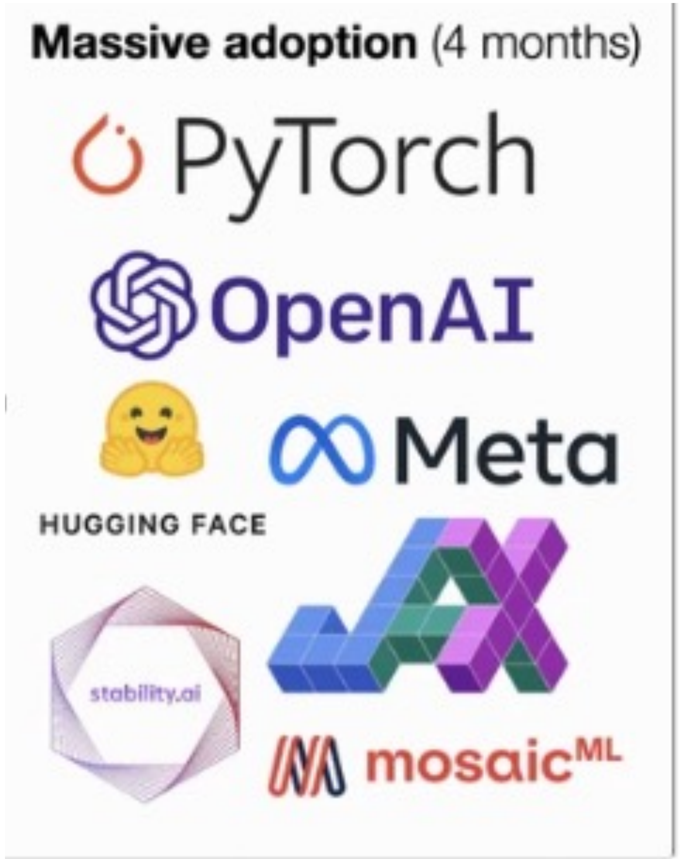


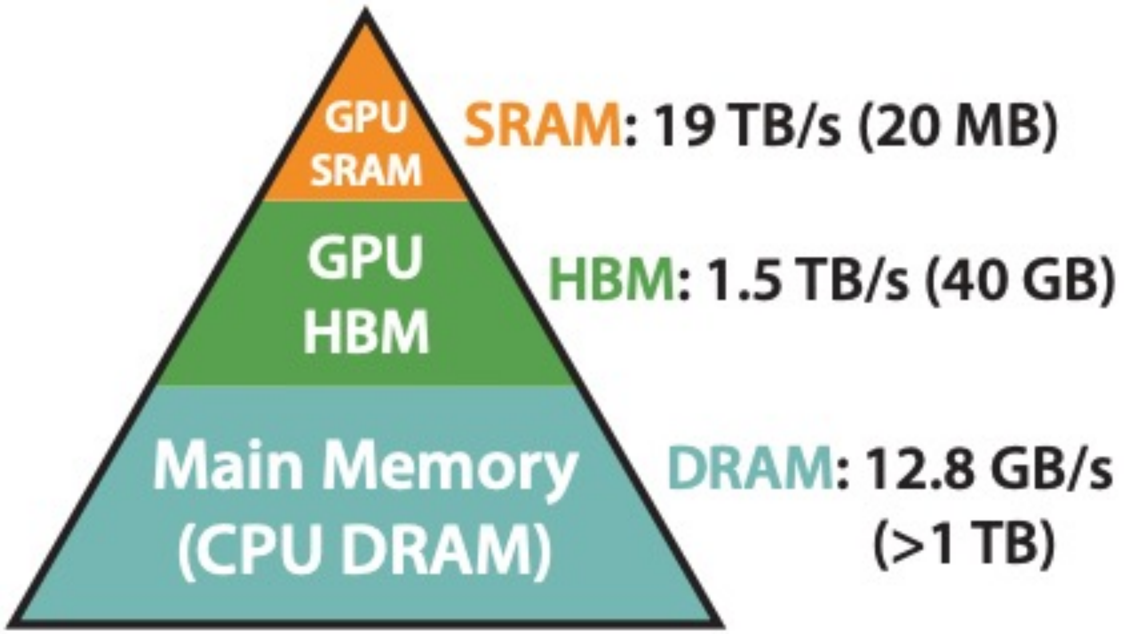
Figure from https://awaisrauf.github.io/deepCuriosity/Attending-NeurIPS2023

Figure from https://neurips.cc/virtual/2022/poster/54008

GPU Memory

Memory is arranged hierarchicaly

- GPU SRAM is small, and supports the fastest access
- GPU HBM is larger but with much slower access
- CPU DRAM is huge, but the slowest of all



Memory Hierarchy with Bandwidth & Memory Size

Figure from https://arxiv.org/pdf/2205.14135

GPU Memory and Transformers

Transformer training is usually memory-bound

- Matrix multiplication takes up 99% of the FLOPS
- But only takes up 61% of the runtime
- Lots of time is wasted moving data around on the GPU
- Instead of doing computation

Table 1. Proportions for operator classes in PyTorch.

Operator class	% flop	% Runtime
△ Tensor contraction	99.80	61.0
□ Stat. normalization	0.17	25.5
○ Element-wise	0.03	13.5

Figure from https://arxiv.org/pdf/2007.00072

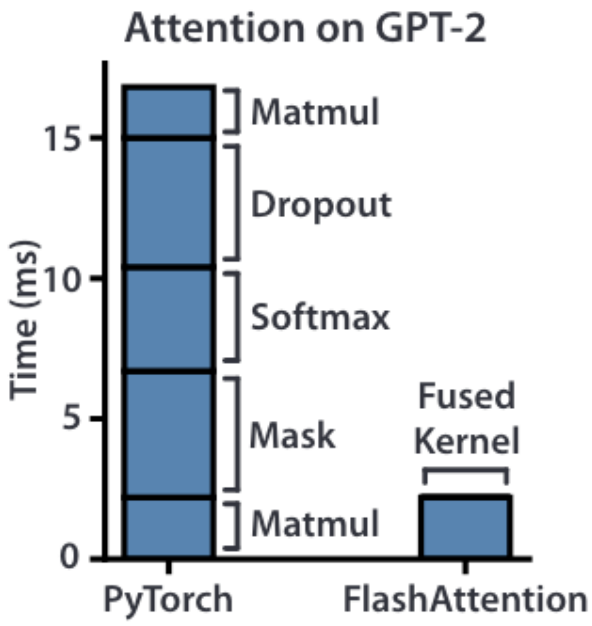


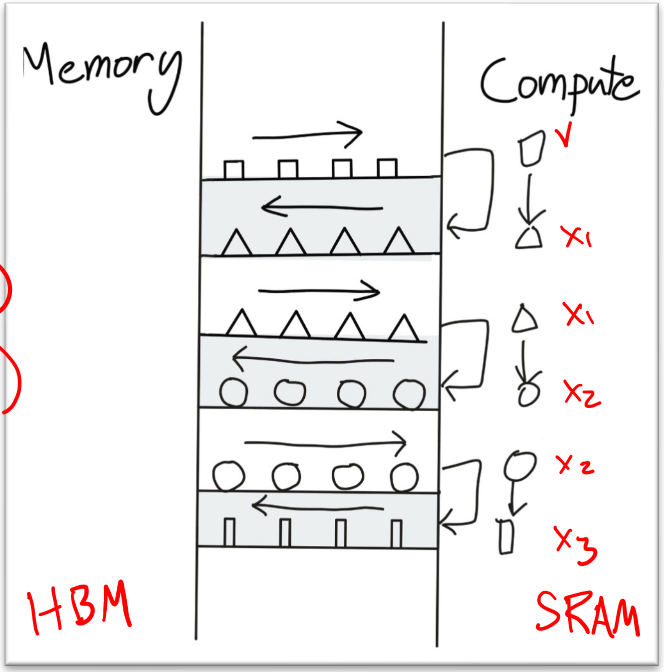
Figure from https://arxiv.org/pdf/2205.14135

Operator Fusion

Version A: Usually, we compute a neural network one layer one at a time by moving the layer input to GPU SRAM (fast/small), doing some computation, then returning the output to GPU HBM (slow/large)

Version B: Operator fusion instead moves the original input to GPU SRAM (fast/small), does a whole sequence of layer computations without ever touching HBM, and then returns the final layer output to GPU HBM (slow/large)

$$x_1 = f_1(v)$$
$$x_2 = f_2(x_1)$$
$$x_3 = f_3(x_2)$$
$$x_4 = f_4(x_3)$$



$$x_4 = f_4(f_3(f_2(f_1(v))))$$

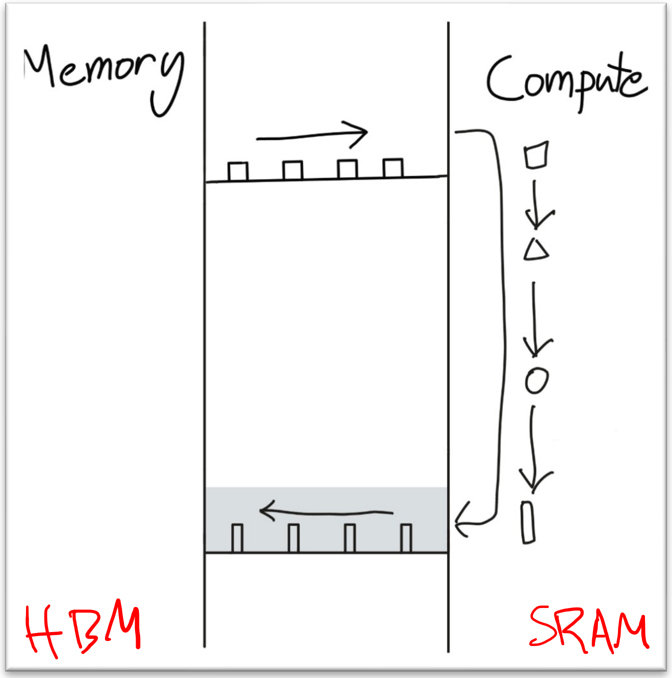


Figure from https://horace.io/brrr_intro.html

Operator Fusion

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Version B: **Operator fusion** instead moves the original input to GPU SRAM (fast/small), does a whole sequence of layer computations without ever touching HBM, and then returns the final layer output to GPU HBM (slow/large)

Version A is exactly how standard attention is implemented

$$\mathbf{S} = \mathbf{Q}\mathbf{K}^\top \in \mathbb{R}^{N\times N}, \quad \mathbf{P} = \mathrm{softmax}(\mathbf{S}) \in \mathbb{R}^{N\times N}, \quad \mathbf{O} = \mathbf{P}\mathbf{V} \in \mathbb{R}^{N\times d},$$

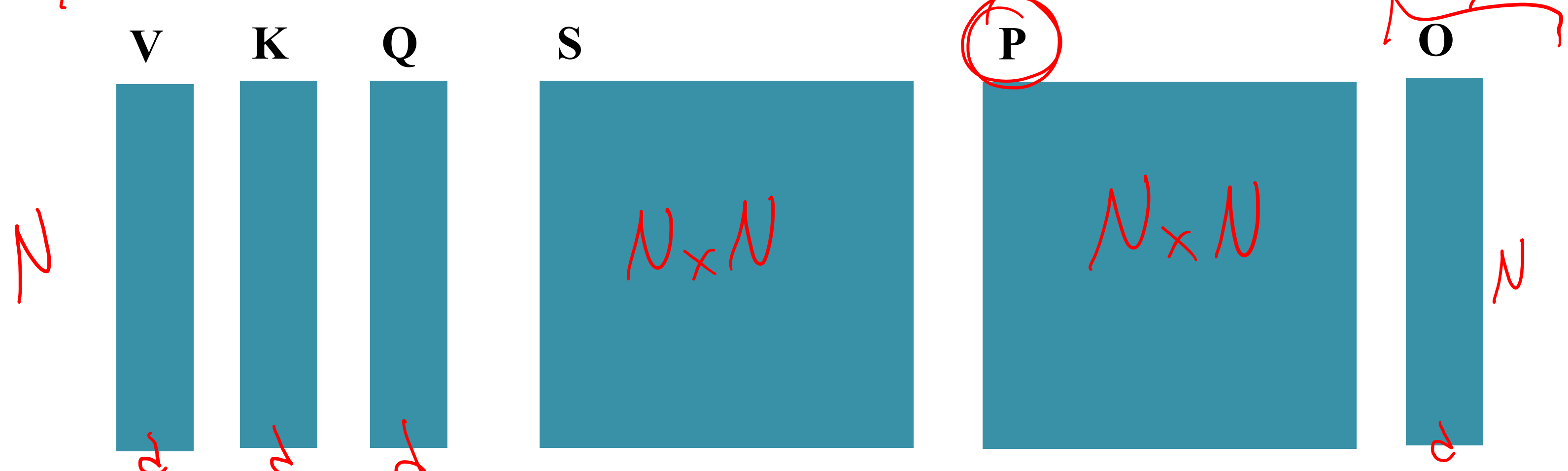
Algorithm 0 Standard Attention Implementation

Require: Matrices $\mathbf{Q}, \mathbf{K}, \mathbf{V} \in \mathbb{R}^{N\times d}$ in HBM.

1: Load $\mathbf{Q}, \mathbf{K}$ by blocks from HBM, compute $\mathbf{S} = \mathbf{Q}\mathbf{K}^\top$, write $\mathbf{S}$ to HBM.
2: Read $\mathbf{S}$ from HBM, compute $\mathbf{P} = \mathrm{softmax}(\mathbf{S})$, write $\mathbf{P}$ to HBM.
3: Load $\mathbf{P}$ and $\mathbf{V}$ by blocks from HBM, compute $\mathbf{O} = \mathbf{P}\mathbf{V}$, write $\mathbf{O}$ to HBM.
4: Return $\mathbf{O}$.

Figure from https://arxiv.org/pdf/2205.14135

Standard Attention



Version A is exactly how standard attention is implemented

$$\mathbf{S} = \mathbf{Q}\mathbf{K}^\top \in \mathbb{R}^{N \times N}, \quad \mathbf{P} = \mathrm{softmax}(\mathbf{S}) \in \mathbb{R}^{N \times N}, \quad \mathbf{O} = \mathbf{P}\mathbf{V} \in \mathbb{R}^{N \times d},$$

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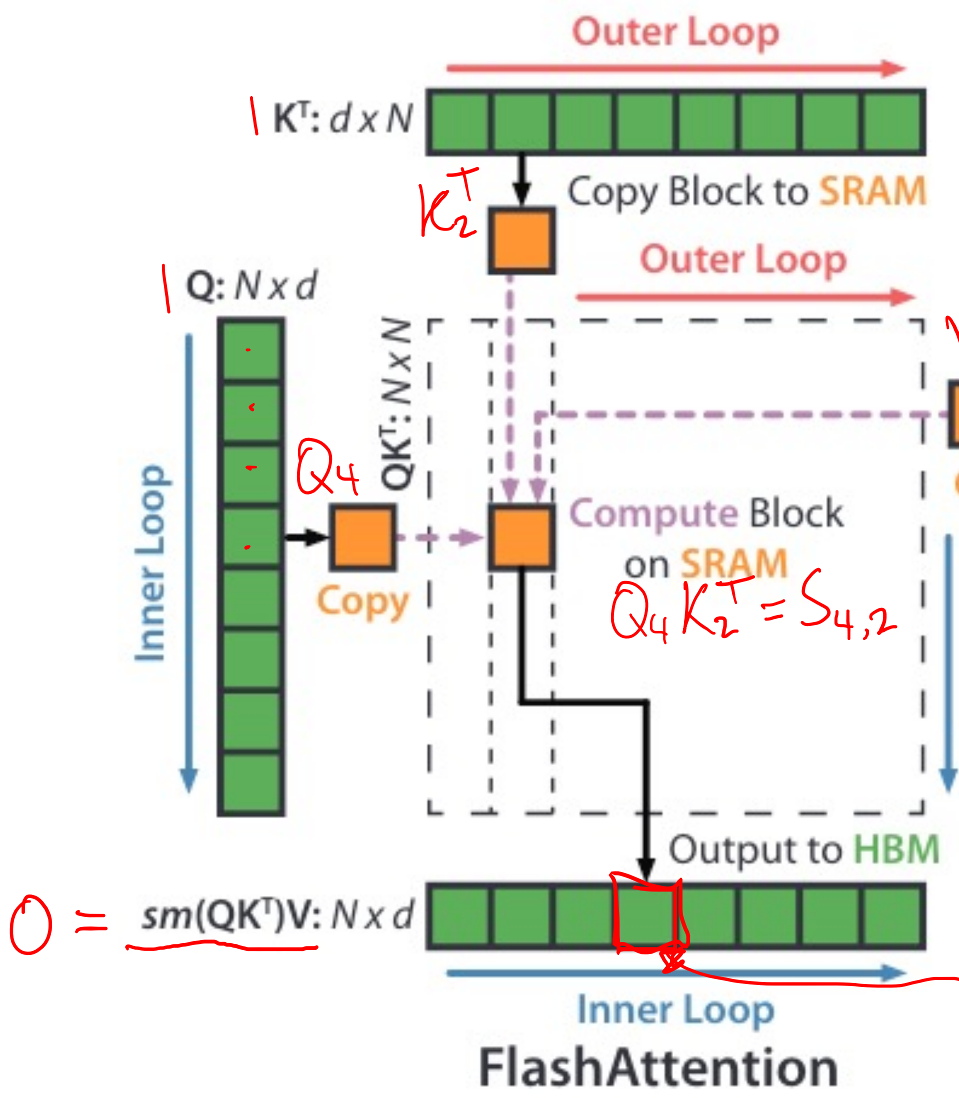
4: Return $\mathbf{O}$.

Figure from https://arxiv.org/pdf/2205.14135

FlashAttention

- Two key ideas are combined to obtain FlashAttention
- Both are well-established ideas, so the interesting part is how they are put together for attention
  1. **Tiling**: compute the attention weights block by block so that we don’t have to load everything into SRAM at once
  2. **Recomputation**: don’t ever store the full attention matrix, but just recompute the parts of it you need during the backward pass

FlashAttention: Tiling



$S = QK^T$ is divided into B^2 blocks

$Q_4 K_2^T = S_{4,2}$

$O = sm(QK^T)V$

$O_i = P_{ij} V_j$

$O_4 \mathrel{+}= \text{cheating\_softmax}(Q_4 K_2^T) V_2$

FlashAttention

Algorithm 1 FLASHATTENTION

Require: Matrices $\mathbf{Q}, \mathbf{K}, \mathbf{V} \in \mathbb{R}^{N \times d}$ in HBM, on-chip SRAM of size M.

Set block sizes $B_c = \left\lceil \frac{M}{4d} \right\rceil, B_r = \min\left(\left\lceil \frac{M}{4d} \right\rceil, d\right)$.

Initialize $\mathbf{O} = (0)_{N \times d} \in \mathbb{R}^{N \times d}, \ell = (0)_N \in \mathbb{R}^N, m = (-\infty)_N \in \mathbb{R}^N$ in HBM.

Divide $\mathbf{Q}$ into $T_r = \left\lceil \frac{N}{B_r} \right\rceil$ blocks $\mathbf{Q}_1, \dots, \mathbf{Q}_{T_r}$ of size $B_r \times d$ each, and divide $\mathbf{K}, \mathbf{V}$ in to $T_c = \left\lceil \frac{N}{B_c} \right\rceil$ blocks $\mathbf{K}_1, \dots, \mathbf{K}_{T_c}$ and $\mathbf{V}_1, \dots, \mathbf{V}_{T_c}$, of size $B_c \times d$ each.

Divide $\mathbf{O}$ into T_r blocks $\mathbf{O}_i, \dots, \mathbf{O}_{T_r}$ of size $B_r \times d$ each, divide ℓ into T_r blocks $\ell_i, \dots, \ell_{T_r}$ of size B_r each, divide m into T_r blocks $m_1, \dots, m_{T_r}$ of size B_r each.

for $1 \le j \le T_c$ **do**

  Load $\mathbf{K}_j, \mathbf{V}_j$ from HBM to on-chip SRAM.

  for $1 \le i \le T_r$ **do**

    Load $\mathbf{Q}_i, \mathbf{O}_i, \ell_i, m_i$ from HBM to on-chip SRAM.

    On chip, compute $\mathbf{S}_{ij} = \mathbf{Q}_i \mathbf{K}_j^T \in \mathbb{R}^{B_r \times B_c}$.

    On chip, compute $\tilde{m}_{ij} = \mathrm{rowmax}(\mathbf{S}_{ij}) \in \mathbb{R}^{B_r}$, $\tilde{\mathbf{P}}_{ij} = \exp(\mathbf{S}_{ij} - \tilde{m}_{ij}) \in \mathbb{R}^{B_r \times B_c}$ (pointwise), $\tilde{\ell}_{ij} = \mathrm{rowsum}(\tilde{\mathbf{P}}_{ij}) \in \mathbb{R}^{B_r}$.

    On chip, compute $m_i^{\mathrm{new}} = \max(m_i, \tilde{m}_{ij}) \in \mathbb{R}^{B_r}$, $\ell_i^{\mathrm{new}} = e^{m_i - m_i^{\mathrm{new}}} \ell_i + e^{\tilde{m}_{ij} - m_i^{\mathrm{new}}} \tilde{\ell}_{ij} \in \mathbb{R}^{B_r}$.

    Write $\mathbf{O}_i \leftarrow \mathrm{diag}(\ell_i^{\mathrm{new}})^{-1}(\mathrm{diag}(\ell_i) e^{m_i - m_i^{\mathrm{new}}} \mathbf{O}_i + e^{\tilde{m}_{ij} - m_i^{\mathrm{new}}} \tilde{\mathbf{P}}_{ij} \mathbf{V}_j)$ to HBM.

    Write $\ell_i \leftarrow \ell_i^{\mathrm{new}}$, $m_i \leftarrow m_i^{\mathrm{new}}$ to HBM.

  end for

end for

Return $\mathbf{O}$.

FlashAttention: Tiling

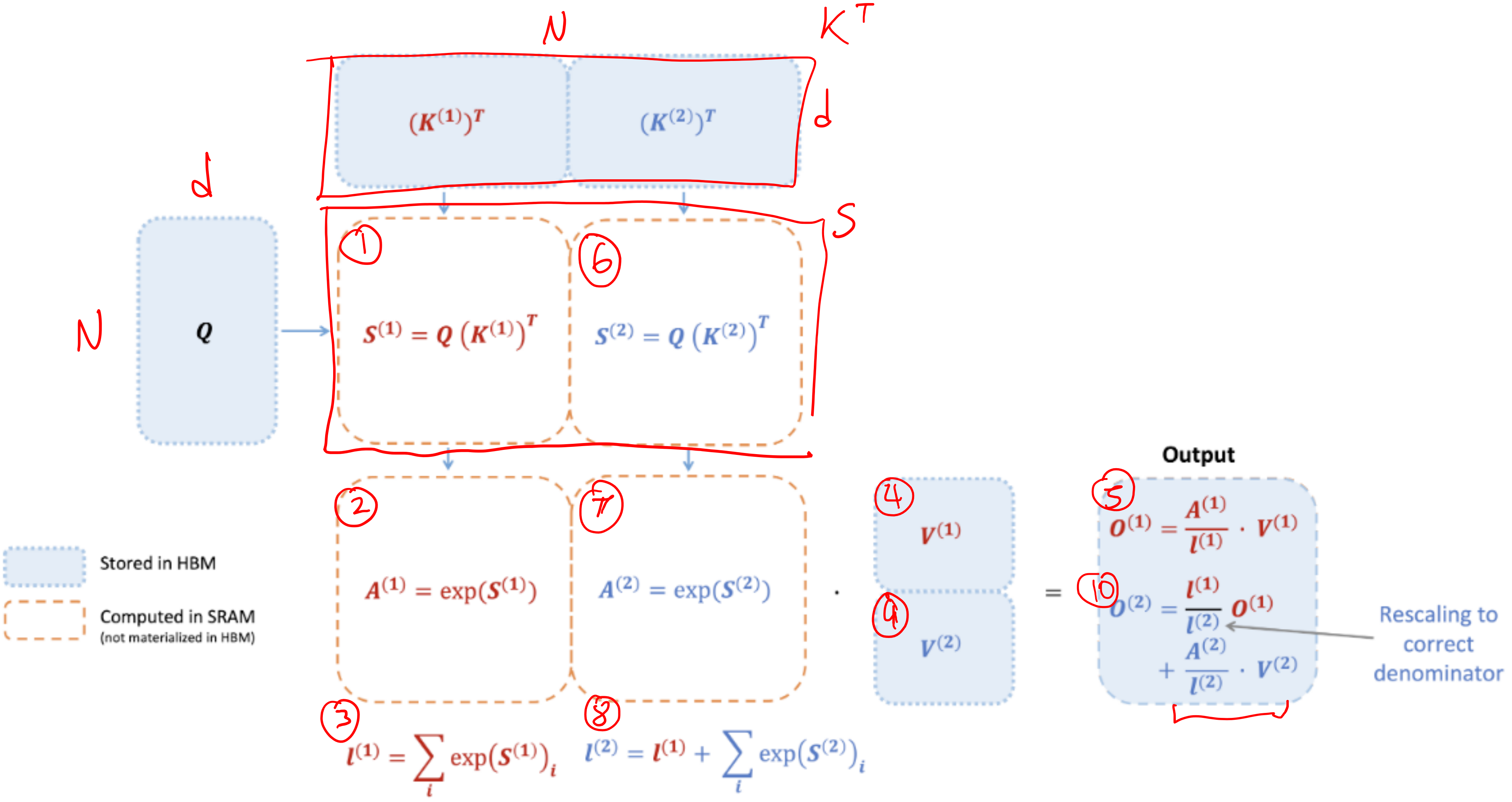


Figure from http://arxiv.org/abs/2307.08691

FlashAttention: Tiling

One of the key challenges is how to compute the softmax since it is inherently going to require working with multiple blocks

$x = [-2, 3, 1]$ $m(x) = 3$ $f(x) = [\exp(-5), \exp(0), \exp(-2)]$ $\ell(x) = \exp(-5) + \exp(0) + \exp(-2)$

For numerical stability, the softmax of vector $x \in \mathbb{R}^B$ is computed as:

$$m(x) := \max_i \; x_i, \quad f(x) := \begin{bmatrix} e^{x_1 - m(x)} & \dots & e^{x_B - m(x)} \end{bmatrix}, \quad \ell(x) := \sum_i f(x)_i, \quad \mathrm{softmax}(x) := \frac{f(x)}{\ell(x)}.$$

Online Softmax

For vectors $x^{(1)}, x^{(2)} \in \mathbb{R}^B$, we can decompose the softmax of the concatenated $x = \begin{bmatrix} x^{(1)} & x^{(2)} \end{bmatrix} \in \mathbb{R}^{2B}$ as:

bigger smaller 1 $m(x^{(2)}) - m(x^{(1)})$

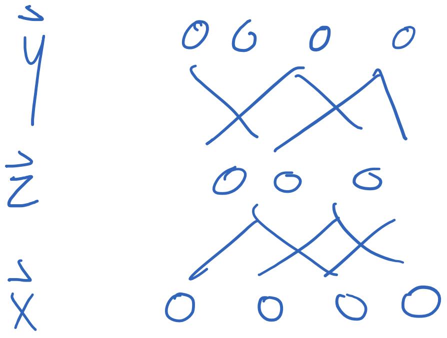
$$m(x) = m(\begin{bmatrix} x^{(1)} & x^{(2)} \end{bmatrix}) = \max(m(x^{(1)}), m(x^{(2)})), \quad f(x) = \begin{bmatrix} e^{m(x^{(1)}) - m(x)} f(x^{(1)}) & e^{m(x^{(2)}) - m(x)} f(x^{(2)}) \end{bmatrix},$$

$$\ell(x) = \ell(\begin{bmatrix} x^{(1)} & x^{(2)} \end{bmatrix}) = e^{m(x^{(1)}) - m(x)} \ell(x^{(1)}) + e^{m(x^{(2)}) - m(x)} \ell(x^{(2)}), \quad \mathrm{softmax}(x) = \frac{f(x)}{\ell(x)}.$$

Therefore if we keep track of some extra statistics $(m(x), \ell(x))$, we can compute softmax one block at a time.[2]

Figure from https://arxiv.org/pdf/2205.14135

Reconstruction for a Feed-Forward MLP



Forward

$z = \sigma(W_1 x + b_1)$

$y = \text{softmax}(W_2 z + b_2)$

Backward

$dJ/dy = \ldots$

$dJ/dz = dJ/dy \cdot dy/dz$

$dJ/dx = dJ/dz \cdot dz/dx$

$z = z(1-z) W_1$

Reconstruction

Forward

$z = \sigma(W_1 x + b_1)$

$y = \text{softmax}(W_2 z + b_2)$

delete z

Backward

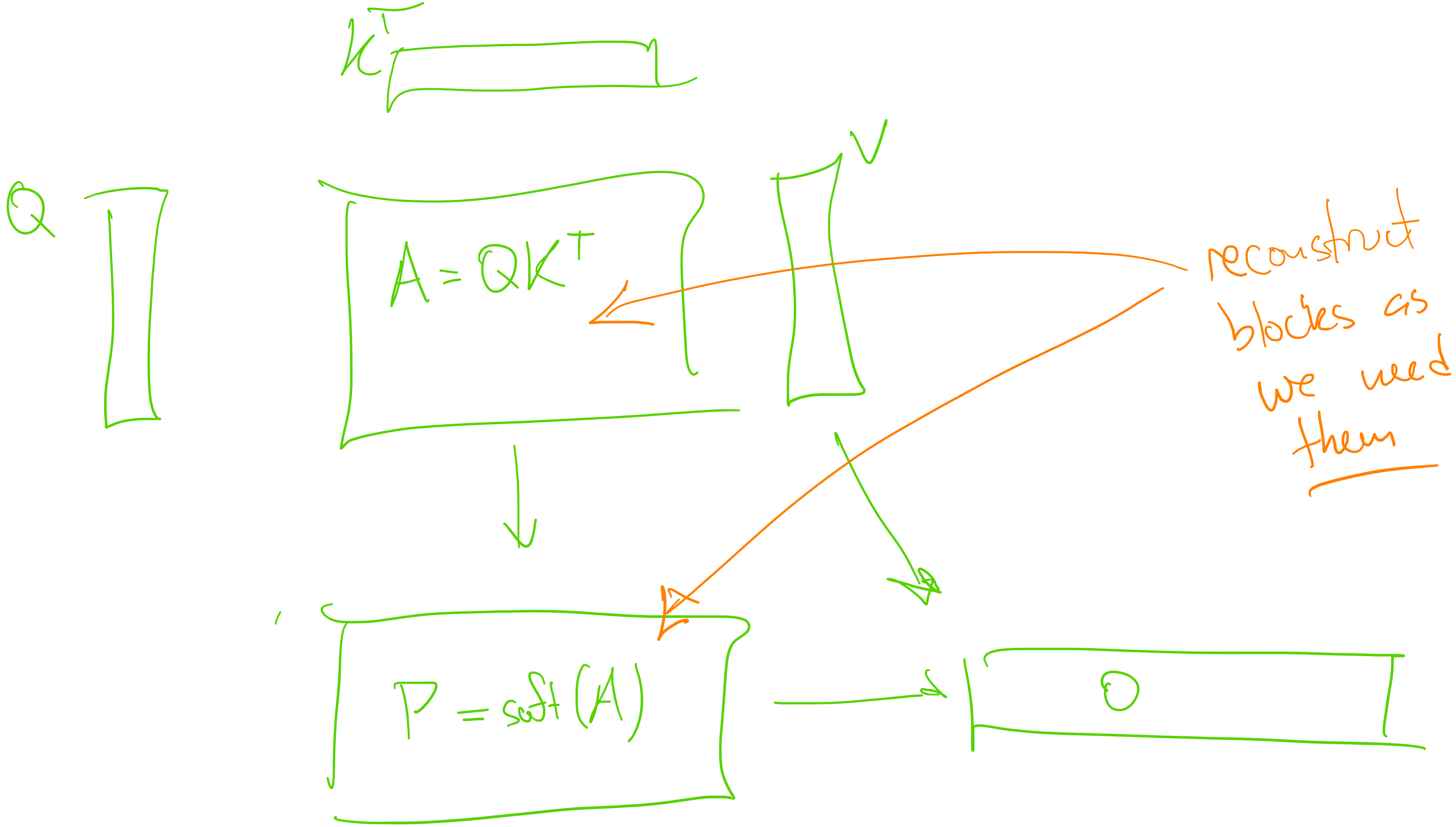
$dJ/dy = \square$

$dJ/dz = \square$

$z = \sigma(W_1 x + b_1)$

$dJ/dx = \square$

FlashAttention: Reconstruction



FlashAttention

Algorithm 1 FlashAttention

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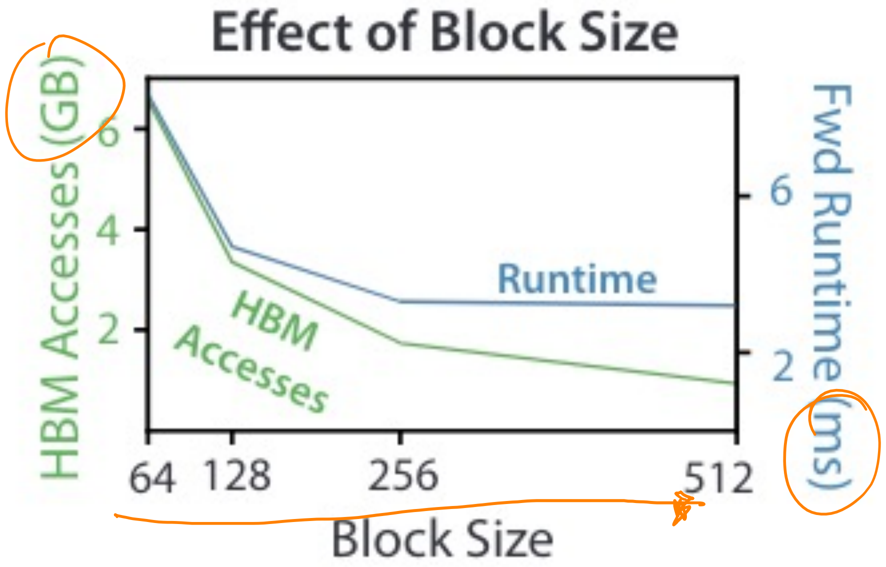
Set block sizes $B_c = \left\lceil \frac{M}{4d} \right\rceil, B_r = \min\left(\left\lceil \frac{M}{4d} \right\rceil, d\right)$.
Initialize $\mathbf{O} = (0)_{N \times d} \in \mathbb{R}^{N \times d}, \ell = (0)_N \in \mathbb{R}^N, m = (-\infty)_N \in \mathbb{R}^N$ in HBM.
Divide $\mathbf{Q}$ into $T_r = \left\lceil \frac{N}{B_r} \right\rceil$ blocks $\mathbf{Q}_1, \ldots, \mathbf{Q}_{T_r}$ of size $B_r \times d$ each, and divide $\mathbf{K}, \mathbf{V}$ in to $T_c = \left\lceil \frac{N}{B_c} \right\rceil$ blocks $\mathbf{K}_1, \ldots, \mathbf{K}_{T_c}$ and $\mathbf{V}_1, \ldots, \mathbf{V}_{T_c}$, of size $B_c \times d$ each.
Divide $\mathbf{O}$ into T_r blocks $\mathbf{O}_i, \ldots, \mathbf{O}_{T_r}$ of size $B_r \times d$ each, divide ℓ into T_r blocks $\ell_i, \ldots, \ell_{T_r}$ of size B_r each, divide m into T_r blocks $m_1, \ldots, m_{T_r}$ of size B_r each.
for $1 \le j \le T_c$ **do**
  Load $\mathbf{K}_j, \mathbf{V}_j$ from HBM to on-chip SRAM.
  for $1 \le i \le T_r$ **do**
    Load $\mathbf{Q}_i, \mathbf{O}_i, \ell_i, m_i$ from HBM to on-chip SRAM.
    On chip, compute $\mathbf{S}_{ij} = \mathbf{Q}_i \mathbf{K}_j^T \in \mathbb{R}^{B_r \times B_c}$.
    On chip, compute $\tilde{m}_{ij} = \mathrm{rowmax}(\mathbf{S}_{ij}) \in \mathbb{R}^{B_r}$, $\tilde{\mathbf{P}}_{ij} = \exp(\mathbf{S}_{ij} - \tilde{m}_{ij}) \in \mathbb{R}^{B_r \times B_c}$ (pointwise), $\tilde{\ell}_{ij} = \mathrm{rowsum}(\tilde{\mathbf{P}}_{ij}) \in \mathbb{R}^{B_r}$.
    On chip, compute $m_i^{\mathrm{new}} = \max(m_i, \tilde{m}_{ij}) \in \mathbb{R}^{B_r}$, $\ell_i^{\mathrm{new}} = e^{m_i - m_i^{\mathrm{new}}} \ell_i + e^{\tilde{m}_{ij} - m_i^{\mathrm{new}}} \tilde{\ell}_{ij} \in \mathbb{R}^{B_r}$.
    Write $\mathbf{O}_i \leftarrow \mathrm{diag}(\ell_i^{\mathrm{new}})^{-1}(\mathrm{diag}(\ell_i) e^{m_i - m_i^{\mathrm{new}}} \mathbf{O}_i + e^{\tilde{m}_{ij} - m_i^{\mathrm{new}}} \tilde{\mathbf{P}}_{ij} \mathbf{V}_j)$ to HBM.
    Write $\ell_i \leftarrow \ell_i^{\mathrm{new}}$, $m_i \leftarrow m_i^{\mathrm{new}}$ to HBM.
  end for
end for
Return $\mathbf{O}$.

FlashAttention: Results

- The algorithm is performing exact attention, so we see no reduction in perplexity or quality of the model
- The key metric is runtime

Attention	Standard	FlashAttention
GFLOPs	66.6	75.2
HBM R/W (GB)	40.3	4.4
Runtime (ms)	41.7	7.3



FlashAttention: Results

- The algorithm is performing exact attention, so we see no reduction in perplexity or quality of the model
- The key metric is runtime

Model implementations	OpenWebText (ppl)	Training time (speedup)
GPT-2 small - Huggingface [87]	18.2	9.5 days (1.0×)
GPT-2 small - Megatron-LM [77]	18.2	4.7 days (2.0×)
GPT-2 small - FlashAttention	18.2	**2.7 days (3.5×)**
GPT-2 medium - Huggingface [87]	14.2	21.0 days (1.0×)
GPT-2 medium - Megatron-LM [77]	14.3	11.5 days (1.8×)
GPT-2 medium - FlashAttention	14.3	**6.9 days (3.0×)**