Machine Learning Department
School of Computer Science
Carnegie Mellon University

State Space Models
+
Hybrid Models

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Lecture 22
Nov. 12, 2025

Motivation

- Transformers are slow at test time: they require a KV cache that grows linearly in size with the sequence length
- State space models (SSMs) are fast at test time: they only hold a fixed size hidden state in memory (like RNNs)
- But we'll see that SSMs can also be trained efficiently with the right tricks
- As well, they elegantly transition between different granularities of representation for the input (e.g. sound at 16KHz vs. 8KHz)

Motivation

- https://www.isattentionallyouneed.com/

Is Attention All You Need?

Current Status: Yes

Time Remaining: 631d 22h 55m 11s

Proposition:

On January 1, 2027, a Transformer-like model will continue to hold the state-of-the-art position in most benchmarked tasks in natural language processing.

Motivation

- https://www.isattentionallyouneed.com/

Is Attention All You Need?

For the Motion

Jonathan Frankle
@jefrankle
Harvard Professor
Chief Scientist Mosaic ML

Against the Motion

Sasha Rush
@srush_nlp
Cornell Professor
Research Scientist Hugging Face 🤗

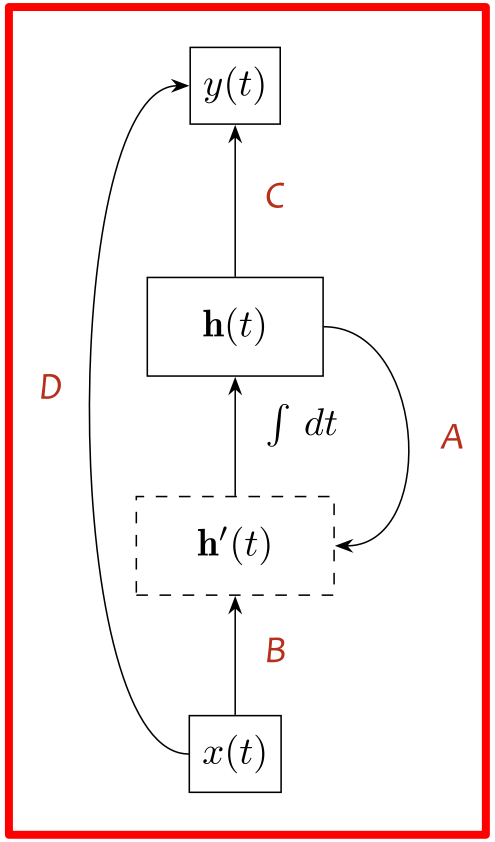
Wager

The wager is for donation of equity in Mosaic ML or Hugging Face to a charity of the winner's choice. Details to come.

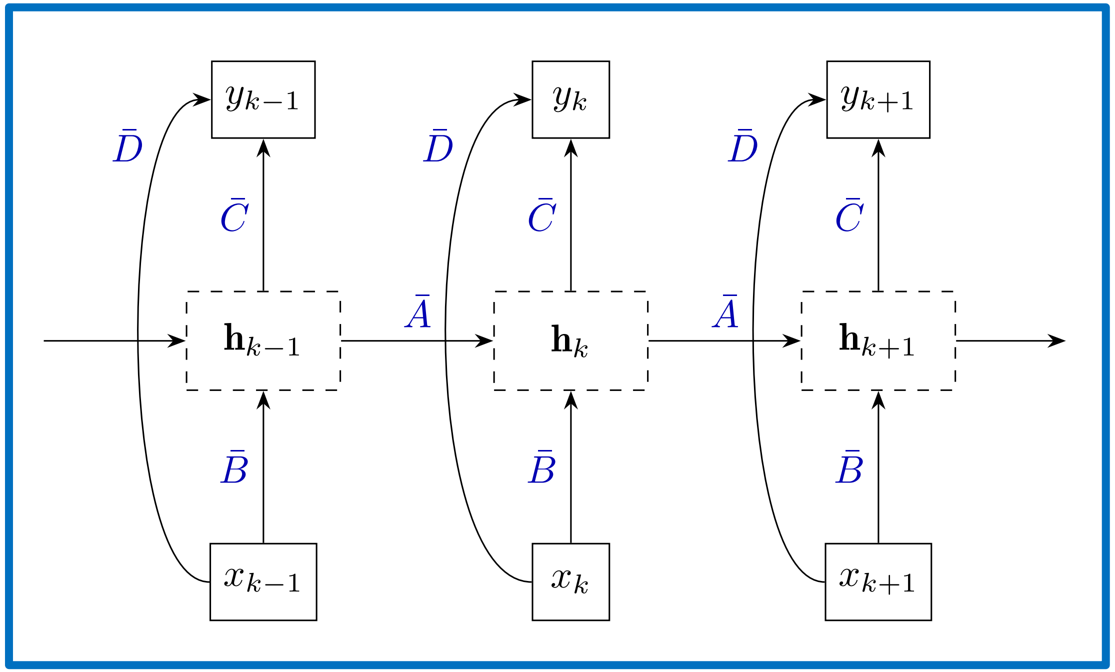
STATE SPACE MODEL (SSM)

Three Representations of a State Space Model

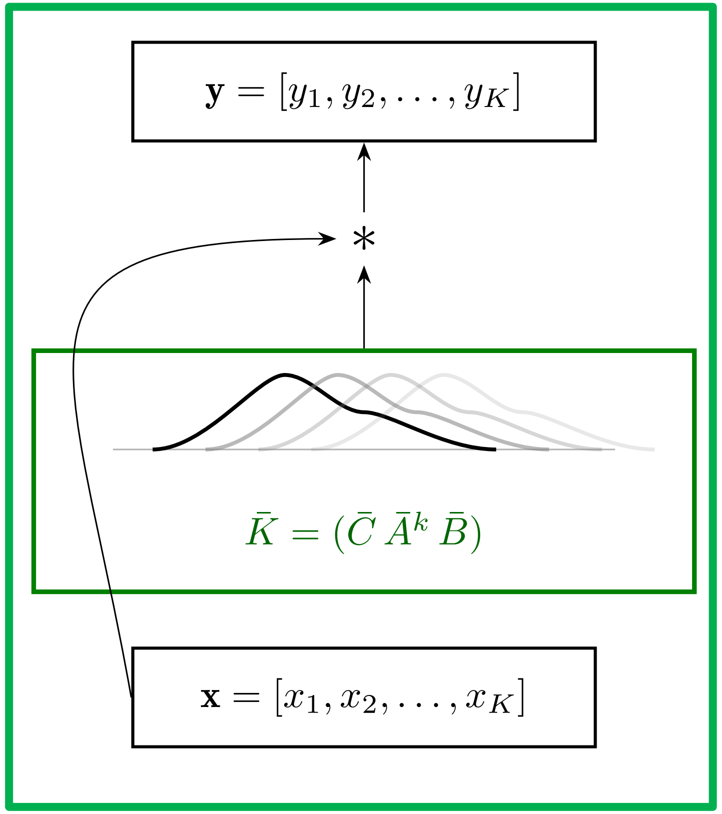
Continuous



Recurrent



Convolutional



Figures inspired by https://hazyresearch.stanford.edu/blog/2022-01-14-s4-3

SSM: 1D Continuous Representation

$x(t) \in \mathbb{R}$

$y(t) \in \mathbb{R}$

$\mathbf{h}(t) \in \mathbb{R}^N$

$A \in \mathbb{R}^{N \times N}$

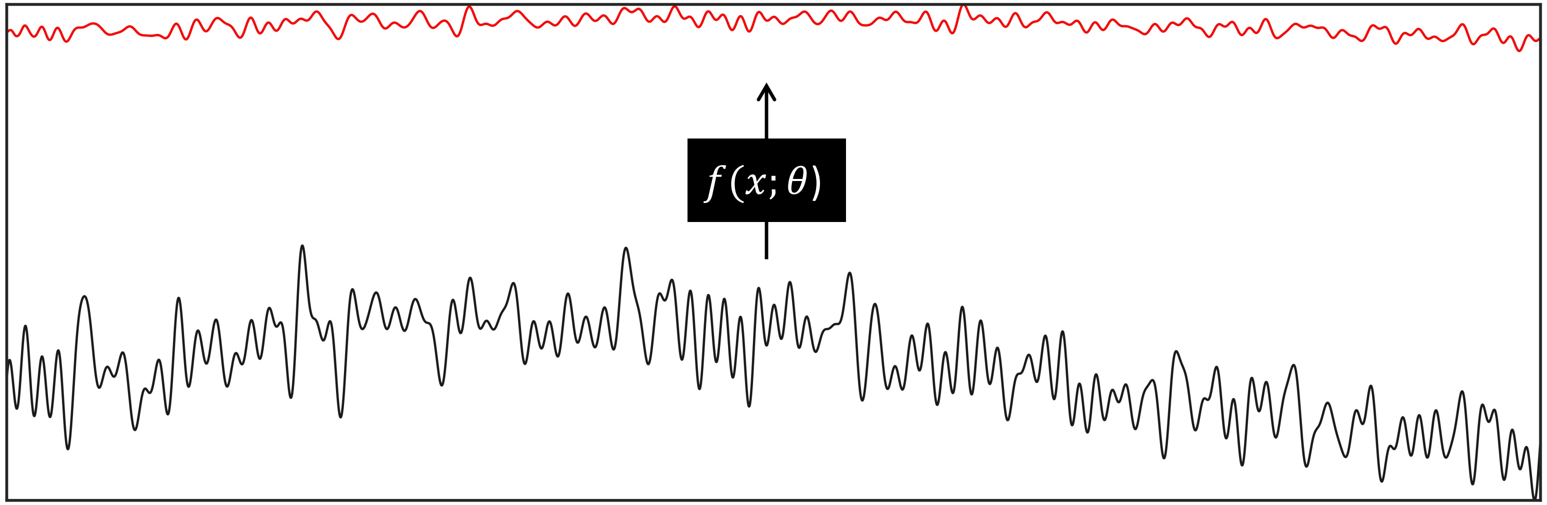
$B \in \mathbb{R}^{N \times 1}$

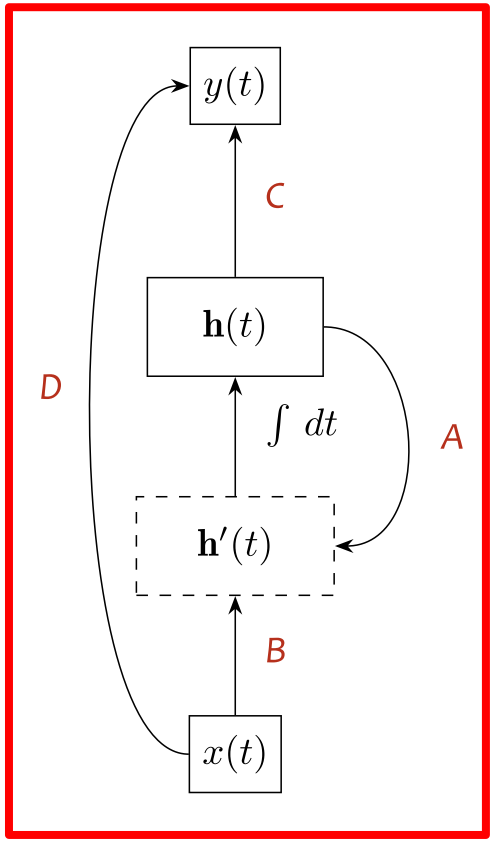
$C \in \mathbb{R}^{1 \times N}$

$D \in \mathbb{R}^{1 \times 1}$

$$\frac{\partial \mathbf{h}(t)}{\partial t} = \mathbf{h}'(t) = \mathbf{A}\mathbf{h}(t) + \mathbf{B}x(t)$$

$$y(t) = \mathbf{C}\mathbf{h}(t) + \mathbf{D}x(t)$$





SSMs map 1D **function** to 1D **function**

Figure from Albert Gu's F24 guest lecture in 10-423/623

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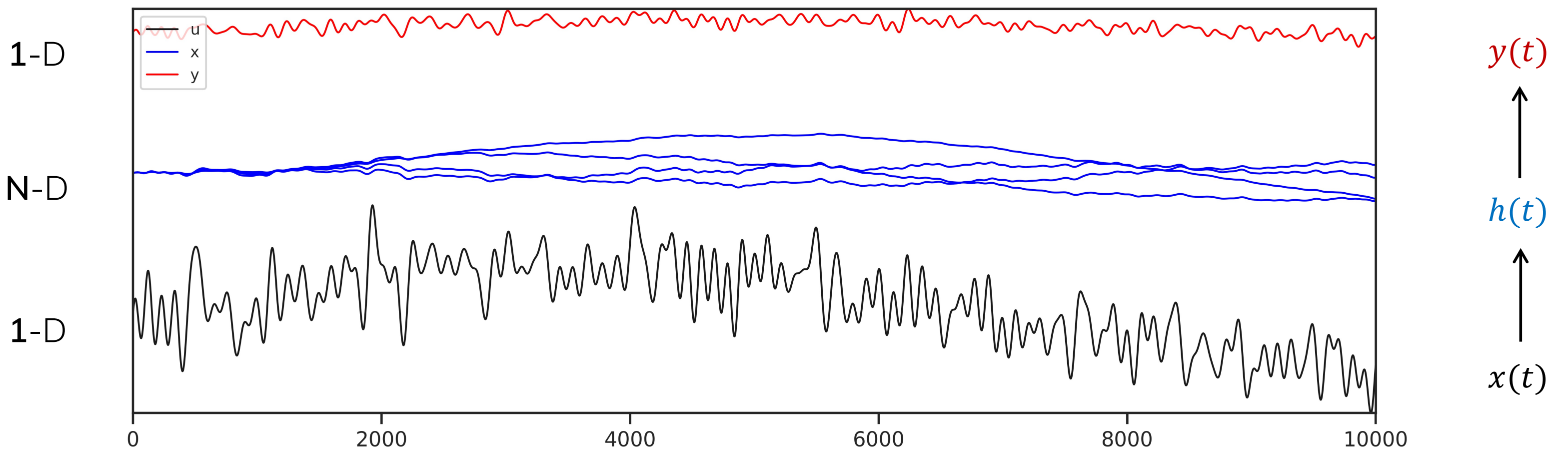


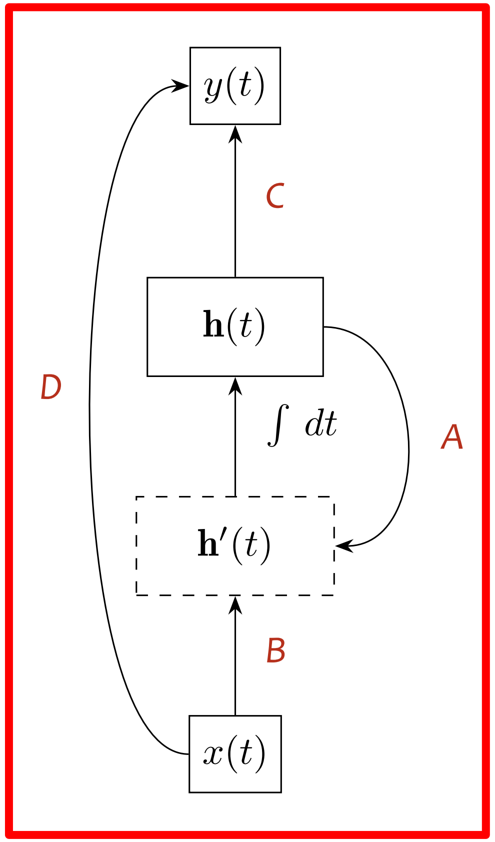


Figure from Albert Gu’s F24 guest lecture in 10-423/623

SSM: 1D Discrete Recurrent Representation

Continuous Representation

- Uses parameters we will actually work with in the end
- Seamlessly represents any continuous 1D → 1D function
- Impractical for real data

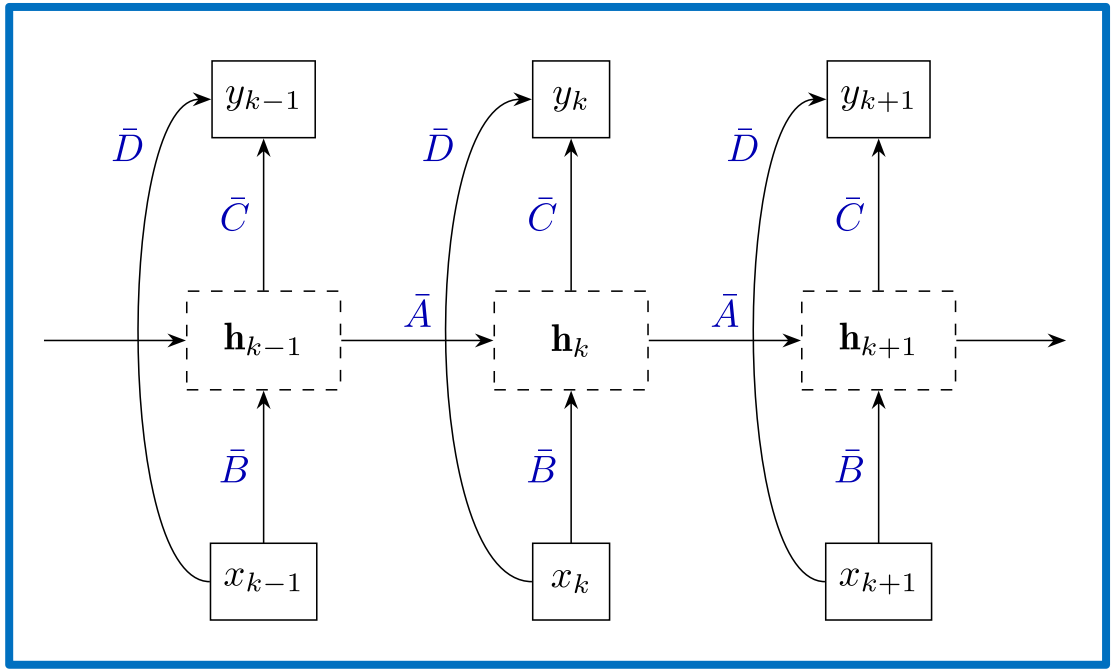
Discrete Recurrent Representation

- A discrete approximation using different parameters which are **functions** of the original parameters A,B,C,D
- Allows us to work with real data

$$
\begin{aligned}
\mathbf{h}'(t) &= \mathbf{A}\mathbf{h}(t) + \mathbf{B}x(t) \\
y(t) &= \mathbf{C}\mathbf{h}(t) + \mathbf{D}x(t)
\end{aligned}
\quad \Longrightarrow \quad
\begin{aligned}
\mathbf{h}_{k+1} &= \bar{\mathbf{A}}\mathbf{h}_k + \bar{\mathbf{B}}x_k \\
y_k &= \bar{\mathbf{C}}\mathbf{h}_k + \bar{\mathbf{D}}x_k
\end{aligned}
$$

SSM: 1D Discrete Recurrent Representation

Question: How can we depict this recurrent computation?



Discrete Recurrent Representation

- A discrete approximation using different parameters which are **functions** of the original parameters A,B,C,D
- Allows us to work with real data

$$\mathbf{h}_{k+1} = \bar{\mathbf{A}}\mathbf{h}_k + \bar{\mathbf{B}}x_k$$
$$y_k = \bar{\mathbf{C}}\mathbf{h}_k + \bar{\mathbf{D}}x_k$$

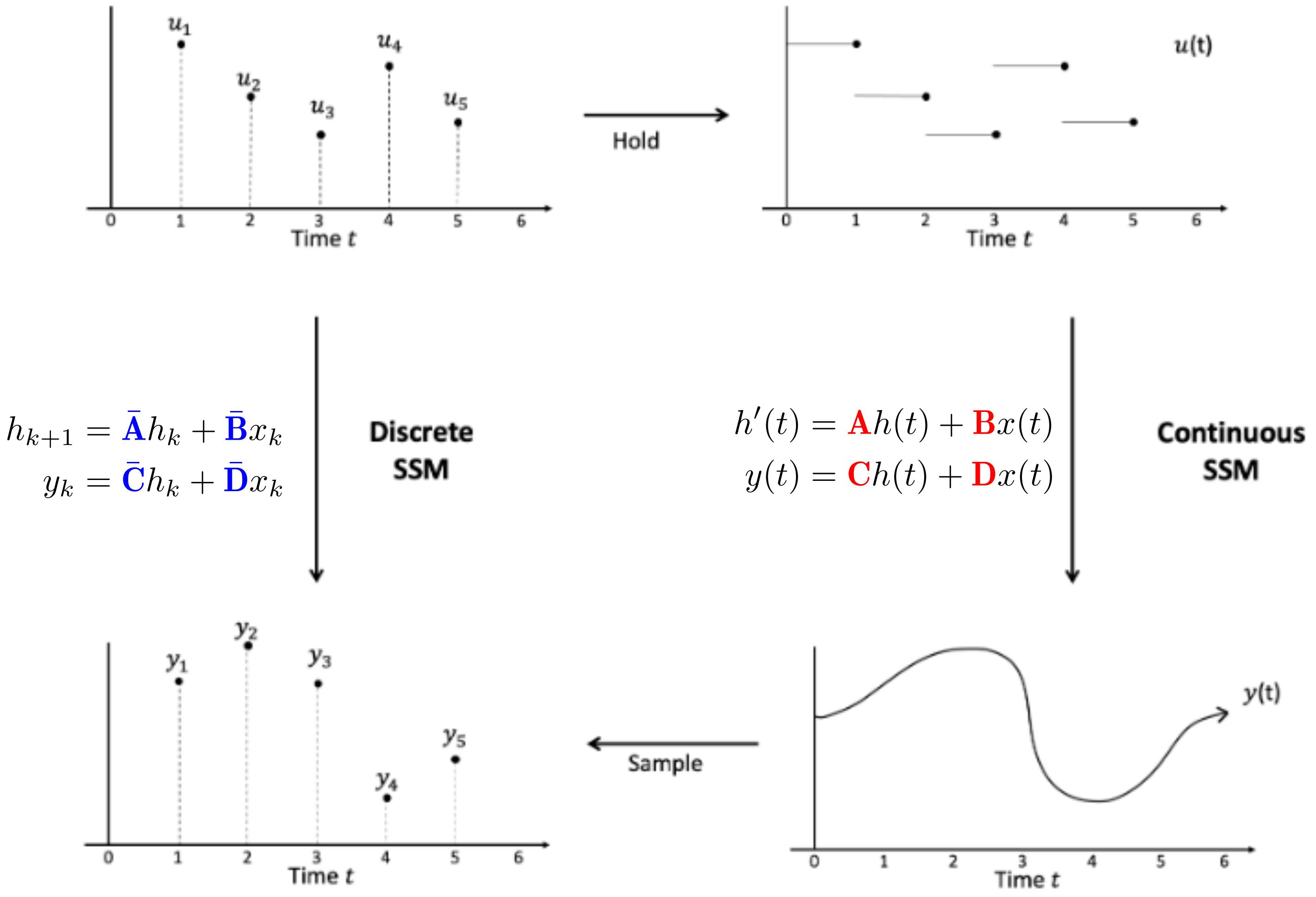
How to discretize a continuous SSM?

S4 uses a bilinear transformation to discretize the continuous SSM

$$\overline{A} = e^{\Delta A}$$
$$\overline{B} = A^{-1}(e^{\Delta A} - I)B$$
$$\overline{C} = C$$
$$\overline{D} = D$$

The bilinear transformation uses a first order Pade approximation:

$$e^x \approx \frac{1+x/2}{1-x/2}$$



The discrete-time SSM (left), a sequence-to-sequence map, is exactly equivalent to applying the continuous-time SSM (right), a function-to-function map, on the held signal.

Figure from https://hazyresearch.stanford.edu/blog/2022-01-14-s4-3

How to discretize a continuous SSM?

S4 uses a bilinear transformation
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$$\overline{\boldsymbol{B}} = \boldsymbol{A}^{-1}(e^{\Delta \boldsymbol{A}} - \boldsymbol{I})\boldsymbol{B}$$
$$\overline{\boldsymbol{C}} = \boldsymbol{C}$$
$$\overline{\boldsymbol{D}} = \boldsymbol{D}$$

$$\overline{\boldsymbol{A}} = (\boldsymbol{I} - \frac{\Delta}{2} \cdot \boldsymbol{A})^{-1}(\boldsymbol{I} + \frac{\Delta}{2} \cdot \boldsymbol{A})$$
$$\overline{\boldsymbol{B}} = (\boldsymbol{I} - \frac{\Delta}{2} \cdot \boldsymbol{A})^{-1}\Delta \boldsymbol{B}$$
$$\overline{\boldsymbol{C}} = \boldsymbol{C}$$
$$\overline{\boldsymbol{D}} = \boldsymbol{D}$$

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$$e^x \approx \frac{1+x/2}{1-x/2}$$

Figure from https://hazyresearch.stanford.edu/blog/2022-01-14-s4-3

SSM: 1D Convolutional Representation

We unroll the recurrent computation as:
Assume a zero initial state: $h_{-1} = 0$.

$$h_0 = \bar{\mathbf{B}} x_0$$
$$h_1 = \bar{\mathbf{A}}\bar{\mathbf{B}} x_0 + \bar{\mathbf{B}} x_1$$
$$h_2 = \bar{\mathbf{A}}^2\bar{\mathbf{B}} x_0 + \bar{\mathbf{A}}\bar{\mathbf{B}} x_1 + \bar{\mathbf{B}} x_2$$
$$\vdots$$
$$y_0 = \bar{\mathbf{C}}\bar{\mathbf{B}} x_0$$
$$y_1 = \bar{\mathbf{C}}\bar{\mathbf{A}}\bar{\mathbf{B}} x_0 + \bar{\mathbf{C}}\bar{\mathbf{B}} x_1$$
$$y_2 = \bar{\mathbf{C}}\bar{\mathbf{A}}^2\bar{\mathbf{B}} x_0 + \bar{\mathbf{C}}\bar{\mathbf{A}}\bar{\mathbf{B}} x_1 + \bar{\mathbf{C}}\bar{\mathbf{B}} x_2$$
$$\vdots$$
$$y_k = \bar{\mathbf{C}}\bar{\mathbf{A}}^k\bar{\mathbf{B}} x_0 + \bar{\mathbf{C}}\bar{\mathbf{A}}^{k-1}\bar{\mathbf{B}} x_1 + \cdots + \bar{\mathbf{C}}\bar{\mathbf{A}}^1\bar{\mathbf{B}} x_{k-1} + \bar{\mathbf{C}}\bar{\mathbf{A}}^0\bar{\mathbf{B}} x_k$$

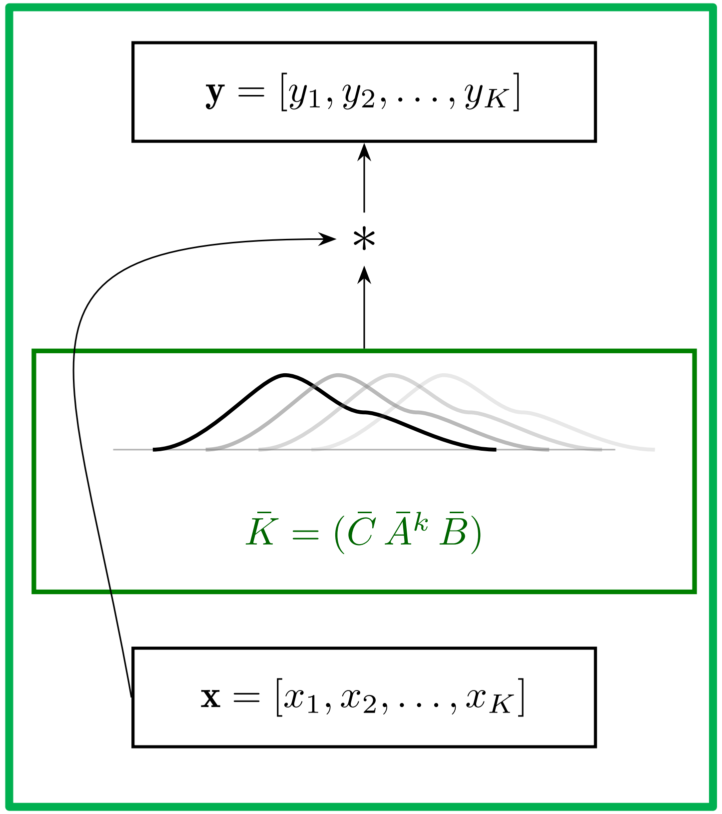
We can represent this as a *global* convolution computation:

$$\mathbf{y} = \bar{\mathbf{K}} * \mathbf{x}$$

where the SSM convolution kernel is:

$$\bar{\mathbf{K}} \in \mathbb{R}^L = \left(\bar{\mathbf{C}}\bar{\mathbf{A}}^0\bar{\mathbf{B}},\ \bar{\mathbf{C}}\bar{\mathbf{A}}^1\bar{\mathbf{B}},\ \ldots,\ \bar{\mathbf{C}}\bar{\mathbf{A}}^{L-2}\bar{\mathbf{B}}, \bar{\mathbf{C}}\bar{\mathbf{A}}^{L-1}\bar{\mathbf{B}}\right)$$

SSM: 1D Convolutional Representation



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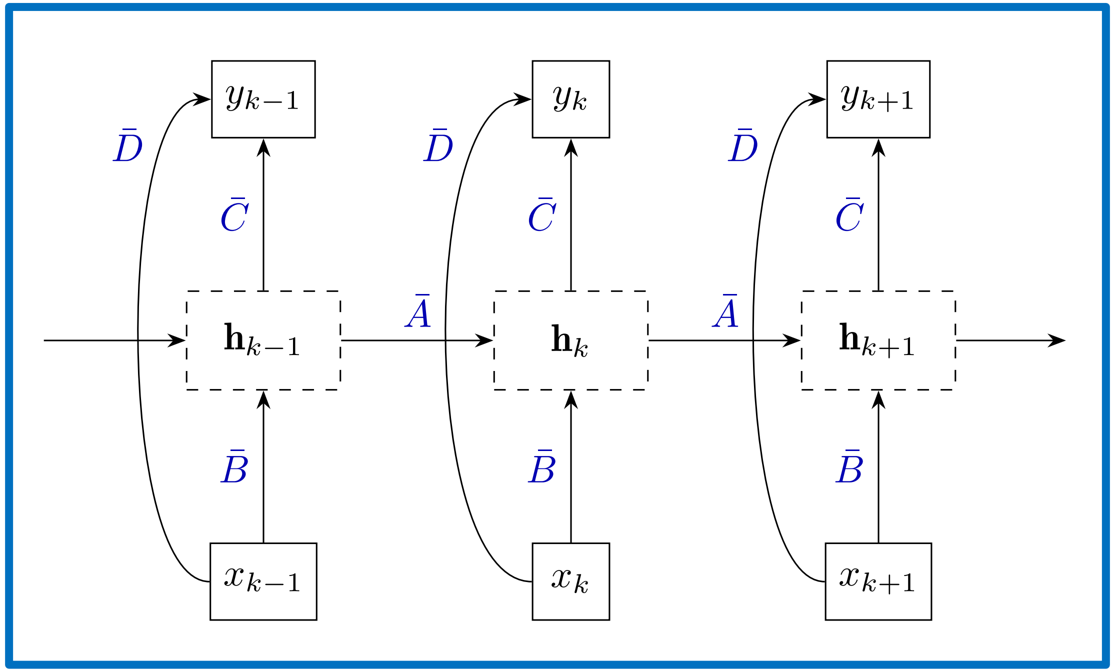
$$y_k = \bar{\mathbf{C}}\bar{\mathbf{A}}^k\bar{\mathbf{B}}x_0 + \bar{\mathbf{C}}\bar{\mathbf{A}}^{k-1}\bar{\mathbf{B}}x_1 + \cdots + \bar{\mathbf{C}}\bar{\mathbf{A}}^1\bar{\mathbf{B}}x_{k-1} + \bar{\mathbf{C}}\bar{\mathbf{A}}^0\bar{\mathbf{B}}x_k$$

THE STRUCTURED STATE SPACE SEQUENCE MODEL (S4)

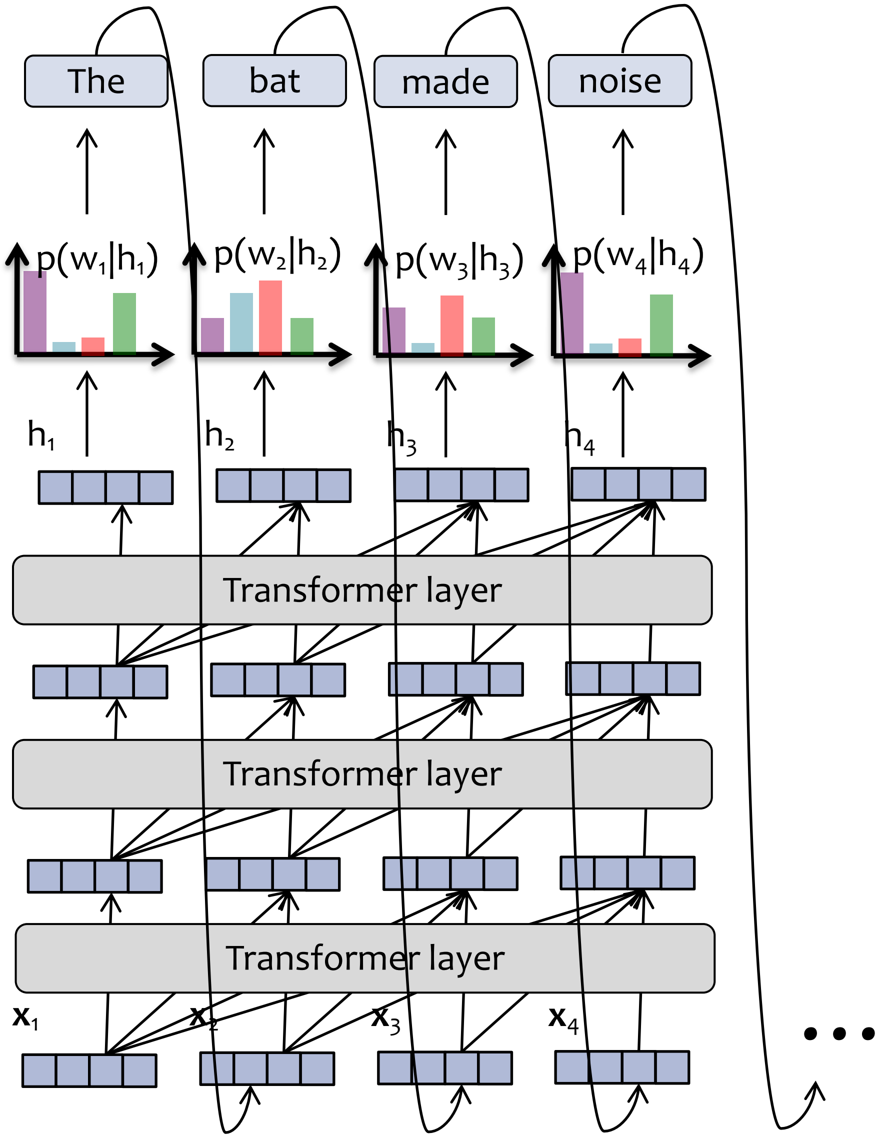
SSM as a Neural Network Layer

- We can take H copies of the 1D recurrent representation
- Let each copy have its own parameters



- This is just like multiple (indep.) heads in Attention
- And just like multiple (indep.) channels in Convolution
- So we get…

Transformer Language Model



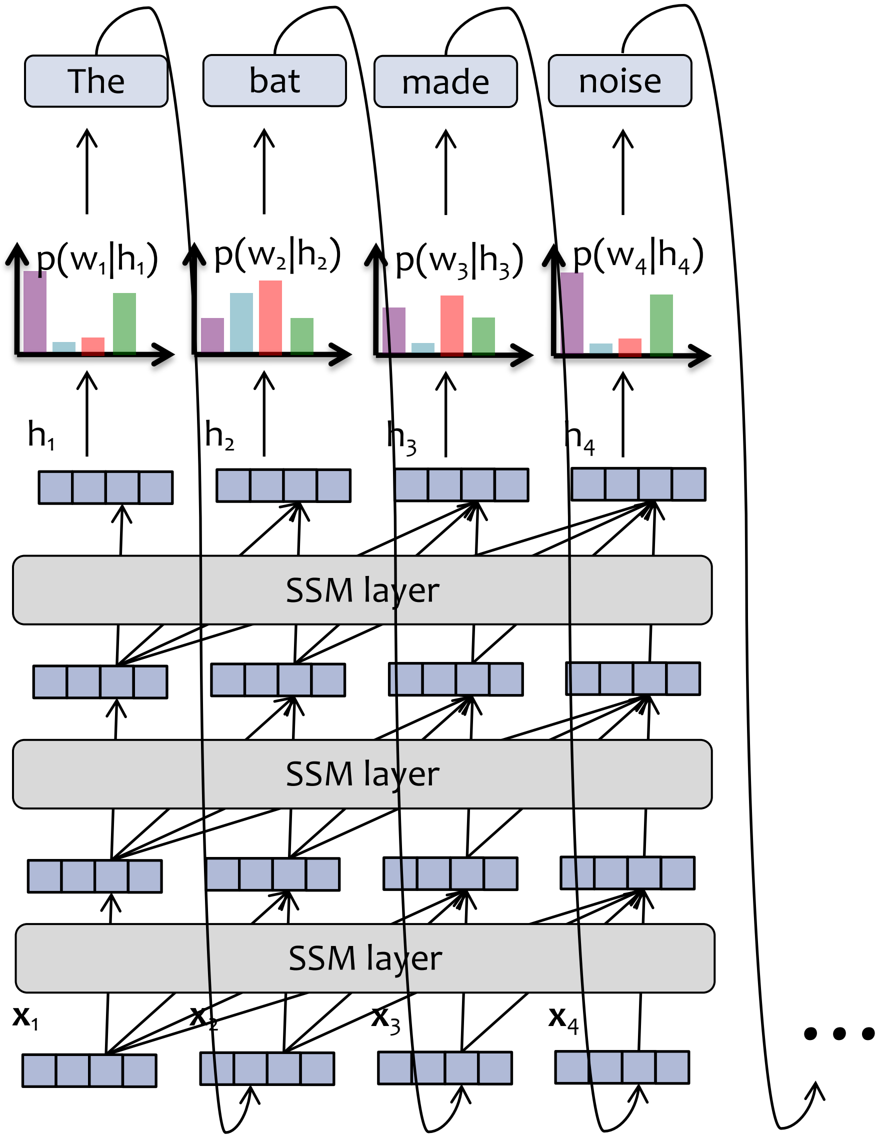
Each layer of a Transformer LM consists of several **sublayers:**

1. attention
2. feed-forward neural network
3. layer normalization
4. residual connections

Each hidden vector looks back at the hidden vectors of the **current and previous timesteps in the previous layer.**

The language model part is just like an RNN-LM.

SSM inside a Deep Language Model



Each layer of an S4 LM consists of several **sublayers as well** including an SSM, nonlinearity, etc.

Each hidden vector looks back at the hidden vectors of the **current and previous timesteps in the previous layer.**

The language model part is just like an RNN-LM or Transformer-LM

Efficiency of SSM, RNN, & Transformer

For SSMs:

1. At test time, generation does NOT need a KV-cache in our **Recurrent representation**, so we can effortlessly generate truly long sequences (unlike Transformers, but just like RNNs)
2. At train time, we can use the **Convolution representation** to do fast parallel training (just like Transformers, but unlike RNNs)

	Train	**Test**
Recurrence		
Attention		
SSM		

S4 Model

We need several additional tricks to get training to work well:

- HiPPO Matrix
  - we initialize the matrix A very carefully
- Efficient computation
  - we decompose A so that we can compute the kernel K very efficiently and in a numerically stable way

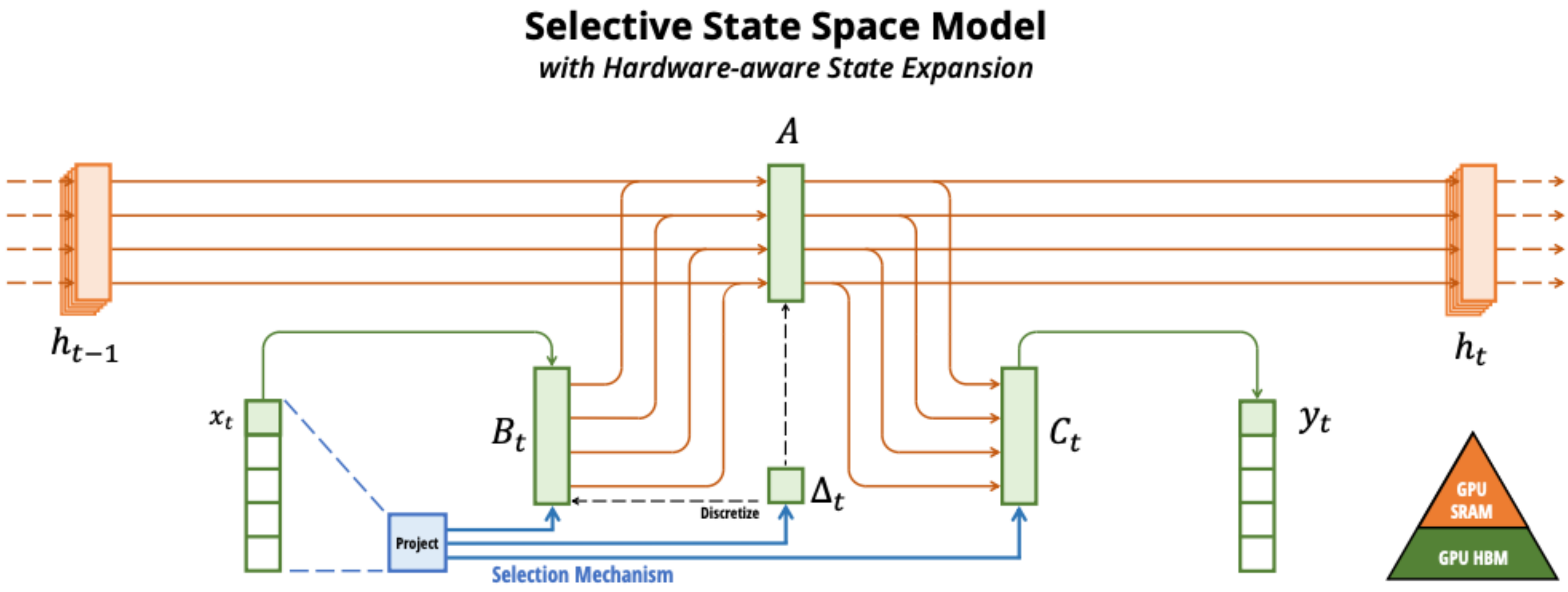
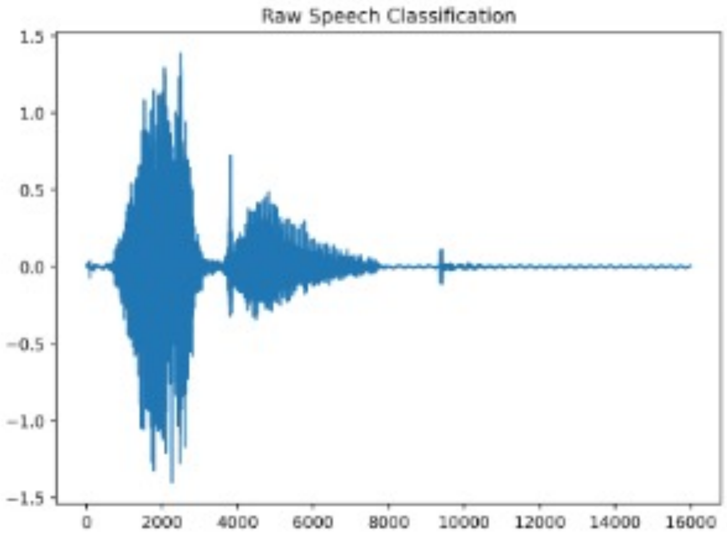


Figure 1: (**Overview**.) Structured SSMs independently map each channel (e.g. $D = 5$) of an input x to output y through a higher dimensional latent state h (e.g. $N = 4$). Prior SSMs avoid materializing this large effective state (DN, times batch size B and sequence length L) through clever alternate computation paths requiring time-invariance: the (Δ, $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$) parameters are constant across time. Our selection mechanism adds back input-dependent dynamics, which also requires a careful hardware-aware algorithm to only materialize the expanded states in more efficient levels of the GPU memory hierarchy.

Figure from https://arxiv.org/pdf/2312.00752

S4 Results: Train and test on different input granularities



		Train: 16K Hz	Test: 8K Hz
	MFCC	RAW	0.5×
Transformer	90.75	✗	✗
Performer	80.85	30.77	30.68
ODE-RNN	65.9	✗	✗
NRDE	89.8	16.49	15.12
ExpRNN	82.13	11.6	10.8
LipschitzRNN	88.38	✗	✗
CKConv	**95.3**	71.66	<u>65.96</u>
WaveGAN-D	✗	<u>96.25</u>	✗
LSSL	93.58	✗	✗
S4	<u>93.96</u>	**98.32**	**96.30**

Figure from https://hazyresearch.stanford.edu/blog/2022-01-14-s4-3

MAMBA

Selective State Space Models

Algorithm 1 SSM (S4)

Input: $x : (\mathsf{B}, \mathsf{L}, \mathsf{D})$
Output: $y : (\mathsf{B}, \mathsf{L}, \mathsf{D})$

$\boldsymbol{A} : (\mathsf{D}, \mathsf{N}) \leftarrow \mathsf{Parameter}$
▷ Represents structured $N \times N$ matrix
$\boldsymbol{B} : (\mathsf{D}, \mathsf{N}) \leftarrow \mathsf{Parameter}$
$\boldsymbol{C} : (\mathsf{D}, \mathsf{N}) \leftarrow \mathsf{Parameter}$
$\Delta : (\mathsf{D}) \leftarrow \tau_\Delta(\mathsf{Parameter})$
$\overline{\boldsymbol{A}}, \overline{\boldsymbol{B}} : (\mathsf{D}, \mathsf{N}) \leftarrow \mathsf{discretize}(\Delta, \boldsymbol{A}, \boldsymbol{B})$
$y \leftarrow \mathsf{SSM}(\overline{\boldsymbol{A}}, \overline{\boldsymbol{B}}, \boldsymbol{C})(x)$
▷ Time-invariant: recurrence or convolution
return y

Algorithm 2 SSM + Selection (S6)

Input: $x : (\mathsf{B}, \mathsf{L}, \mathsf{D})$
Output: $y : (\mathsf{B}, \mathsf{L}, \mathsf{D})$

$\boldsymbol{A} : (\mathsf{D}, \mathsf{N}) \leftarrow \mathsf{Parameter}$
▷ Represents structured $N \times N$ matrix
$\boldsymbol{B} : (\mathsf{B}, \mathsf{L}, \mathsf{N}) \leftarrow s_B(x)$
$\boldsymbol{C} : (\mathsf{B}, \mathsf{L}, \mathsf{N}) \leftarrow s_C(x)$
$\Delta : (\mathsf{B}, \mathsf{L}, \mathsf{D}) \leftarrow \tau_\Delta(\mathsf{Parameter} + s_\Delta(x))$
$\overline{\boldsymbol{A}}, \overline{\boldsymbol{B}} : (\mathsf{B}, \mathsf{L}, \mathsf{D}, \mathsf{N}) \leftarrow \mathsf{discretize}(\Delta, \boldsymbol{A}, \boldsymbol{B})$
$y \leftarrow \mathsf{SSM}(\overline{\boldsymbol{A}}, \overline{\boldsymbol{B}}, \boldsymbol{C})(x)$
▷ Time-varying: recurrence (*scan*) only
return y

- **Selective** state space models differ from S4 in that they let the parameters B and C vary at each timestep

Selective State Space Models

Algorithm 1 SSM (S4)

Input: $x : (\mathsf{B}, \mathsf{L}, \mathsf{D})$
Output: $y : (\mathsf{B}, \mathsf{L}, \mathsf{D})$

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▷ Time-varying: recurrence (*scan*) only
return y

$$h_t = Ah_{t-1} + Bx_t$$
$$y_t = C^\top h_t$$

$$h_t = A_t h_{t-1} + B_t x_t$$
$$y_t = C_t^\top h_t$$

Mamba's Scan Implementation

- We can no longer compute the kernel K once up front
- Instead we perform an efficient scan implementation

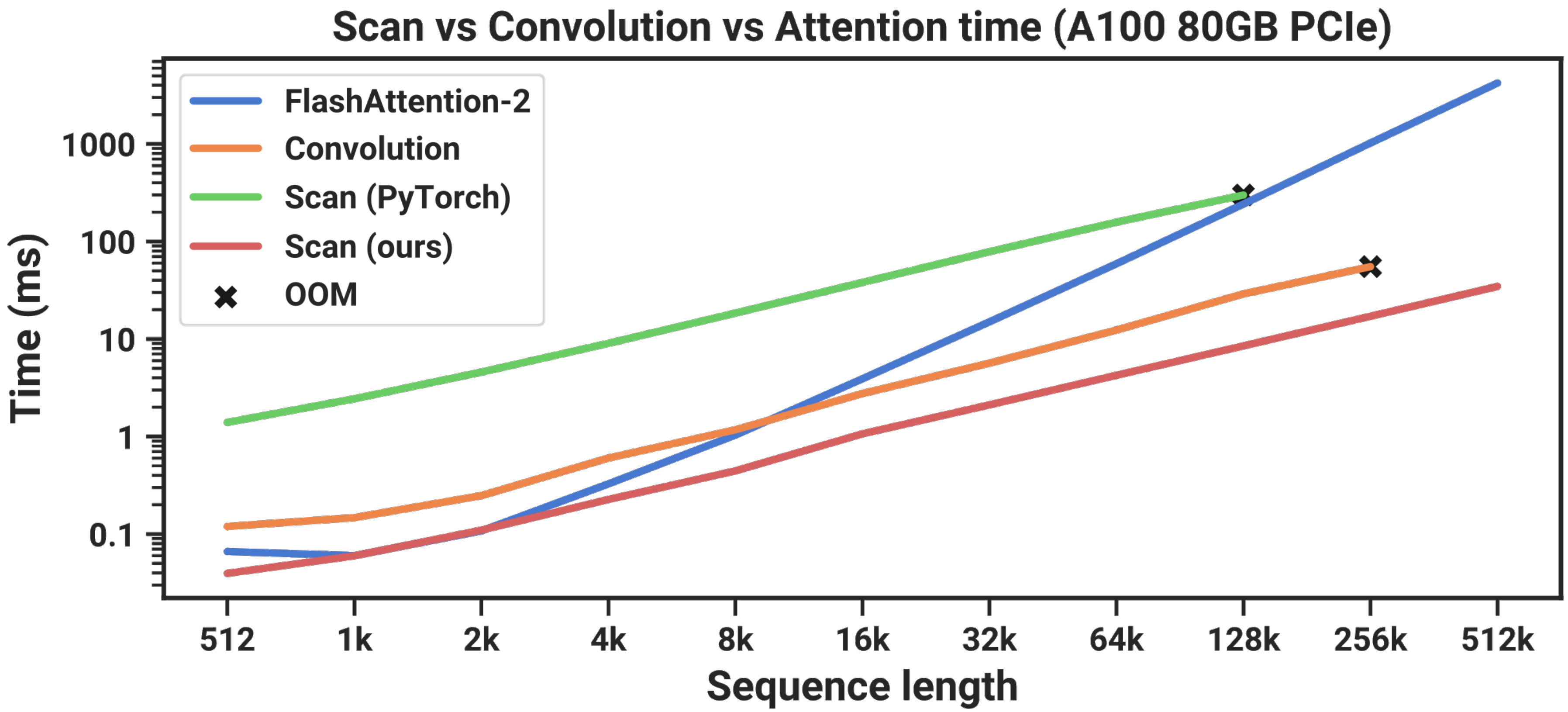


Figure from https://jackcook.com/2024/02/23/mamba.html

Mamba Results

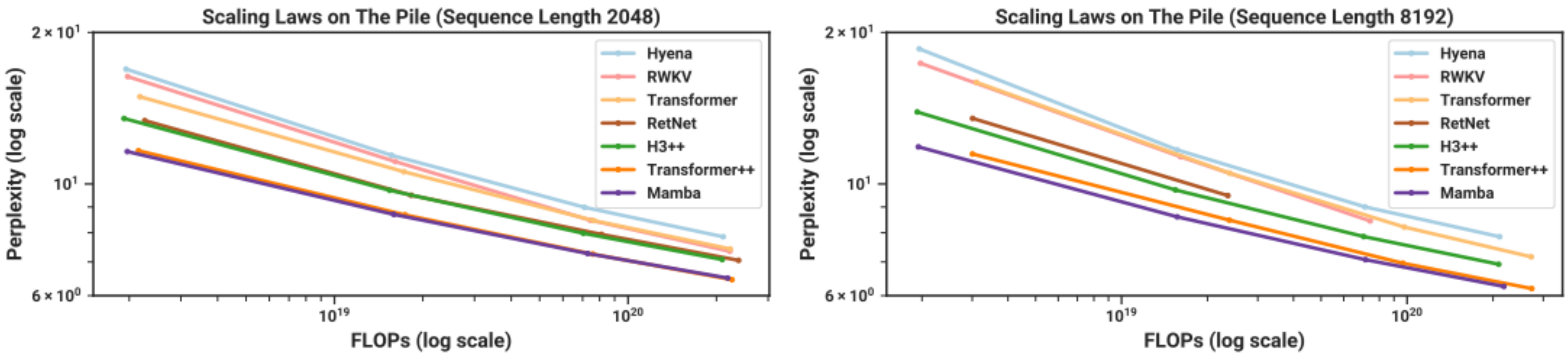
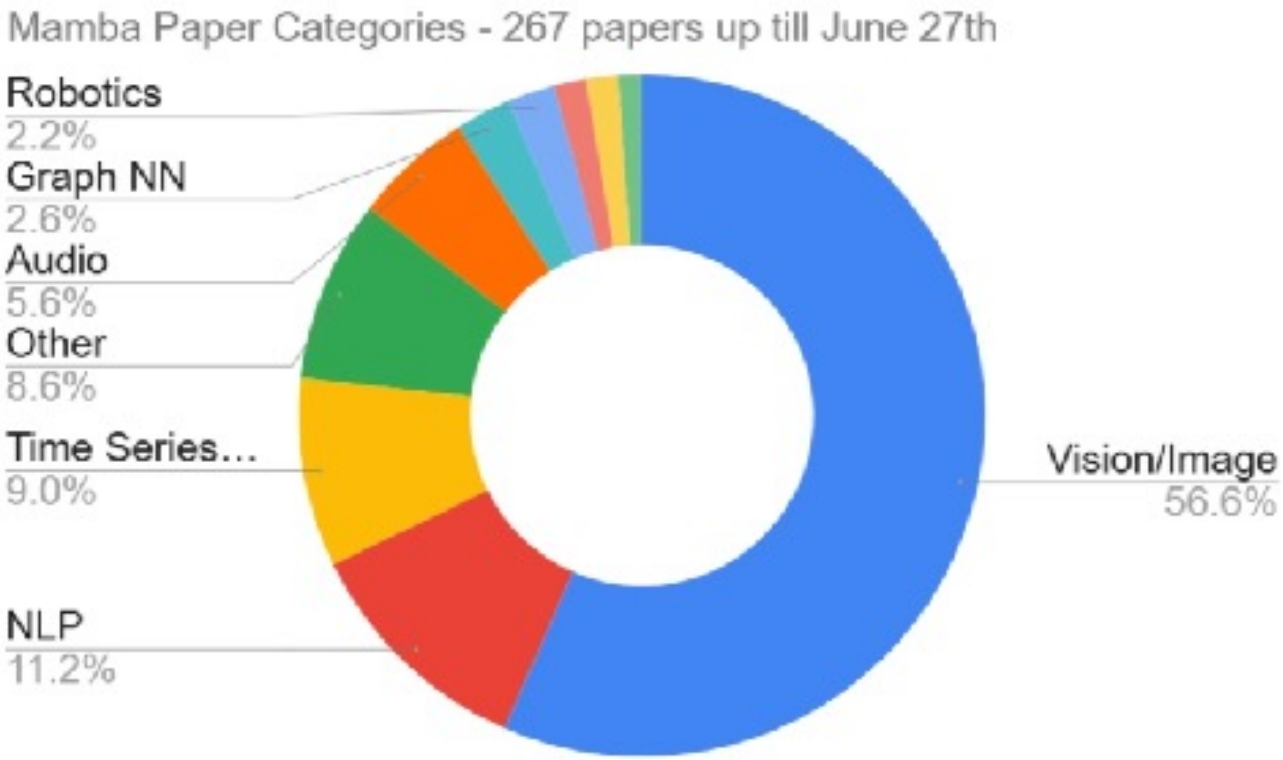


Figure 4: (**Scaling Laws.**) Models of size $\approx 125M$ to $\approx 1.3B$ parameters, trained on the Pile. Mamba scales better than all other attention-free models and is the first to match the performance of a very strong "Transformer++" recipe that has now become standard, particularly as the sequence length grows.

- Main takeaway: Mamba was the first non-attention based LM to challenge a Transformer

Figure from https://arxiv.org/pdf/2312.00752

Mamba Use in the Real World



Strong out-of-the-box on **general modalities**

(not just language!)

Figure from Albert Gu's F24 guest lecture in 10-423/623

LINEAR ATTENTION AND SSMS

Linear Attention

- Linear attention is identical to standard attention, but drops the softmax: (below M is the causal mask)

$$\mathbf{O} = (\mathbf{Q}\mathbf{K}^{\intercal} \odot \mathbf{M})\mathbf{V} \in \mathbb{R}^{L \times d_v} \qquad \boldsymbol{o}_t = \sum_{i=1}^{t} (\boldsymbol{v}_i \boldsymbol{k}_i^{\intercal}) \boldsymbol{q}_t = \sum_{i=1}^{t} \boldsymbol{v}_i (\boldsymbol{k}_i^{\intercal} \boldsymbol{q}_t) \in \mathbb{R}^{d_v},$$

- And can be expressed as a recurrence:

$$\mathbf{S}_t = \mathbf{S}_{t-1} + \boldsymbol{v}_t \boldsymbol{k}_t^{\intercal} \in \mathbb{R}^{d_v \times d_k}, \qquad \boldsymbol{o}_t = \mathbf{S}_t \boldsymbol{q}_t \in \mathbb{R}^{d_v}$$

- Draws a direct connection between Transformers and SSMs

Figure from https://arxiv.org/pdf/2412.06464

Linear Attention

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$$\mathbf{O} = (\mathbf{Q}\mathbf{K}^\intercal \odot \mathbf{M})\mathbf{V} \in \mathbb{R}^{L \times d_v} \qquad \boldsymbol{o}_t = \sum_{i=1}^{t} (\boldsymbol{v}_i \boldsymbol{k}_i^\intercal)\boldsymbol{q}_t = \sum_{i=1}^{t} \boldsymbol{v}_i (\boldsymbol{k}_i^\intercal \boldsymbol{q}_t) \in \mathbb{R}^{d_v},$$

- And can be expressed as a recurrence:

$$\mathbf{S}_t = \mathbf{S}_{t-1} + \boldsymbol{v}_t \boldsymbol{k}_t^\intercal \in \mathbb{R}^{d_v \times d_k}, \qquad \boldsymbol{o}_t = \mathbf{S}_t \boldsymbol{q}_t \in \mathbb{R}^{d_v}$$

- Various forms of linear attention have been proposed:

Method	Online Update
LA	$\mathbf{S}_t = \mathbf{S}_{t-1} + \boldsymbol{v}_t \boldsymbol{k}_t^T$
Mamba2	$\mathbf{S}_t = \alpha_t \mathbf{S}_{t-1} + \boldsymbol{v}_t \boldsymbol{k}_t^T$
Longhorn	$\mathbf{S}_t = \mathbf{S}_{t-1}(\mathbf{I} - \epsilon \boldsymbol{k}_t \boldsymbol{k}_t^T) + \epsilon_t \boldsymbol{v}_t \boldsymbol{k}_t^T, \epsilon_t = \frac{\beta_t}{1 + \beta_t \boldsymbol{k}_t^\top \boldsymbol{k}_t}$
DeltaNet	$\mathbf{S}_t = \mathbf{S}_{t-1}(\mathbf{I} - \beta_t \boldsymbol{k}_t \boldsymbol{k}_t^T) + \beta_t \boldsymbol{v}_t \boldsymbol{k}_t^T$
Gated DeltaNet	$\mathbf{S}_t = \mathbf{S}_{t-1}\left(\alpha_t(\mathbf{I} - \beta_t \boldsymbol{k}_t \boldsymbol{k}_t^T)\right) + \beta_t \boldsymbol{v}_t \boldsymbol{k}_t^T$

Figure from https://arxiv.org/pdf/2412.06464

HYBRID MODELS

Hybrid Models

Hybrid models combine the best of Transformers and SSMs

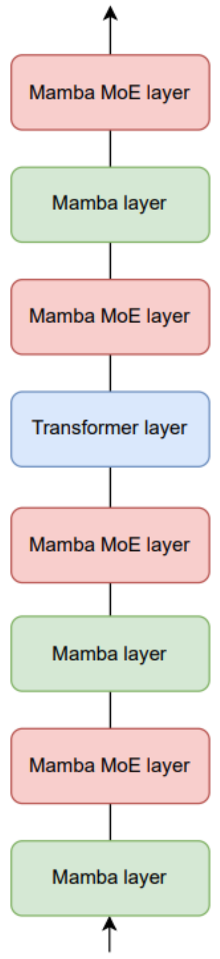
Motivation:

- *Long-context scalability:* Reduce the quadratic cost of self-attention while maintaining or improving modeling ability for sequences spanning tens or hundreds of thousands of tokens.
- *Better hardware utilization:* Exploit architectures that stream activations sequentially (like SSMs) to improve throughput and fit larger contexts on fixed-memory GPUs.
- *Robust generalization:* Mix inductive biases—e.g., attention for global dependencies, convolutional or state-space layers for local and temporal structure—to handle diverse data types (text, audio, vision).

Examples: Jamba, Nemotron-H, Qwen 3 Next

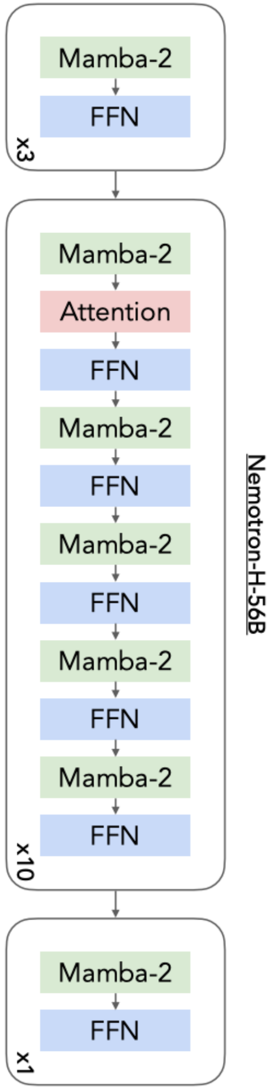
Hybrid Models

Jamba

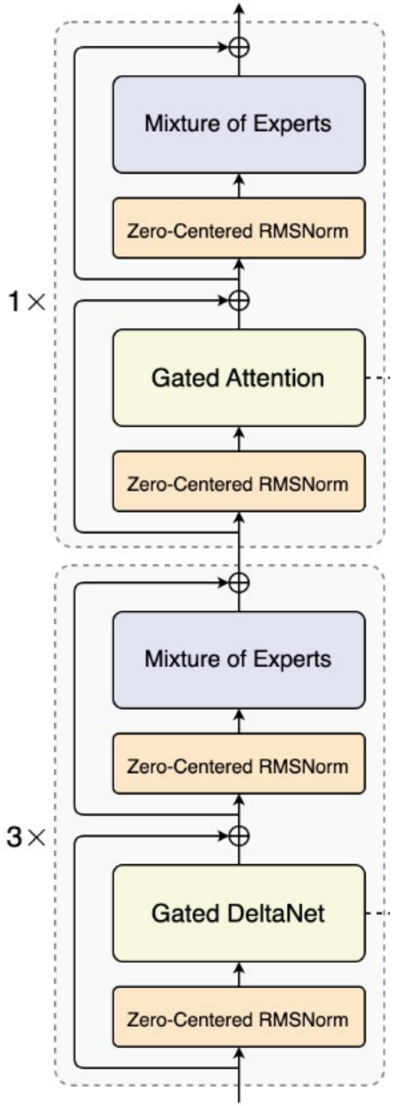


(a) Jamba block

Nemotron-H



Qwen 3 Next

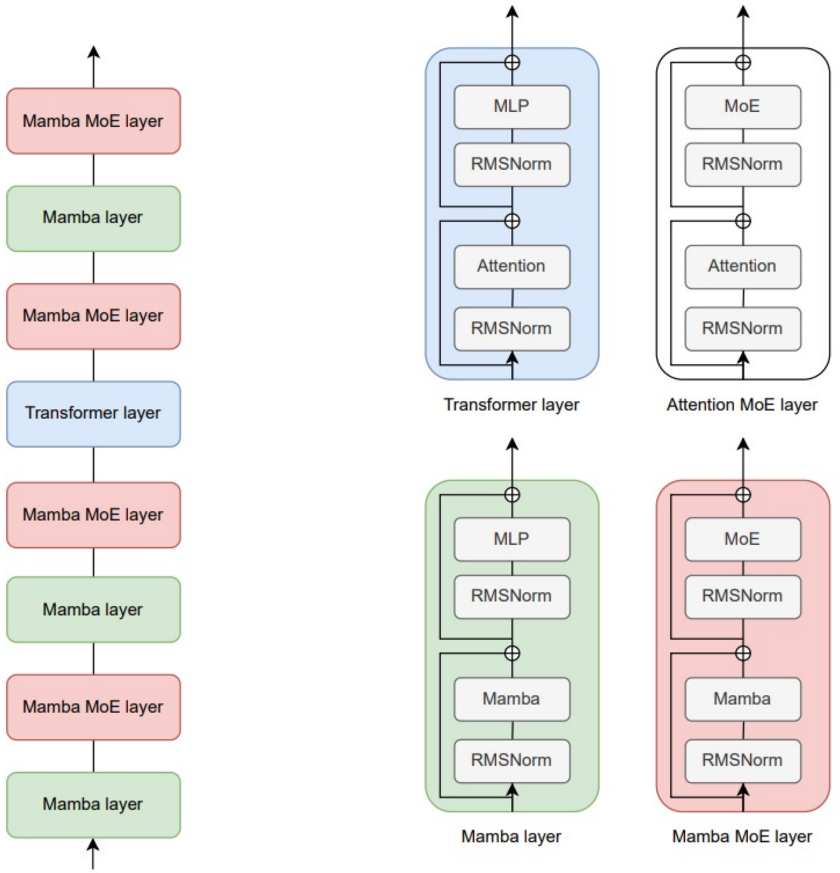


- *Common to all these models:* standard attention layers are **interspersed** with linear attention layers
- The standard attention has **quadratic complexity,** and the linear attention has **linear complexity** in the sequence length
- **Goal:** reduce memory requirements and speed up generation (all of these models accomplish this)

Figure from https://arxiv.org/pdf/2403.19887 Figure from https://arxiv.org/pdf/2504.03624 Figure from https://qwen.ai/blog?id=4074cca80393150c248e508aa62983f9cb7d27cd

Jamba

- Jamba was the **first** hybrid model to combine Transformer layers with SSMs layers (2024)
- The architecture dramatically **reduces the size of the KV cache** for long contexts
- Jamba enables dramatically **longer contexts** to fit on a single GPU than traditional Tranformer LMs
- This enables much **higher throughput** (measured in tokens per second) at generation time



(a) Jamba block

(b) Different types of layers

	Available params	Active params	KV cache (256K context, 16bit)
LLAMA-2	6.7B	6.7B	128GB
Mistral	7.2B	7.2B	32GB
Mixtral	46.7B	12.9B	32GB
Jamba	52B	12B	4GB

Figure from https://arxiv.org/pdf/2403.19887

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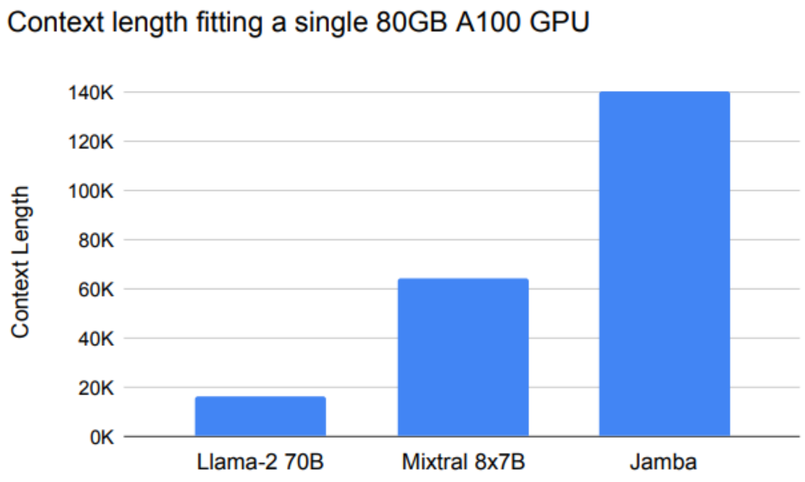
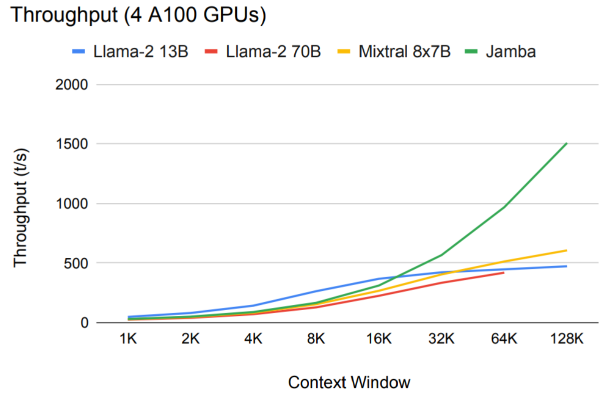
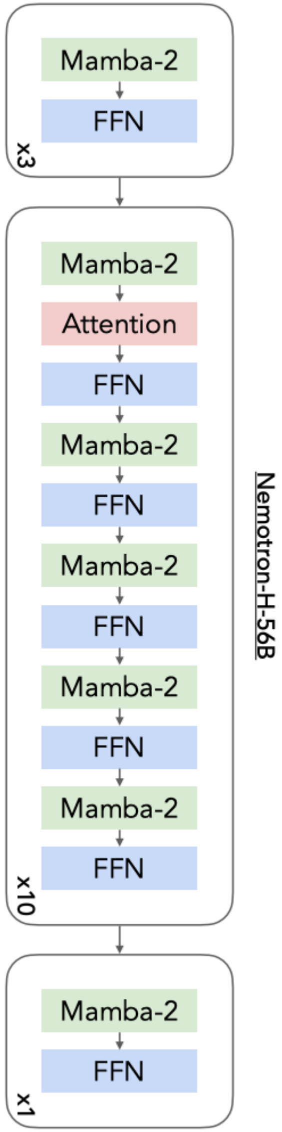




(b) Throughput at different context lengths (single batch, 4 A100 GPUs). With a context of 128K tokens, Jamba obtains 3x the throughput of Mixtral, while Llama-2-70B does not fit with this long context.

Figure from https://arxiv.org/pdf/2403.19887

Nemotron-H



- Nemotron-H demonstrated that performance of a standard Transformer (Nemotron-T) could be retained while gaining dramatic throughput improvements
- Also introduced a vision-language model (VLM) variant

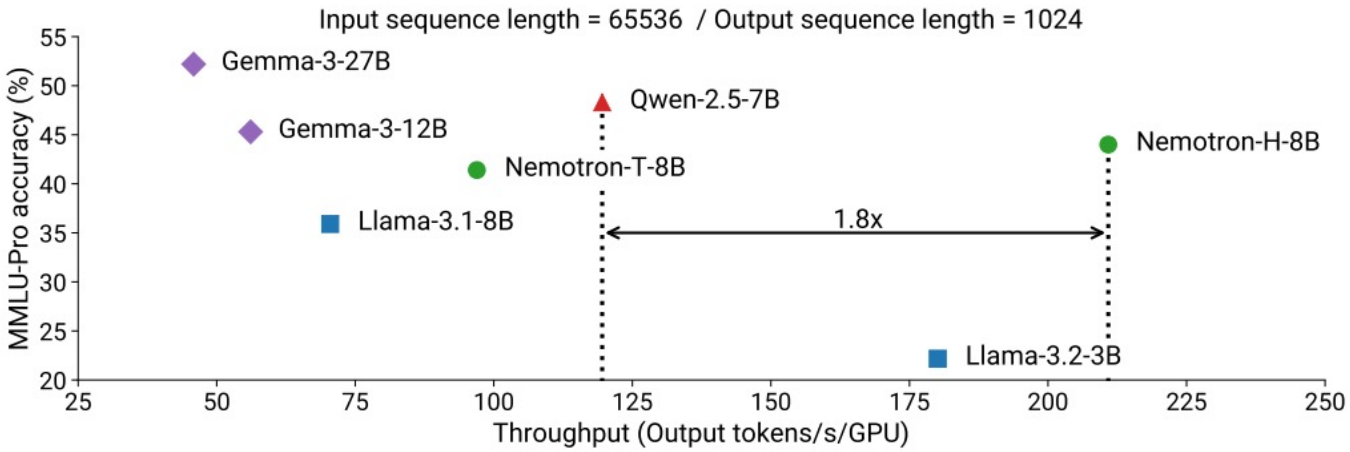
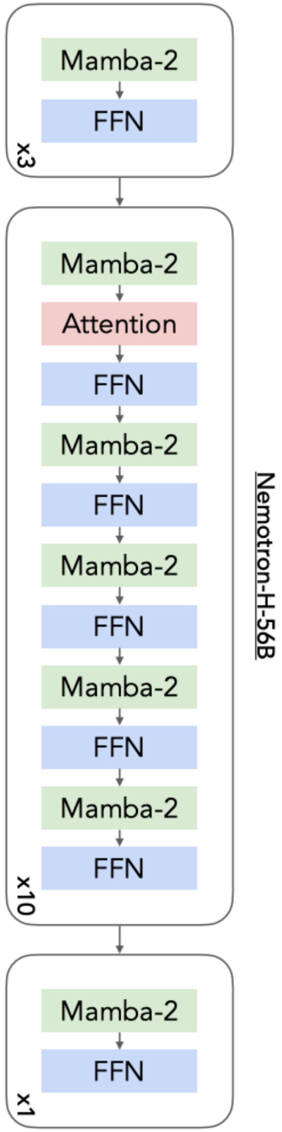


Figure 7 | MMLU-Pro accuracy versus inference throughput (normalized by number of GPUs used) for Nemotron-H-8B-Base compared to existing similarly-sized Transformer models.

Figure from https://arxiv.org/pdf/2504.03624

Nemotron-H



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Task	Nemotron-H 56B-VLM	VLM w/ Qwen2.5 72B-Instruct	NVLM-D-1.0 72B (2024-09-17)
MMMU (val)	63.6	**65.1**	62.6†
MathVista	**70.7**	70.5	66.7†
ChartQA	**89.4**	88.9	86.0
AI2D	94.7	**94.9**	94.2
OCRBench	862	**869**	853
TextVQA	81.1	**83.5**	82.1
RealWorldQA	68.4	**71.4**	69.7
DocVQA	**93.2**	92.0	92.6

Figure from https://arxiv.org/pdf/2504.03624

Qwen-3-Next

- Qwen-3-Next uses Gated Delta Net layers instead of standard linear attention layers

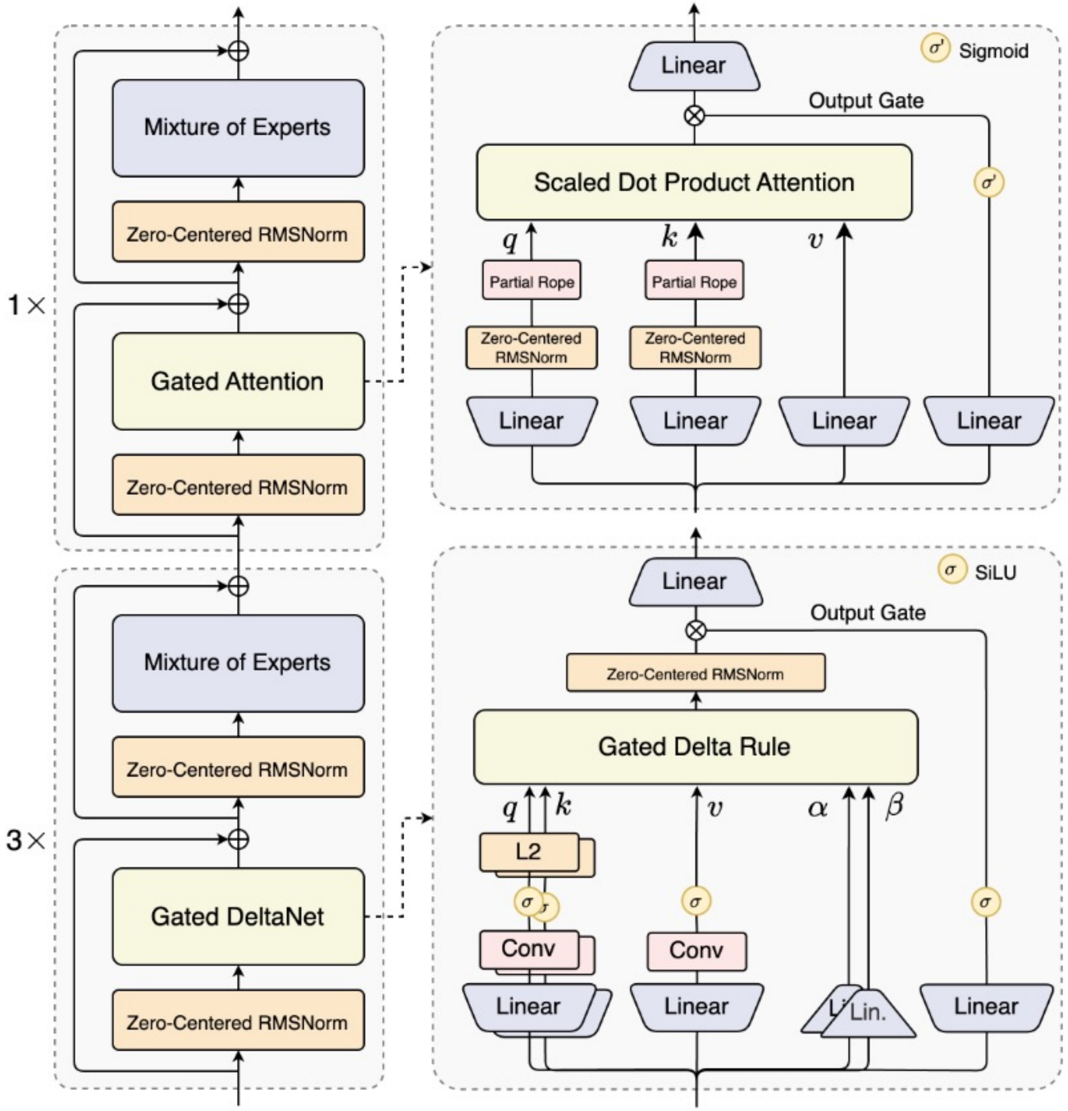

Figure from https://qwen.ai/blog?id=4074cca80393150c248e508aa62983f9cb7d27cd

Qwen-3-Next

- Qwen-3-Next uses Gated Delta Net layers instead of standard linear attention layers
- Both training time and generation are dramatically reduced, compared to dense models of comparable size

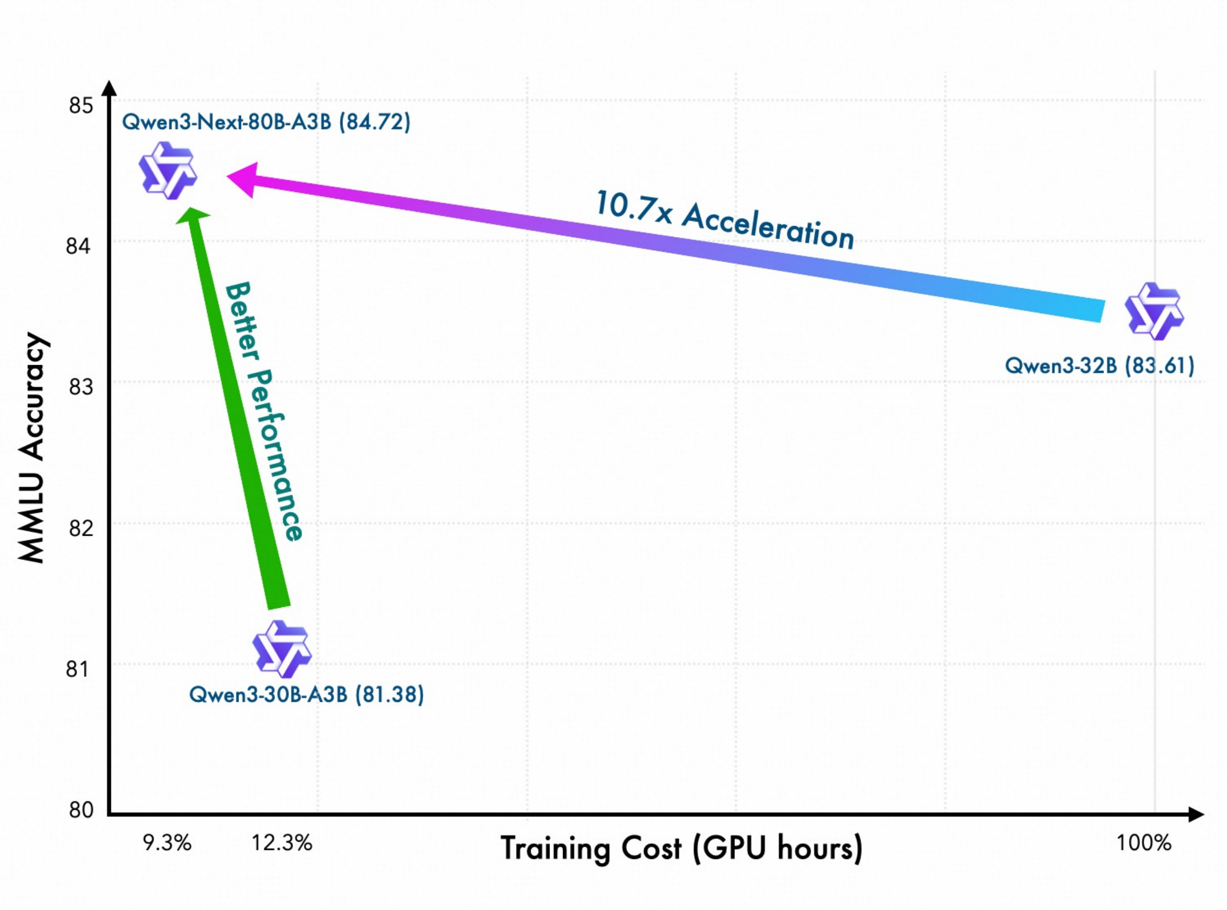

Figure from https://qwen.ai/blog?id=4074cca80393150c248e508aa62983f9cb7d27cd

Qwen-3-Next

- Qwen-3-Next uses Gated Delta Net layers instead of standard linear attention layers
- Both training time and generation are dramatically reduced, compared to dense models of comparable size
- Impressive performance across standard benchmarks

	Qwen3-30B-A3B Base	Qwen3-32B Base	Qwen3-Next-80B-A3B Base	Qwen3-235B-A22B Base
Architecture	MoE	Dense	MoE	MoE
# Total Params	30B	32B	80B	235B
# Activated Params	3B	32B	3B	22B
General Tasks				
MMLU	81.38	83.61	84.72	**87.81**
MMLU-Redux	81.17	83.41	83.80	**87.40**
MMLU-Pro	61.49	65.54	66.05	**68.18**
SuperGPQA	35.72	39.78	41.52	**44.06**
BBH	81.54	87.38	87.13	**88.87**
Math, STEM & Coding Tasks				
GPQA	43.94	**49.49**	43.43	47.47
GSM8K	91.81	93.40	90.30	**94.39**
MATH	59.04	61.62	62.36	**71.84**
EvalPlus	71.45	72.05	72.89	**77.60**
CRUX-O	67.20	72.50	74.25	**79.00**
Multilingual Tasks				
MGSM	79.11	83.06	81.28	**83.53**
MMMLU	81.46	83.83	84.43	**86.70**
INCLUDE	67.00	67.87	69.79	**73.46**

Figure from https://qwen.ai/blog?id=4074cca80393150c248e508aa62983f9cb7d27cd

Motivation

- https://www.isattentionallyouneed.com/

Is Attention All You Need?

Current Status: Yes

Time Remaining: 631d 22h 55m 11s

Proposition:

On January 1, 2027, a Transformer-like model will continue to hold the state-of-the-art position in most benchmarked tasks in natural language processing.