Automated Program Verification and Testing 15414/15614 Fall 2016 Lecture 26: Counterexamples & Abstraction Refinement

Matt Fredrikson
mfredrik@cs.cmu.edu

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Abstraction (Review)

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But it might introduce **spurious counterexamples**

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The states in our abstraction will be: $S = \mathsf{Loc} \times \{0,1\}^m$

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Why is this conservative?

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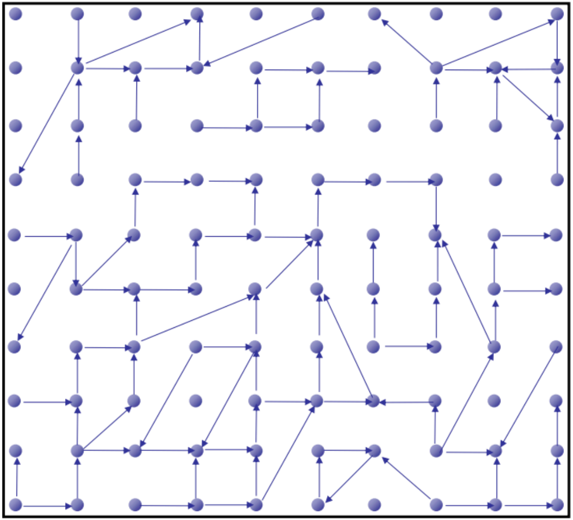

Image Credit: Tom Henzinger, Ranjit Jhala, Rupak Majumdar

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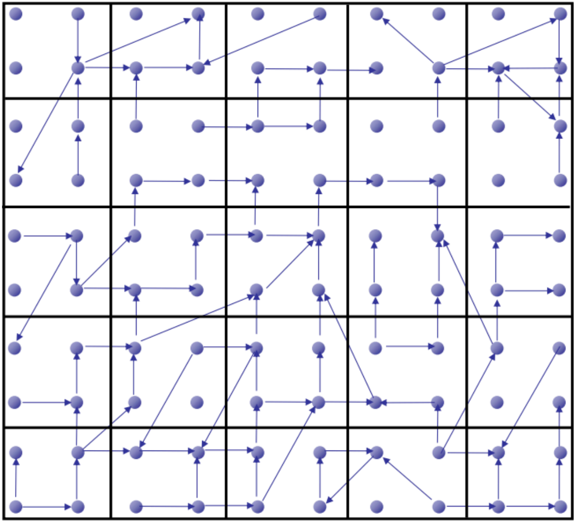

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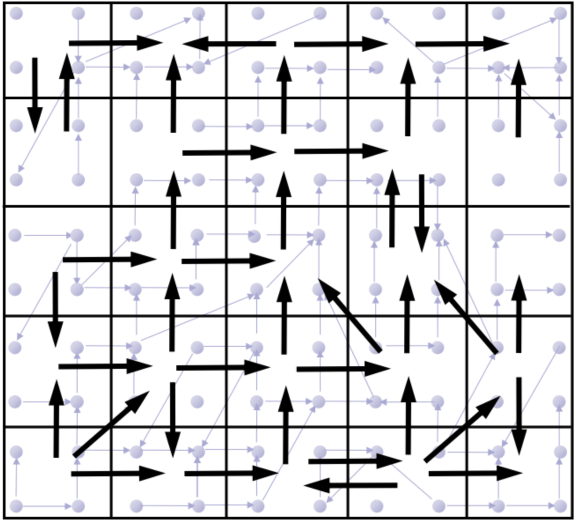

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Just predicates. **Idea**: Use predicate transformers

Strengthening Predicates (Review)

Given $E = \{\phi_1, \ldots, \phi_n\}$, let $\mathsf{Pred}(\phi, E)$:

- The **weakest** DNF over E,
- that is at least as strong as ϕ,
- where each clause has n literals

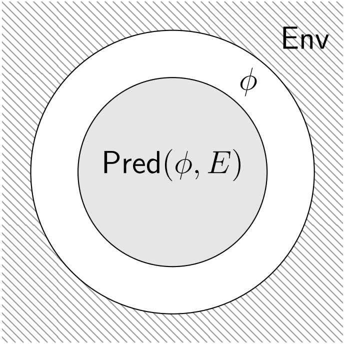
Notice: $\mathsf{Pred}(\phi, E) \Rightarrow \phi$

Compute this by querying SMT solver

- What's the complexity of this?
- $O(2^n)$
- Need to query each:

$$p_1 \wedge \cdots \wedge p_n \Rightarrow \phi$$

where p_i is ϕ_i or $\neg\phi_i$



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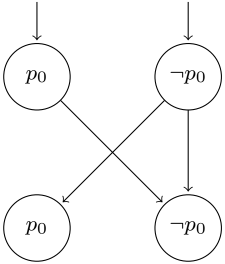

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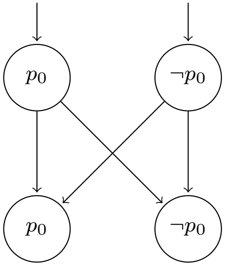

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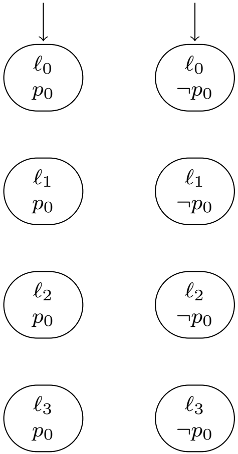

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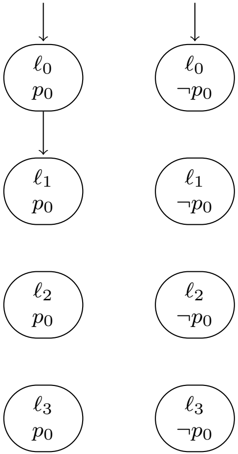

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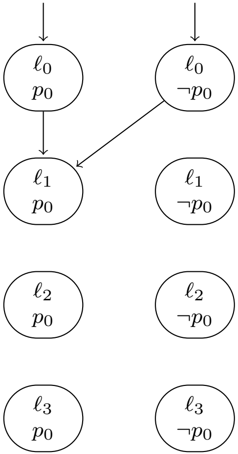

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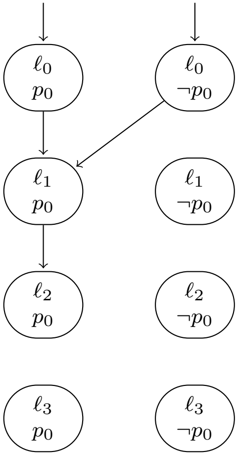

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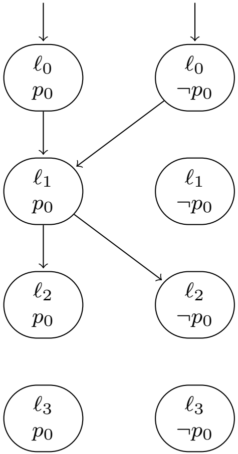

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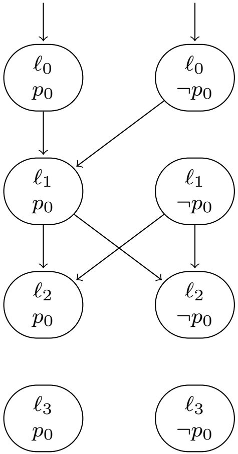

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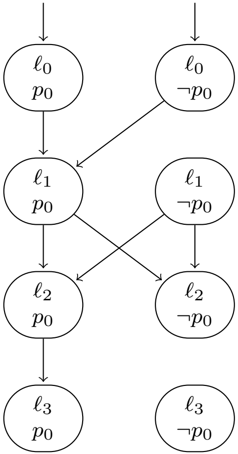

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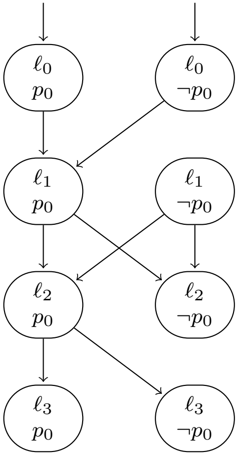

Example: Predicate Abstraction

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\begin{aligned}
\ell_0 :&\quad i := 1; \\
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Suppose we check:

$$\mathbf{G}\ (\neg \ell_0 \to 0 \le i)$$

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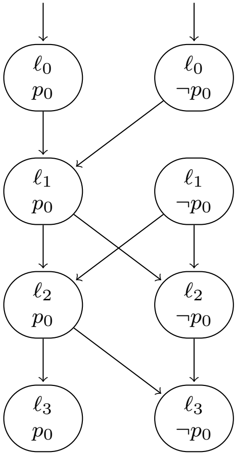

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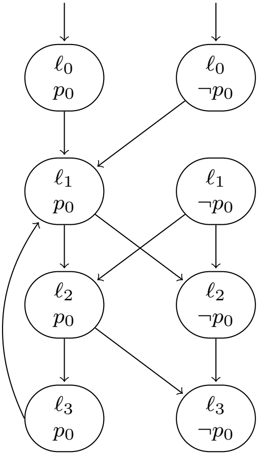

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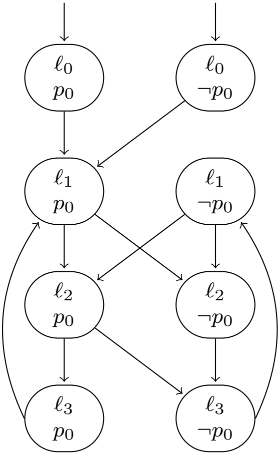

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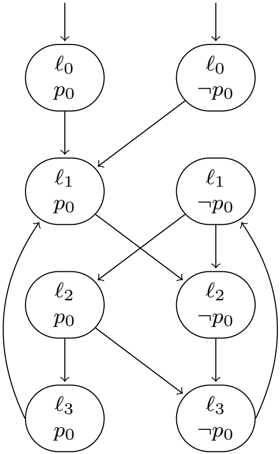

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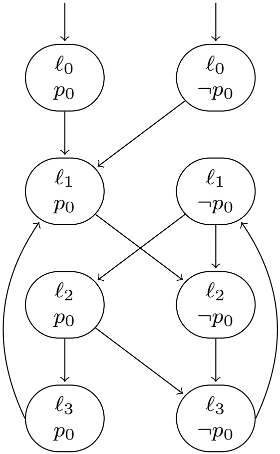

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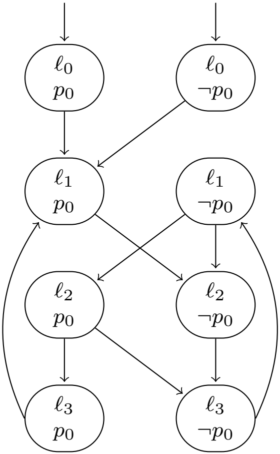

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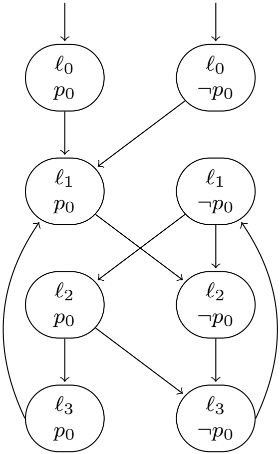

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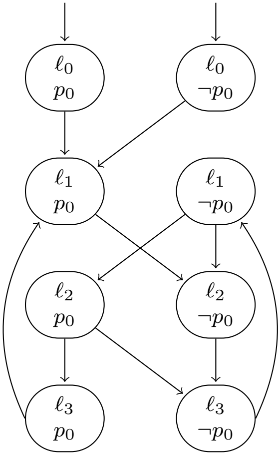

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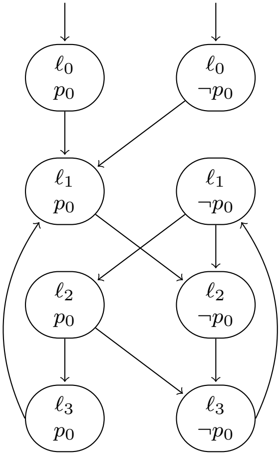

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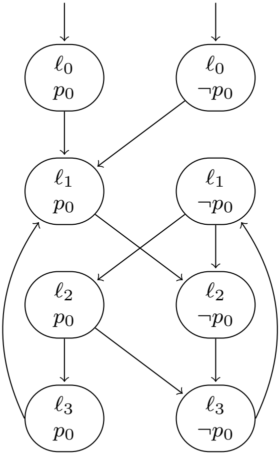

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$$

Spurious Counterexamples

$$\begin{array}{ll} \ell_0: & i := 1; \\ \ell_1: & \textbf{while}(0 \leq x < 1)\ \{ \\ \ell_2: & \quad i := i - 1; \\ \ell_3: & \quad x := x + 1; \\ & \} \end{array}$$

Consider the KS path:

$$\begin{array}{l} (\ell_0, p_0) \\ (\ell_1, p_0) \\ (\ell_2, \neg p_0) \end{array}$$

(recall that $p_0 \Leftrightarrow 0 \leq i$)

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\begin{array}{l}
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(recall that $p_0 \Leftrightarrow 0 \leq i$)

Consider the corresponding program path

Spurious Counterexamples

ℓ_0 : $i := 1;$
ℓ_1 : **while**$(0 \leq x < 1)$ {
ℓ_2 : $i := i - 1;$
ℓ_3 : $x := x + 1;$
}

ℓ_0 : $i := 1;$

Consider the KS path:

(ℓ_0, p_0)
(ℓ_1, p_0)
$(\ell_2, \neg p_0)$

(recall that $p_0 \Leftrightarrow 0 \leq i$)

Consider the corresponding program path

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$\ell_0 :\quad i := 1;$
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$\quad\quad \}$

$\ell_0 :\quad i := 1;$
$\ell_1 :\quad \textbf{assume}(0 \leq x < 1)$

Consider the KS path:

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Consider the KS path:

$$(\ell_0, p_0)$$
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(recall that $p_0 \Leftrightarrow 0 \leq i$)

Is this a valid Hoare triple?

Consider the corresponding program path

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Is this a valid Hoare triple?

1. $\{0 \leq i\}\ i := 1\ \{0 \leq i\}$

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Spurious Counterexamples

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\ell_0: & i := 1; \\
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 & \{i < 0\}
\end{array}
$$

Consider the KS path:

$$
\begin{array}{c}
(\ell_0, p_0) \\
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(recall that $p_0 \Leftrightarrow 0 \le i$)

Is this a valid Hoare triple?

1. $\{0 \le i\}\ i := 1\ \{0 \le i\}$ **Yes**

Consider the corresponding program path

Spurious Counterexamples

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1. $\{0 \leq i\}\ i := 1\ \{0 \leq i\}$ **Yes**
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Spurious Counterexamples

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1. $\{0 \le i\}\ i := 1\ \{0 \le i\}$ **Yes**
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Consider the corresponding program path

This is how we know that the counterexample is spurious

Abstraction Refinement

We want to make the abstraction more precise: add more predicates

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Main technique behind all modern software model checking

1. Start with a simple, automatic abstraction
2. Search for counterexamples
3. Refine spurious counterexamples, building model on-demand
4. Continue until real counterexample, or property holds

Cause and Refinement

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\begin{array}{ll}
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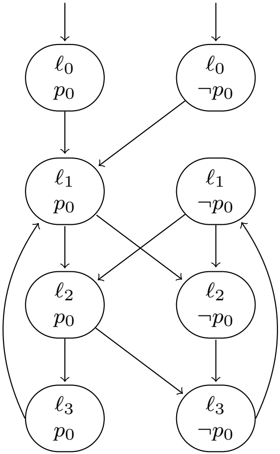

What caused this?

$$(\ell_0, p_0)\ (\ell_1, p_0)\ (\ell_2, \neg p_0)$$

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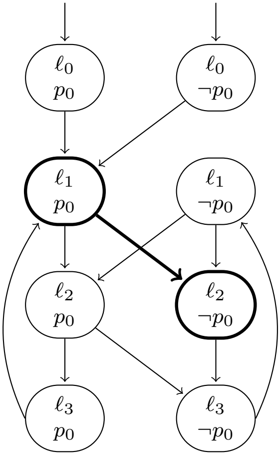

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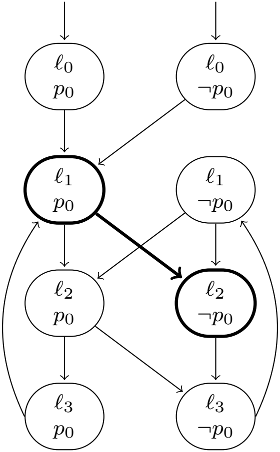

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We had $\neg$Pred$(0 \leq x < 1, \{p_0\})$ = *true*

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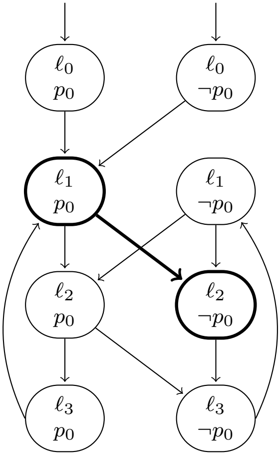

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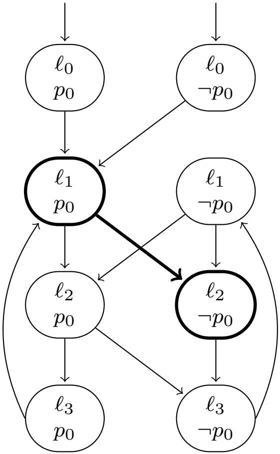

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How do we fix it?

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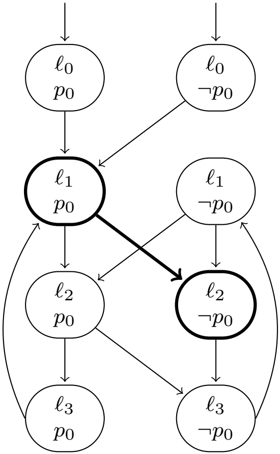
How do we fix it?

$$E = \{\underbrace{0 \leq i}_{p_0}, \underbrace{0 \leq x < 1}_{p_1}\}$$



Example: Abstraction Refinement

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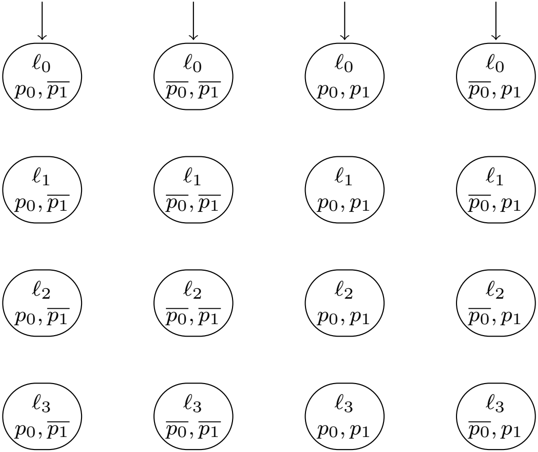

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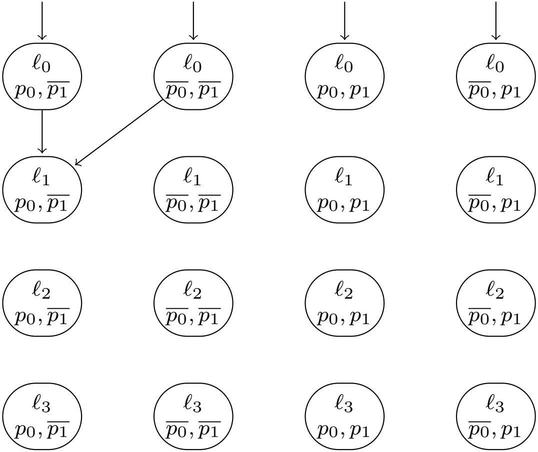

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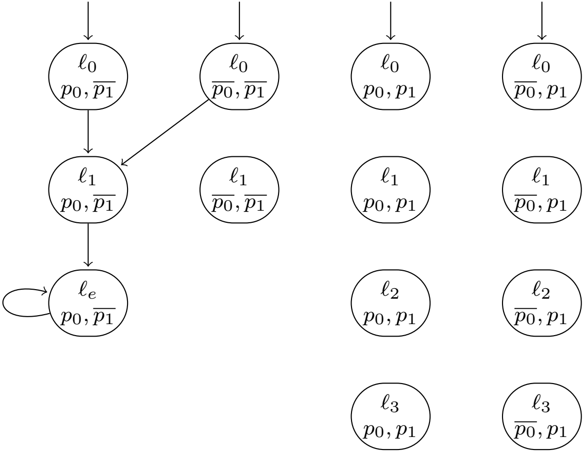

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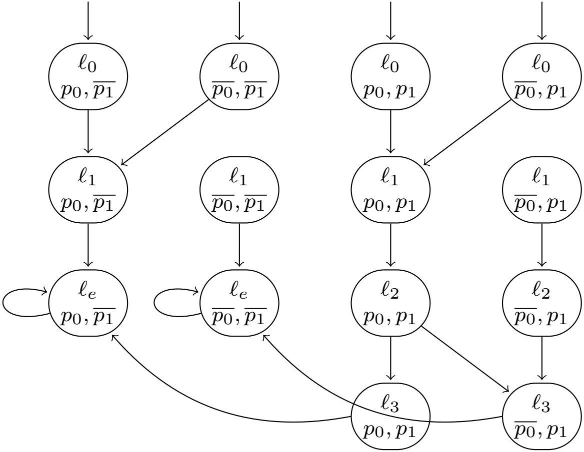

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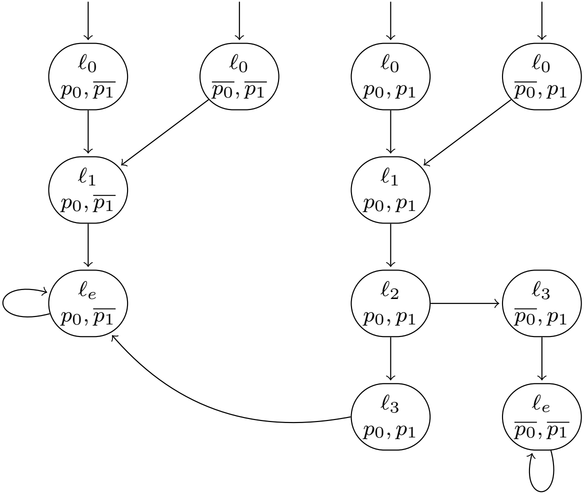

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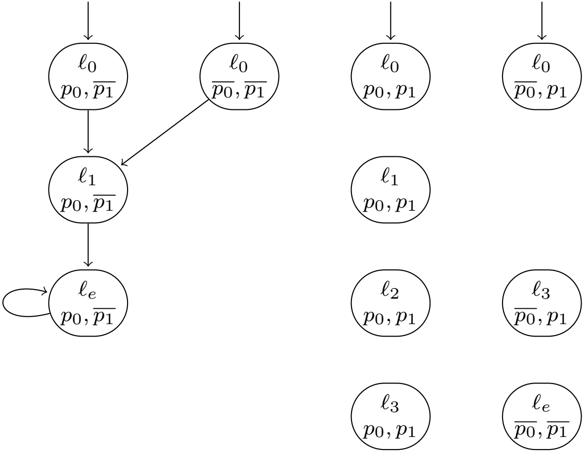
Is there a counterexample?



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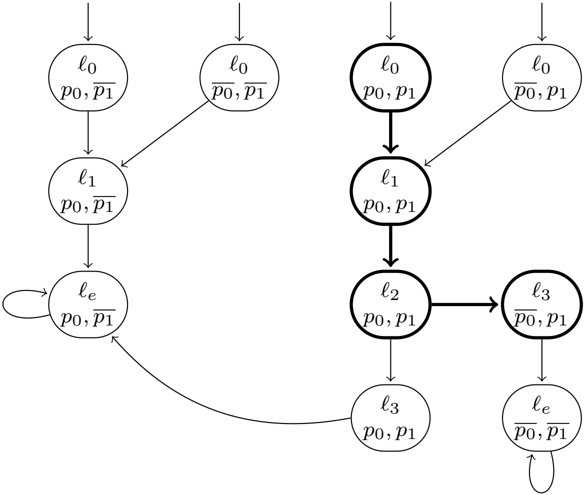

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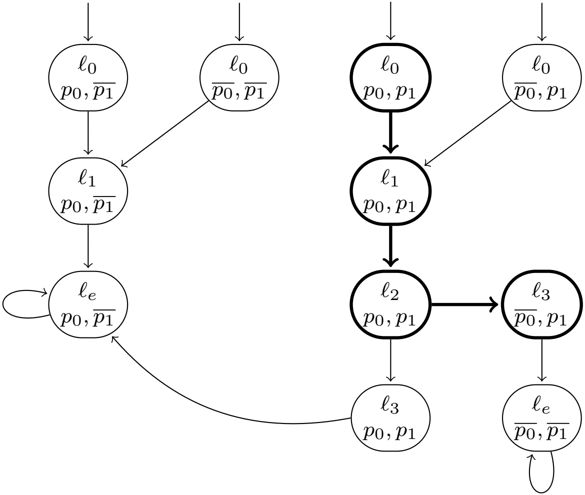
Is this valid?

$\{0 \le i \land 0 \le x < 1\}$
$i := 1$;
assume $(0 \le x < 1)$
$i := i - 1$;
$\{i < 0 \land 0 \le x < 1\}$



Lazy Abstraction

After the second counterexample, it seems $x = 1$ is relevant

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- There must be a better way!

Idea: Don't refine error-free parts of the abstraction

Example: Lazy Abstraction

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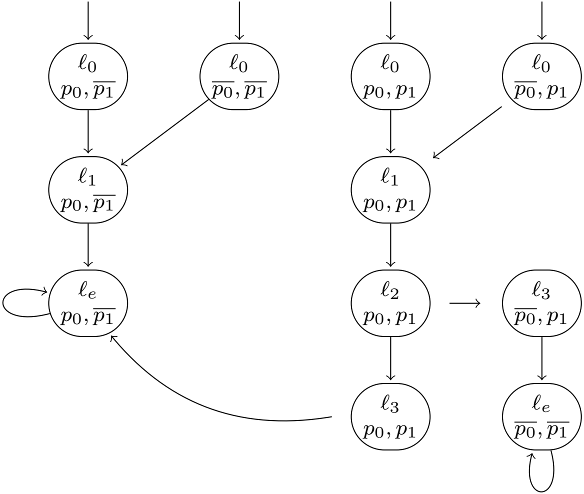

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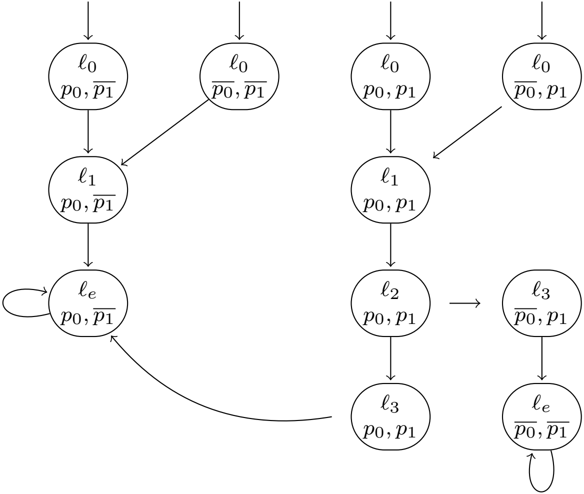
Don't need to update left side with

$$p_2 \Leftrightarrow i = 1$$



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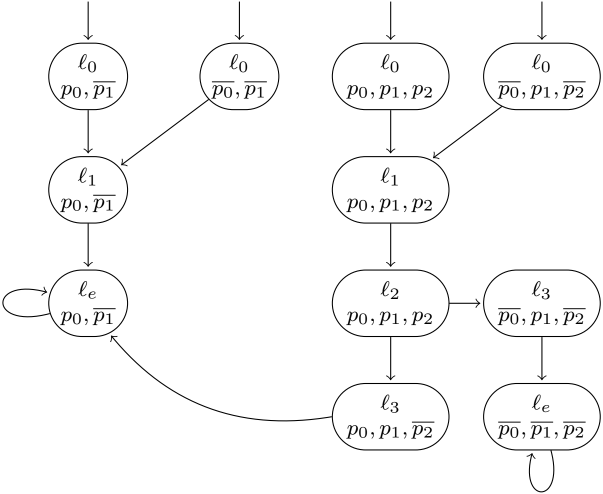

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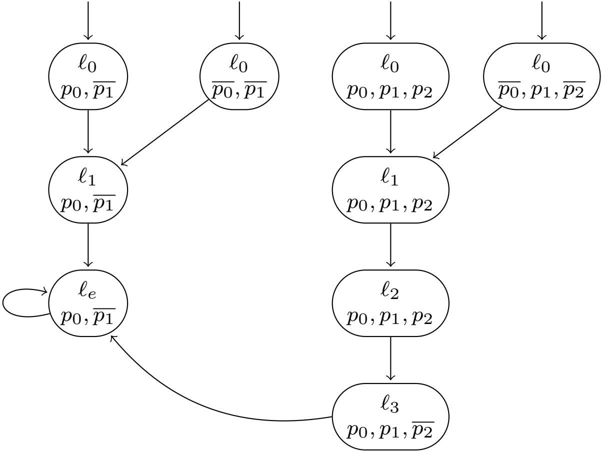

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Don't need to update left side with

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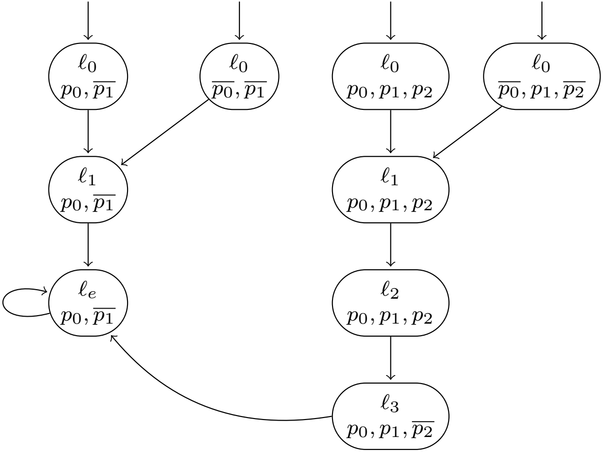
Now there's no counterexample



More on Lazy Abstraction

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In practice, this approach gives drastic performance improvements

Proofs from Abstractions

ℓ_0 : $i := 1$;
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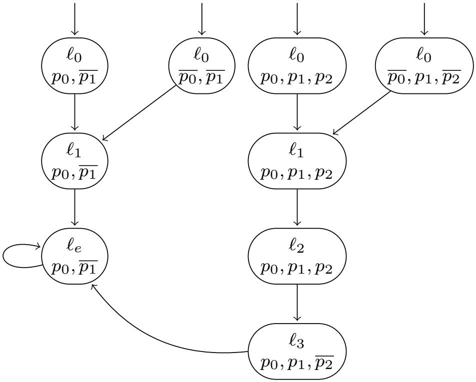

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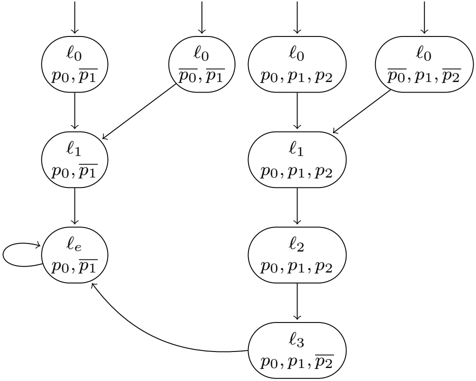

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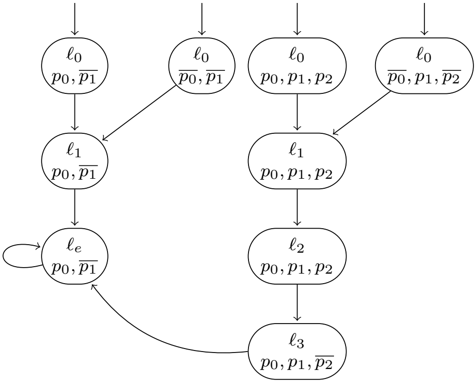

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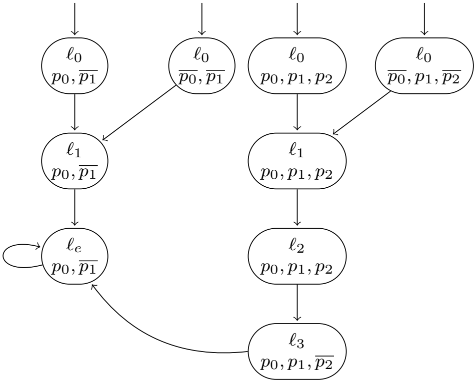

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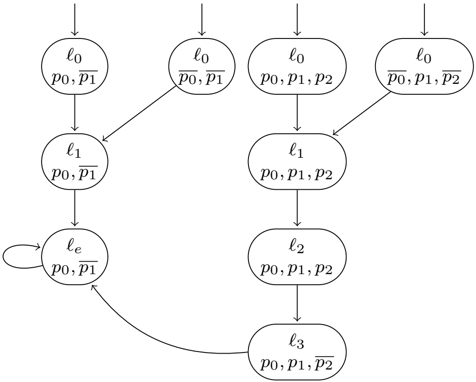

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\lor & (0 \le i \land \neg(0 \le x < 1) \land i \ne 1) \\
\Leftrightarrow & \\
0 \le i &
\end{array}
$$

CEGAR automatically constructs deductive proofs!

Limitations

Suppose we wanted to verify:

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How would CEGAR do it?

Limitations

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ℓ_e : **skip**
$\{0 \leq i\}$

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How would CEGAR do it?

- Find $i = 10, x = 9$

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& \{0 \le i\}
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How would we do it by hand?

- Find the invariant $0 \le i - x$

How would CEGAR do it?

- Find $i = 10, x = 9$
- Find $i = 10, x = 8$

Limitations

Suppose we wanted to verify:

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Finding the right predicates early is crucial

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Before, we found new predicates by intuition

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- Otherwise, take an observation that explains why the path won't execute

SSA Form

To build ϕ_{path}, we'll put path in **static single-assignment** (SSA) form

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$i := 1$
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wp$(\ldots,$ *true*$) = 0 \leq i_0 \wedge i_1 = 1 \wedge 0 \leq i_1 \wedge 0 \leq x_0 < 1 \wedge \neg(0 \leq i_1) \wedge i_2 = i_1 - 1$

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We have a counterexample, path formula pair

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Is the path formula satisfiable?

No. We already knew this path was invalid

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How do we automatically find such an explanation?

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1. First, enumerate every $l, l' \in C$ where $l \neq l'$

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We'll modify this slightly:

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2. If $l \Rightarrow l'$, then remove l'

Example: Learning New Predicates

Initial formula:

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\begin{array}{ll}
0 \leq i_0 & \wedge \\
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$i_1 = 1$ wasn't previously in our set, so we refine by adding it

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Path formulas give us a way to check invariants on individual paths

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- If *sat*, then this path is a valid counterexample

Bounded Model Checking

Path formulas give us a way to check invariants on individual paths

1. Given an invariant property **G** ϕ,
2. Enumerate a sequence of statements $\ell_1, \ldots, \ell_n$
3. Create the "counterexample" $(\ell_1, \textit{true}), \ldots, (\ell_n, \textit{true}), (\textbf{skip}, \neg\phi)$
4. Generate the path formula ϕ_{path}
5. Check ϕ_{path} for satisfiability

The results tell us everything:

- If *unsat*, there's no way to execute $\ell_1, \ldots, \ell_n$ satisfying $\neg\phi$
- If *sat*, then this path is a valid counterexample

sat assignment to initial SSA variables is an input to the program

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- When run on these inputs, the property will be violated

Bounded Model Checking: Example

ℓ_0 : $i := 1;$
ℓ_1 : **while**$(0 \leq x < 2)$ $\{$
ℓ_2 : $i := i - 1;$
ℓ_3 : $x := x + 1;$
$\}$
ℓ_e : **assert**$(0 \leq i)$

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We suspect the path:

$i := 1;$
assume$(0 \leq x < 2)$
$i := i - 1;$
$x := x + 1;$
assume$(0 \leq x < 2)$
$i := i - 1;$
$x := x + 1;$
assume$(\neg(0 \leq x < 2))$
assert$(0 \leq i)$

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$x := x + 1;$
assume$(\neg(0 \leq x < 2))$
assert$(0 \leq i)$

After SSA, assumption encoding:

assume $i_1 = 1;$
assume $0 \leq x_0 < 2;$
assume $i_2 = i_1 - 1;$
assume $x_1 = x_0 + 1;$
assume $0 \leq x_1 < 2;$
assume $i_3 = i_2 - 1;$
assume $x_2 = x_1 + 1;$
assume $\neg(0 \leq x_2 < 2);$
assert $0 \leq i_3;$

Bounded Model Checking: Example

$$\begin{aligned}
&\ell_0 : i := 1;\\
&\ell_1 : \textbf{while}(0 \le x < 2)\ \{\\
&\ell_2 : \quad i := i - 1;\\
&\ell_3 : \quad x := x + 1;\\
&\quad\ \}\\
&\ell_e : \textbf{assert}(0 \le i)
\end{aligned}$$

We suspect the path:

$$\begin{aligned}
&i := 1;\\
&\textbf{assume}(0 \le x < 2)\\
&i := i - 1;\\
&x := x + 1;\\
&\textbf{assume}(0 \le x < 2)\\
&i := i - 1;\\
&x := x + 1;\\
&\textbf{assume}(\neg(0 \le x < 2))\\
&\textbf{assert}(0 \le i)
\end{aligned}$$

Path formula:

$$\begin{aligned}
&i_1 = 1 && \wedge\\
&0 \le x_0 < 2 && \wedge\\
&i_2 = i_1 - 1 && \wedge\\
&x_1 = x_0 + 1 && \wedge\\
&0 \le x_1 < 2 && \wedge\\
&i_3 = i_2 - 1 && \wedge\\
&x_2 = x_1 + 1 && \wedge\\
&\neg(0 \le x_2 < 2) && \wedge\\
&0 \le i_3
\end{aligned}$$

Bounded Model Checking: Example

ℓ_0 : $i := 1;$
ℓ_1 : **while** $(0 \le x < 2)$ {
ℓ_2 : $i := i - 1;$
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}
ℓ_e : **assert** $(0 \le i)$

We suspect the path:

$i := 1;$
assume $(0 \le x < 2)$
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$x := x + 1;$
assume $(0 \le x < 2)$
$i := i - 1;$
$x := x + 1;$
assume $(\neg(0 \le x < 2))$
assert $(0 \le i)$

Path formula:

$$
\begin{array}{ll}
i_1 = 1 & \wedge \\
0 \le x_0 < 2 & \wedge \\
i_2 = i_1 - 1 & \wedge \\
x_1 = x_0 + 1 & \wedge \\
0 \le x_1 < 2 & \wedge \\
i_3 = i_2 - 1 & \wedge \\
x_2 = x_1 + 1 & \wedge \\
\neg(0 \le x_2 < 2) & \wedge \\
0 \le i_3 &
\end{array}
$$

Is this satisfiable?

Bounded Model Checking: Example

$$
\begin{aligned}
\ell_0 &: i := 1; \\
\ell_1 &: \textbf{while}(0 \le x < 2)\ \{ \\
\ell_2 &: \quad i := i - 1; \\
\ell_3 &: \quad x := x + 1; \\
&\quad \} \\
\ell_e &: \textbf{assert}(0 \le i)
\end{aligned}
$$

We suspect the path:

$$
\begin{aligned}
& i := 1; \\
& \textbf{assume}(0 \le x < 2) \\
& i := i - 1; \\
& x := x + 1; \\
& \textbf{assume}(0 \le x < 2) \\
& i := i - 1; \\
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& \textbf{assume}(\neg(0 \le x < 2)) \\
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Path formula:

$$
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i_1 = 1 & \wedge \\
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i_3 = i_2 - 1 & \wedge \\
x_2 = x_1 + 1 & \wedge \\
\neg(0 \le x_2 < 2) & \wedge \\
0 \le i_3 &
\end{array}
$$

Is this satisfiable?

$i_1 = 1, x_0 = 0, i_2 = 0, x_1 = 1, i_3 = -1, x_2 = 2$

Bounded Model Checking: Example

$$\begin{aligned}
\ell_0 &: i := 1; \\
\ell_1 &: \textbf{while}(0 \leq x < 2)\ \{ \\
\ell_2 &: \quad i := i - 1; \\
\ell_3 &: \quad x := x + 1; \\
&\quad \} \\
\ell_e &: \textbf{assert}(0 \leq i)
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&\textbf{assume}(0 \leq x < 2) \\
&i := i - 1; \\
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&\textbf{assume}(\neg(0 \leq x < 2)) \\
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&i_3 = i_2 - 1 && \wedge \\
&x_2 = x_1 + 1 && \wedge \\
&\neg(0 \leq x_2 < 2) && \wedge \\
&0 \leq i_3
\end{aligned}$$

Is this satisfiable?

$$i_1 = 1, x_0 = 0, i_2 = 0, x_1 = 1, i_3 = -1, x_2 = 2$$

We can use $x = 0$ as an initial test case

Next Lecture

Go over homeworks

Next Lecture

Go over homeworks

Review for the final

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Go over homeworks

Review for the final

Last homework due on Friday evening, 11:59

- No late days!
- University policy...