Automated Program Verification and Testing 15414/15614 Fall 2016 Lecture 24: Symbolic Model Checking 2, Spin

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Symbolic Transition Systems (Recap)

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Recall: this is similar to how we treated assertions in Hoare logic

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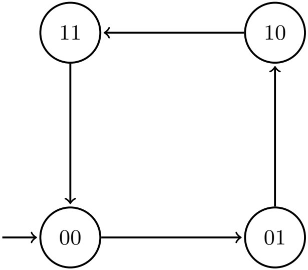
Example: Symbolic Representation



Symbolic transitions:

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\begin{array}{ll}
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Initial state: $v_0 = 0 \wedge v_0 = 1$

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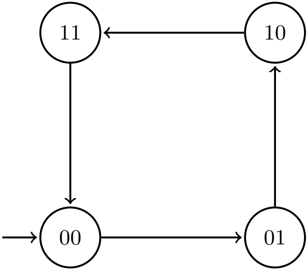

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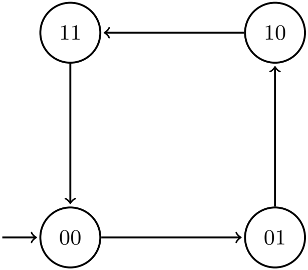

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The transitions are a predicate

$$\psi_R(v_0, v_1, v_0', v_1')$$

- Over four Boolean $\{0, 1\}$ variables
- Variables completely determine state of system

Same for the initial state: $\psi_I(v_0, v_1)$

Fixpoints

Let $\tau : 2^S \mapsto 2^S$ be a predicate transfomer

- τ is **monotonic** iff $P \subseteq Q$ implies $\tau(P) \subseteq \tau(Q)$
- A **fixpoint** of τ is a predicate (set) Z where $\tau(Z) = Z$
- A **least fixpoint** of τ, written $\mu Z.\tau(Z)$, is:
  1. A fixpoint of τ, so $\tau(\mu Z.\tau(Z)) = Z$
  2. A subset of any other fixpoint
- A **greatest fixpoint** of τ, written $\nu Z.\tau(Z)$, is:
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Computing Fixpoints

We have a simple algorithm that gives us fixpoints

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\begin{array}{l}
\textbf{function } \textit{lfp}(\tau) \; \{ \\
\quad Q := \textit{false}; \\
\quad Q' := \tau(Q); \\
\quad \textbf{while}(Q \neq Q') \; \{ \\
\qquad Q := Q'; \\
\qquad Q := \tau(Q'); \\
\quad \} \\
\quad \textbf{return } Q; \\
\}
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function lfp(τ) {
  Q := false;
  Q′ := τ(Q);
  while(Q ≠ Q′) {
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  return Q;
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```

```
function gfp(τ) {
  Q := true;
  Q′ := τ(Q);
  while(Q ≠ Q′) {
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  return Q;
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Fixpoint Semantics of CTL

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Our "base" operator is **EX** ϕ, given by the predicate transformer:

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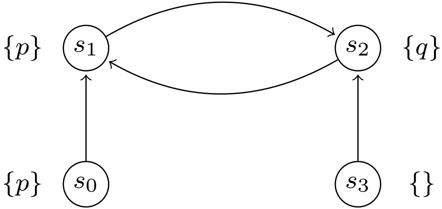
- **EG** $\phi = \nu Z.\phi \wedge \textbf{EX}\ Z$
- **E** $(\phi_1\ \textbf{U}\ \phi_2) = \mu Z.\phi_2 \vee (\phi_1 \wedge \textbf{EX}\ Z)$

Example: **E** $(p$ **U** $q)$

$$\tau(Z) = q \vee (p \wedge \textbf{EX}\ Z)$$



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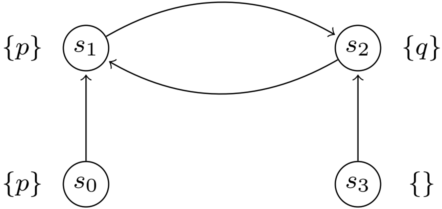

First compute $\tau($*false*$) = \tau(\varnothing)$

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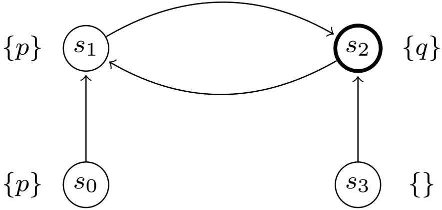

Then $\tau^1(\textit{false}) = \tau(\{s_2\})$

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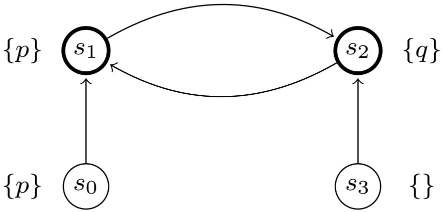

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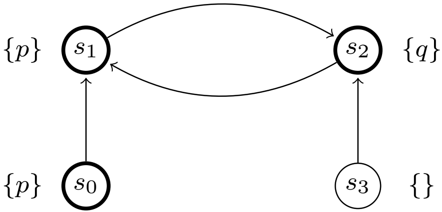

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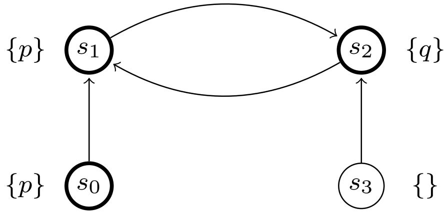

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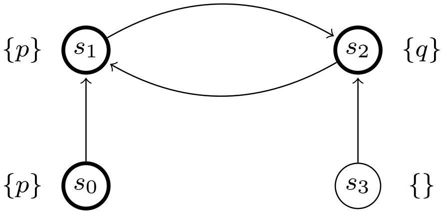

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We've reached the fixpoint $\mu Z.\tau(Z)$

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If it is, then the corresponding set is non-empty, and ϕ holds

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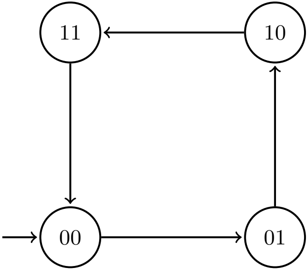

Suppose we want to check **EX** $v_0 = 1$

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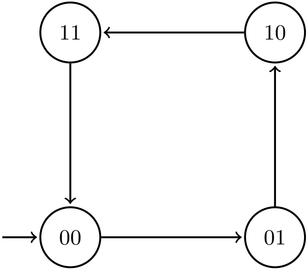

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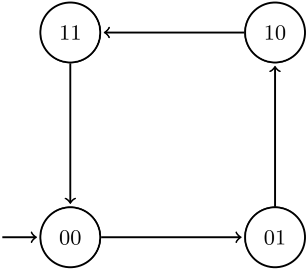

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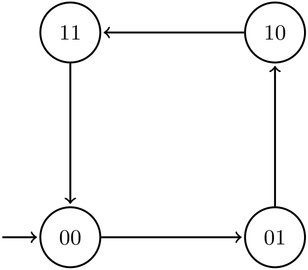

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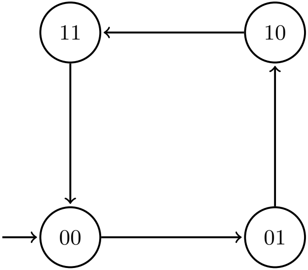

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This formula is *false*, so there are no states that satisfy

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We know that we can compute greatest fixpoints by:

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2. Repeating, until the predicate doesn't change

Symbolic Model Checking (**EG**)

We have that **EG** ϕ = $\nu Z.\phi \wedge$ **EX** Z

So to check **EG** ϕ:

1. Find the fixpoint of τ = $\nu Z.\phi \wedge$ **EX** Z
2. Conjoin ψ_I
3. Check for satisfiability

We know that we can compute greatest fixpoints by:

1. Applying the predicate transformer to *true*
2. Repeating, until the predicate doesn't change

But before we can do this, must show $\nu Z.\phi \wedge$ **EX** Z is monotonic

Symbolic Model Checking (**E** $(\phi_1$ **U** $\phi_2))$

We have that **E** $(\phi_1$ **U** $\phi_2) = \mu Z.\phi_2 \vee (\phi_1 \wedge$ **EX** $Z)$

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But what have we gained by doing it this way?

Efficient Propositional Encodings

Given a predicate $\phi(x_1, \ldots, x_n) \mapsto \{0, 1\}$

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This gives us an easy way to test fixpoints

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Consider the two-bit comparator:

$$\phi(x_1, x_2, y_1, y_2) = (x_1 \leftrightarrow y_1) \wedge (x_2 \leftrightarrow y_2)$$

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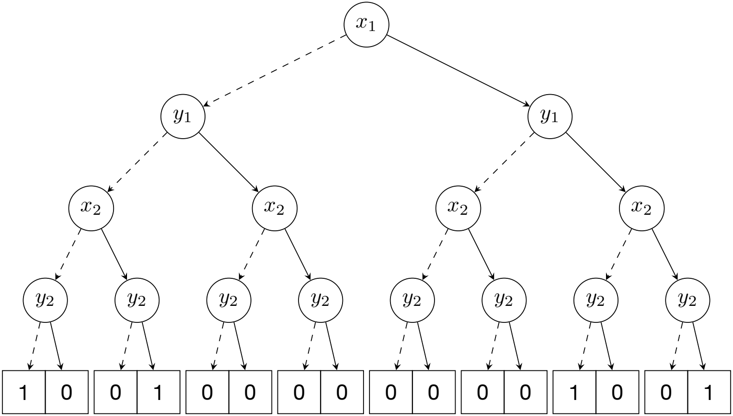

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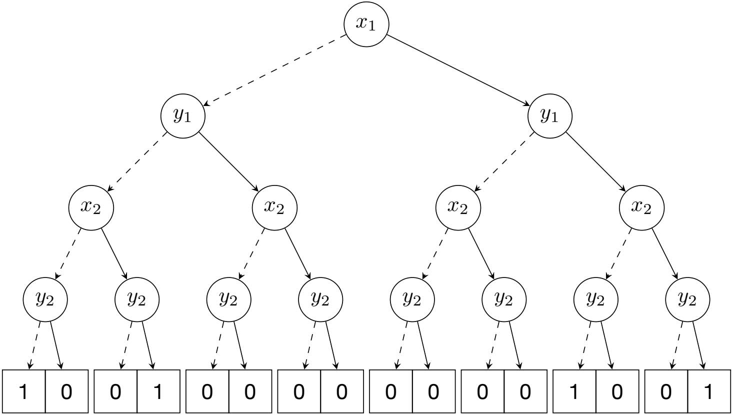
These are called **Ordered Binary Decision Diagrams** (OBDDs)

Ordered Binary Decision Trees



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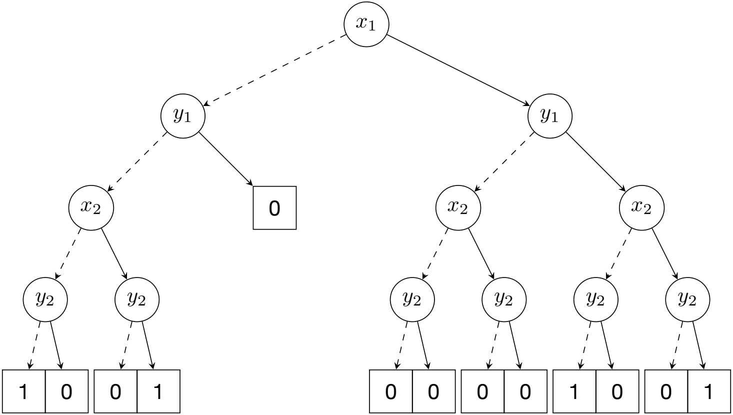

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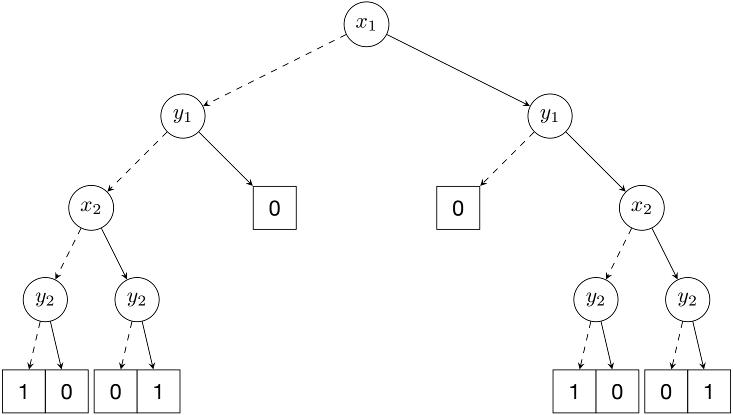

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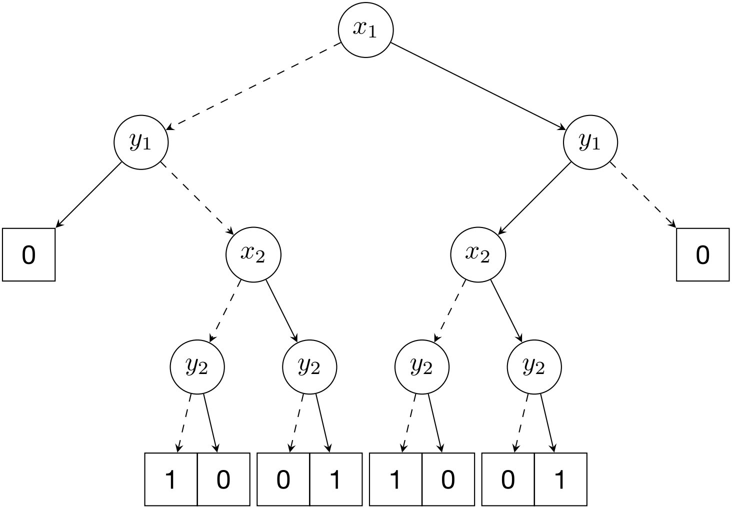

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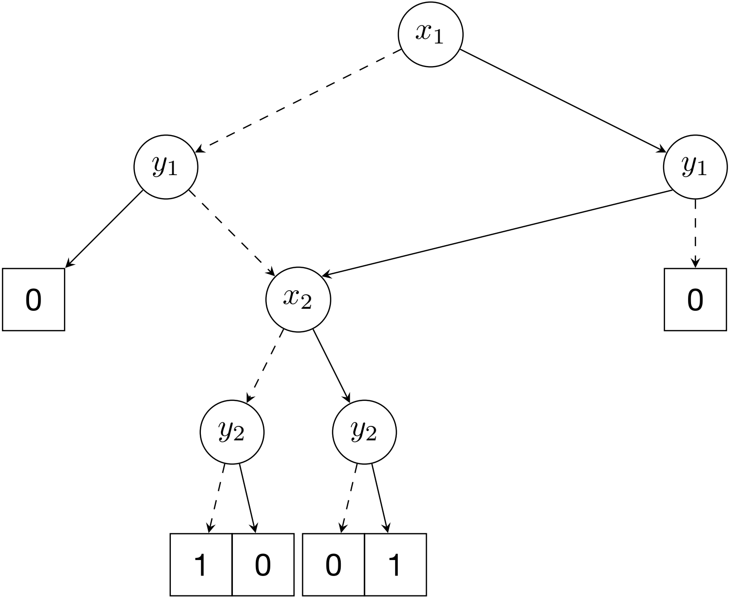

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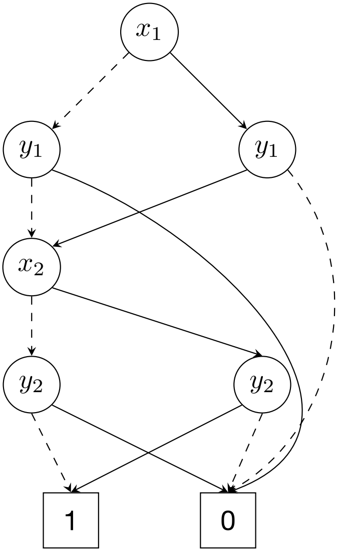

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- $\sim$ order of magnitude savings on many real examples

Introduction to Spin

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Tool you'll use for the final homework

Why Spin?

Mature implementation

1. Under development since 1980, freely-available since 1991
2. Winner of ACM Software Systems Award (others include Unix, TCP/IP, GCC, LLVM, `make`, ...)
3. Lots of real applications and successes (see previous slide)
4. Several projects extend Spin with frontends and other utilities
5. Based on concepts we've covered: ω-automata and LTL

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Good documentation

1. Several books (see Holzmann 2003, Ben-Ari 2008)
2. Annual workshops since 1995
3. Used extensively in other courses
4. Google turns up many hits when looking for specific info

Spin

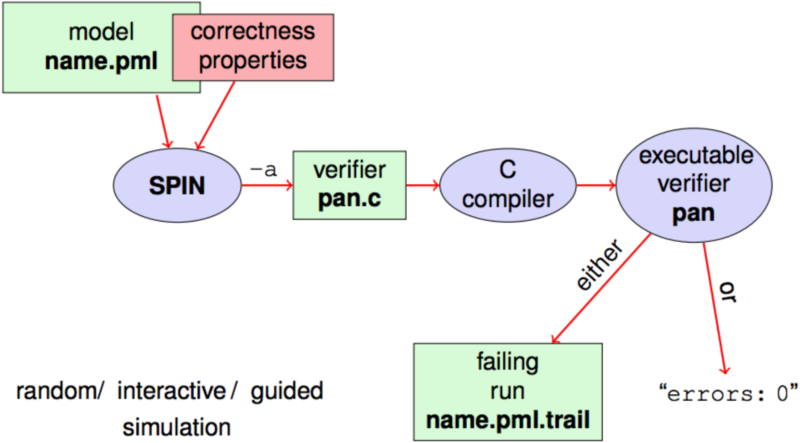

Image credit: Bernhard Beckert and Vladimir Klebanov

Promela

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Not an implementation language

- No libraries
- No pointers
- No standard input
- ...

Promela: Hello World

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active proctype P() {
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To run:

```
> spin hellow.pml
 Hello world!
```

Data types

```
bit                         {0,1}
bool                        {0,1}
byte                      [0..255]
short              [-2^15..2^15-1]
int                [-2^31..2^31-1]

#define N 10
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```
if
:: (a == b) -> state = state + 1
:: else -> state = state - 1
fi

if
:: x = 0
:: x = 1
fi
```

Blocking

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If an expression-statement evaluates to 0, then it is blocked

```
byte state = 1;

proctype A()
{ byte tmp;
  (state==1) -> tmp = state; tmp = tmp+1; state = tmp
}

proctype B()
{ byte tmp;
  (state==1) -> tmp = state; tmp = tmp-1; state = tmp
}

init
{ run A(); run B()
}
```

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```
proctype Euclid(int x, y)
  {
    do
    :: (x > y) -> x = x - y
    :: (x < y) -> y = y - x
    :: (x == y) -> break
    od;
  }
```

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```
mtype = {ack, err, accept};

chan c1 = [16] of { mtype };          // store up to 16 messages
chan c2 = [16] of { int, mtype };     // two fields per message

// rendez-vous channel for synchronous communication
// size 0: can transmit but not store a message
chan port = [0] of { short };
```

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- Reads the head of `channel` into `var`

Process Communications

Sending a message: `channel!expr`

- Can specify multiple fields with `channel!expr1,expr2`
- Appends the value of `expr` to the end of `channel`
- If `channel` is full, statement blocks

Receiving a message: `channel?var`

- Can specify multiple fields with `channel?expr1,expr2`
- Reads the head of `channel` into `var`
- If `channel` is empty, statement blocks

Process Communications

Sending a message: `channel!expr`

- Can specify multiple fields with `channel!expr1,expr2`
- Appends the value of `expr` to the end of `channel`
- If `channel` is full, statement blocks

Receiving a message: `channel?var`

- Can specify multiple fields with `channel?expr1,expr2`
- Reads the head of `channel` into `var`
- If `channel` is empty, statement blocks

The expression `len(channel)` returns # of messages on `channel`

Channels: Example

```
#define msgtype 33

chan name = [0] of { byte, byte };

active proctype A()
{ name!msgtype,124;
  // synchronous channel, no second receive in B
  // process will block here forever
  name!msgtype,121;
}

active proctype B()
{ byte state;
  name?msgtype(state)
}
```

Atomicity

Basic statements execute atomically

- Assignments, expressions, `goto`, `skip`

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Guarded commands are **not atomic**

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- Assignments, expressions, `goto`, `skip`

Guarded commands are **not atomic**

```
int a, b, c;

active proctype P1() {
  a = 1; b = 5;
  if
  :: a != 0 -> c = b / a;  // this can be #div0!
  :: else -> c = b;
  fi
}

active proctype P2() {
  a = 0;
}
```

Atomicity

Use an `atomic` block to prevent bad interleavings

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```
int a, b, c;

active proctype P1() {
  a = 1; b = 5;
  atomic {
    if
    :: a != 0 -> c = b / a;
    :: else -> c = b;
    fi
  }
}

active proctype P2() {
  a = 0;
}
```

Stating Correctness Properties: `assert`

Option 1: `assert` statements

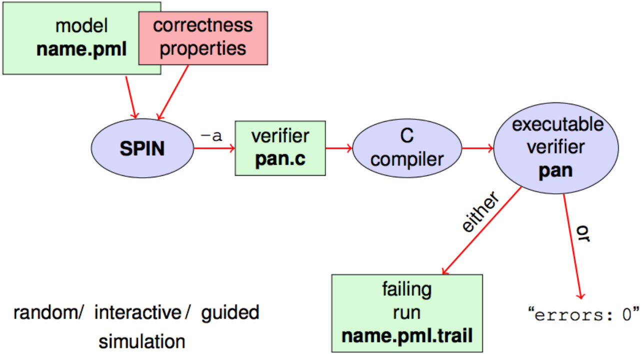
```
bool flag[2];
bool turn;
byte cnt = 0;

active [2] proctype user()
{
  flag[_pid] = true;
  turn = _pid;
  (flag[1-_pid] == false || turn == 1-_pid);

  cnt++;
crit: assert(cnt == 1);  // critical section
  cnt--;

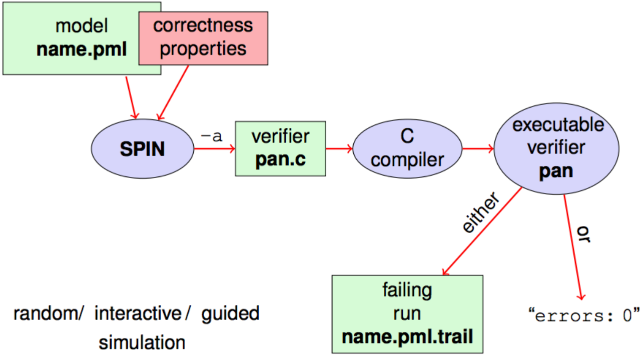
  flag[_pid] = false;
}
```

Checking the property



random/ interactive / guided
simulation

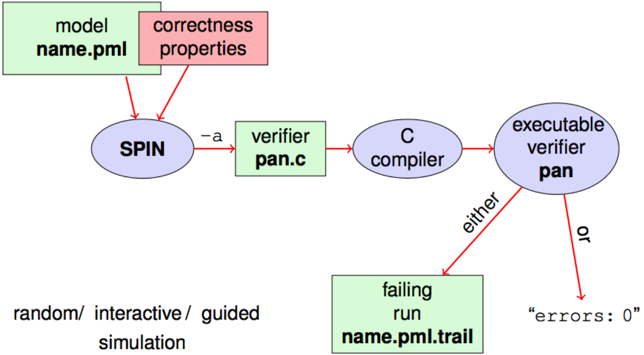
Checking the property



Step 1: Generate a verifier

```
> spin -a mutex.pml          // spin generates pan.c
```

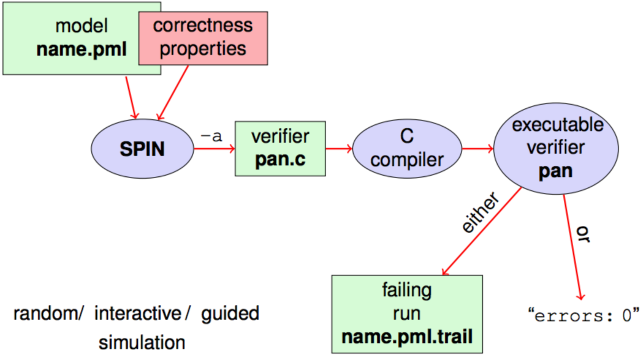
Checking the property



Step 2: Compile the verifier

```
> gcc -o pan pan.c        // output in pan
```

Checking the property



Step 3: Run the verifier to do exhaustive model checking

```
> ./pan
```

Verification Results

```
(Spin Version 6.4.5 -- 1 January 2016)
  + Partial Order Reduction

Full statespace search for:
  never claim           - (none specified)
  assertion violations  +
  acceptance   cycles   - (not selected)
  invalid end states  +

State-vector 28 byte, depth reached 16, errors: 0
       56 states, stored
       21 states, matched
       77 transitions (= stored+matched)
        0 atomic steps
hash conflicts:         0 (resolved)

Stats on memory usage (in Megabytes):
    0.003 equivalent memory usage for states
    0.292 actual memory usage for states
  128.000 memory used for hash table (-w24)
    0.534 memory used for DFS stack (-m10000)
  128.730 total actual memory usage

unreached in proctype user
  (0 of 8 states)
```

Stating Correctness Properties: LTL

Option 2: Write an LTL formula

```
bool flag[2];
bool turn;
byte cnt = 0;

active [2] proctype user()
{
  flag[_pid] = true;
  turn = _pid;
  (flag[1-_pid] == false || turn == 1-_pid);

crit: skip;  // critical section

  flag[_pid] = false;
}

ltl mutex { [] (!p[0]@crit || !p[1]@crit) }
```

LTL in Spin

```
Grammar:
  ltl ::= opd | ( ltl ) | ltl binop ltl | unop ltl

Operands (opd):
  true, false, user-defined names starting with a lower-case letter,
  or embedded expressions inside curly braces, e.g.,: { a+b>n }.

Unary Operators (unop):
  []  (the temporal operator always)
  <>  (the temporal operator eventually)
  !   (the boolean operator for negation)

Binary Operators (binop):
  U   (the temporal operator strong until)
  W (the temporal operator weak until
  V   (the dual of U): (p V q) means !(!p U !q))
  &&  (the boolean operator for logical and)
  ||  (the boolean operator for logical or)
  /\  (alternative form of &&)
  \/  (alternative form of ||)
  ->  (the boolean operator for logical implication)
  <-> (the boolean operator for logical equivalence)
```

Counterexamples

Let's introduce the bug from the previous homework

```
bool flag[2];
bool turn;
byte cnt = 0;

active [2] proctype user()
{
  turn = _pid;
  flag[_pid] = true;
  (flag[1-_pid] == false || turn == 1-_pid);

crit: skip;  // critical section

  flag[_pid] = false;
}

ltl mutex { [] (!p[0]@crit || !p[1]@crit) }
```

Generating counterexmaples

```
> spin -a mutex.pml; gcc -o pan pan.c; ./pan
> spin -t -p -l mutex.pml

using statement merging
  1:  proc  1 (user:1) mutex.pml:8 (state 1)  [turn = _pid]
  2:  proc  0 (user:1) mutex.pml:8 (state 1)  [turn = _pid]
  3:  proc  0 (user:1) mutex.pml:9 (state 2)  [flag[_pid] = 1]
  4:  proc  0 (user:1) mutex.pml:10 (state 3) [(((flag[(1-_pid)]==0)||(
       turn==(1-_pid))))]
  5:  proc  1 (user:1) mutex.pml:9 (state 2)  [flag[_pid] = 1]
  6:  proc  1 (user:1) mutex.pml:10 (state 3) [(((flag[(1-_pid)]==0)||(
       turn==(1-_pid))))]
  7:  proc  1 (user:1) mutex.pml:12 (state 4) [cnt = (cnt+1)]
  8:  proc  1 (user:1) mutex.pml:13 (state 5) [assert((cnt==1))]
  9:  proc  0 (user:1) mutex.pml:12 (state 4) [cnt = (cnt+1)]
spin: mutex.pml:13, Error: assertion violated
spin: text of failed assertion: assert((cnt==1))
 10:  proc  0 (user:1) mutex.pml:13 (state 5) [assert((cnt==1))]
spin: trail ends after 10 steps
#processes: 2
    flag[0] = 1
    flag[1] = 1
    turn = 0
    cnt = 2
 10:  proc  1 (user:1) mutex.pml:14 (state 6)
 10:  proc  0 (user:1) mutex.pml:14 (state 6)
2 processes created
```

Generating counterexmaples

```
> spin -t -p -l mutex.pml
```

Generating counterexmaples

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- Failed verification produces `mutex.pml.trail`

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- Basically, inject the discovered fault into execution

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- Basically, inject the discovered fault into execution
- `-p` option prints all statements in the execution

Generating counterexmaples

```
> spin -t -p -l mutex.pml
```

- Failed verification produces `mutex.pml.trail`
- `-t` option tells Spin to use `mutex.pml.trail` to guide simulation
- Basically, inject the discovered fault into execution
- `-p` option prints all statements in the execution
- `-l` option prints the values of local variables

Next Lecture

Last assignment goes out today

Due at midnight on last day of classes

Next class: Software Model Checking