Automated Program Verification and Testing 15414/15614 Fall 2016 Lecture 20: Explicit-State Model Checking, Part 1

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Useful Temporal Properties (Review)

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These are inherently linear-time properties

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If ϕ is a propositional formula over P, then the LTL formula

$$\mathbf{G}\ \phi$$

is an invariant property

Example: Mutual Exclusion (Review)

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Think of a bad prefix as a witness of a violating instance

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- Liveness properties can only be violated in **infinite time**

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This implies that there are no finite witnesses for liveness

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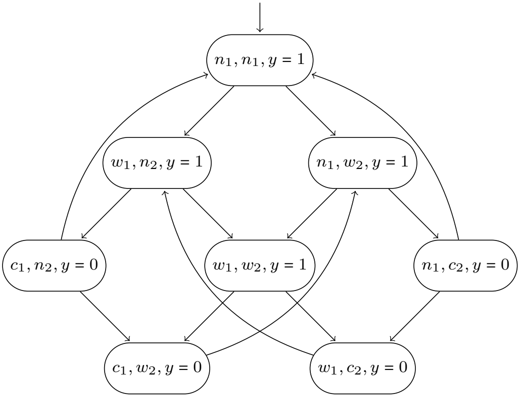
Any finite prefix can always be extended to satisfy these properties

Liveness: “Repeated Eventually”



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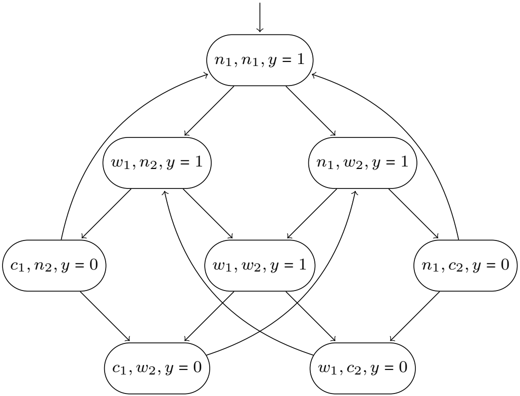

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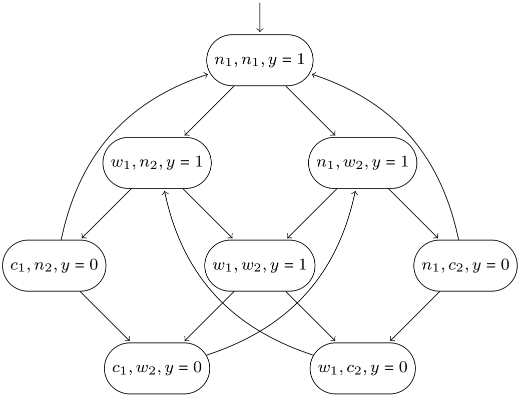

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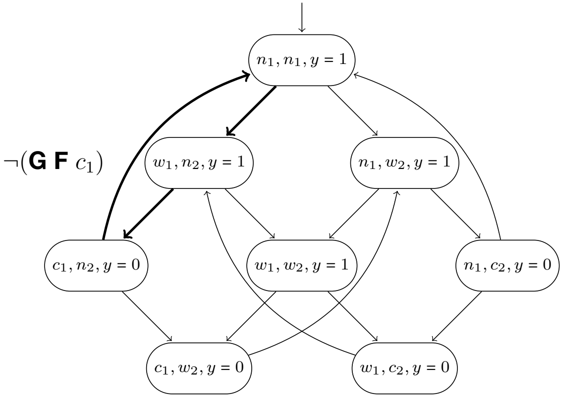

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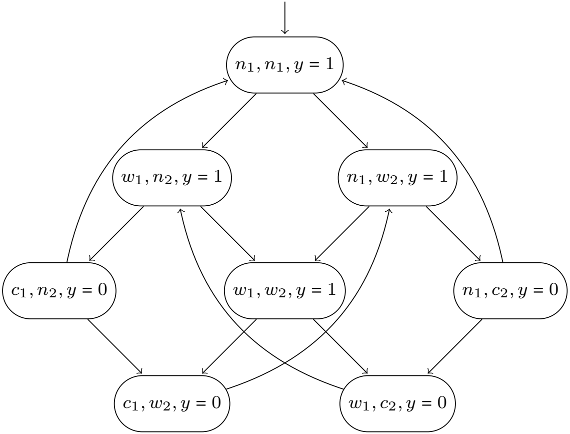

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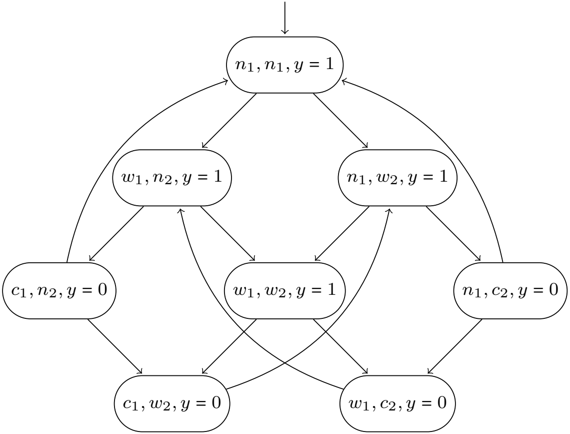

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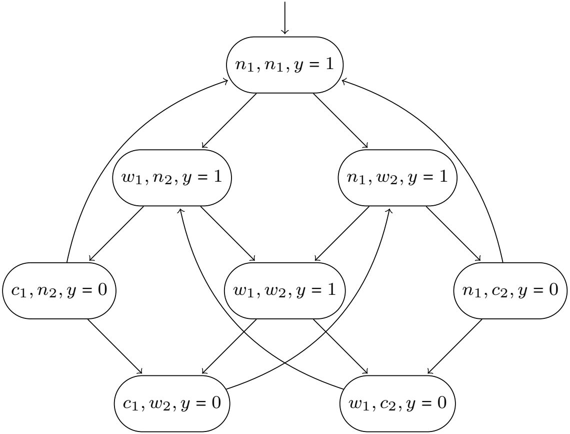

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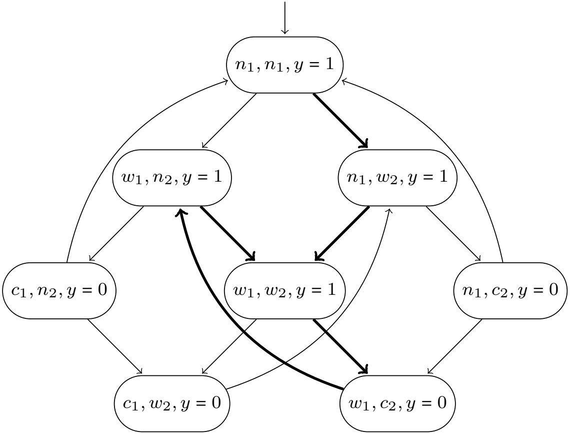

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Fairness constraints allow us to do this

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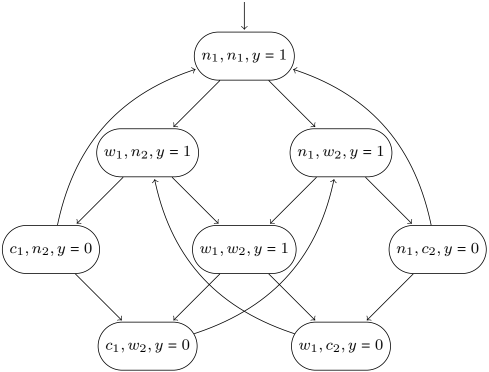
"Enabled" means "able to execute" a particular transition

Example: Weak Fairness

Consider the mutual exclusion example

A process is **enabled** if able to enter its critical section immediately



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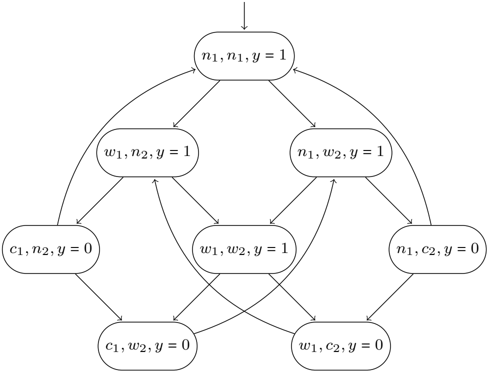

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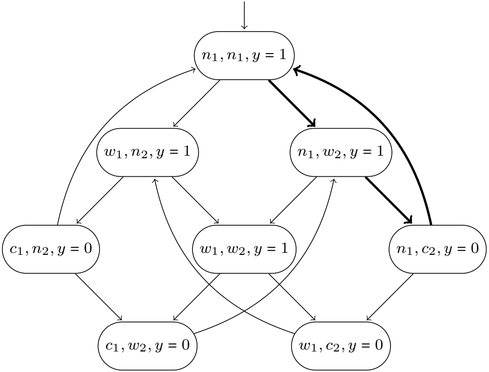

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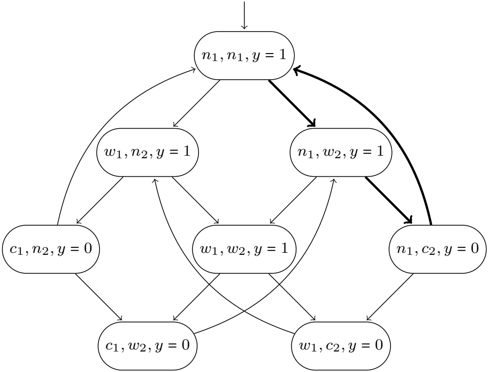

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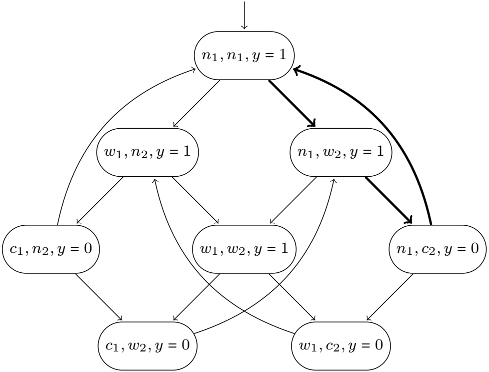

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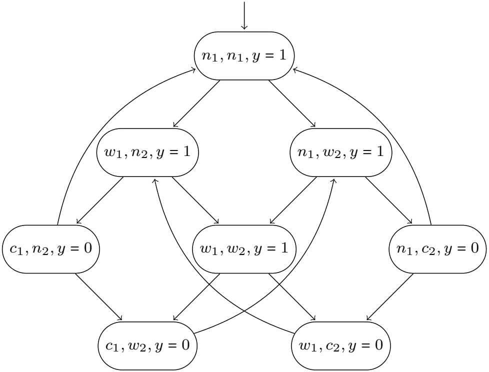

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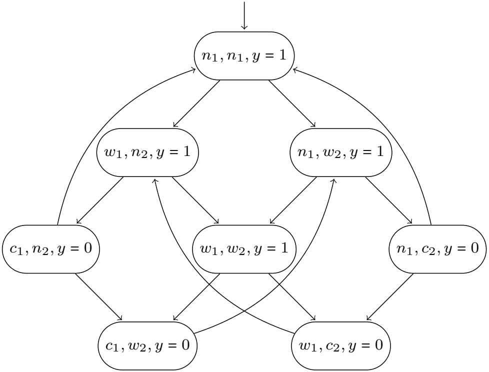

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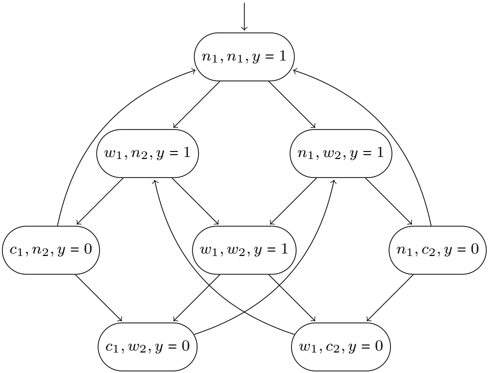

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So a strongly-fair scheduler lets both execute infinitely often

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In fact, we can say that:

Unconditional Fairness $\Longrightarrow$ Strong Fairness $\Longrightarrow$ Weak Fairness

Checking CTL

CTL Model Checking Problem

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Sometimes called "global" model checking

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- *Sat*$(\neg\phi) = S \setminus$ *Sat*(ϕ)

Satisfiable Set: *Sat*(ϕ)

To reduce the number of cases, we'll assume a normal form

Existential Normal Form (ENF)

$$\phi ::= \textit{true} \mid \phi_1 \wedge \phi_2 \mid \neg\phi \mid \textbf{EX}\ \phi \mid \textbf{E}\ (\phi_1\ \textbf{U}\ \phi_2) \mid \textbf{EG}\ \phi$$

Exercise: Convince yourself we can rewrite formulas in ENF

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We say T is a **greatest fixed point solution** of the above equation

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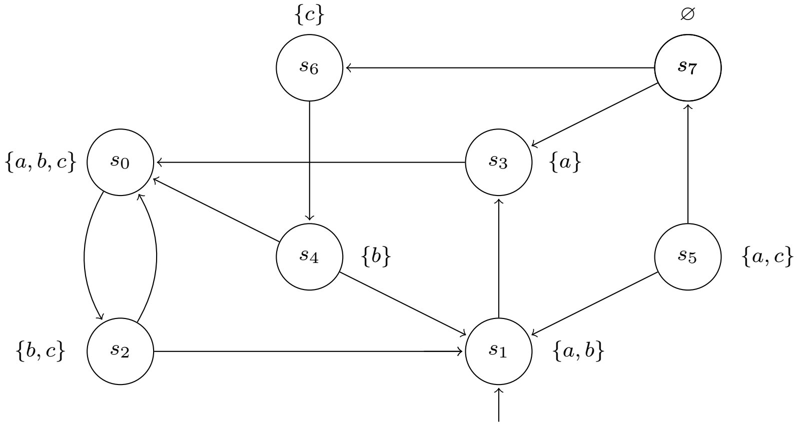

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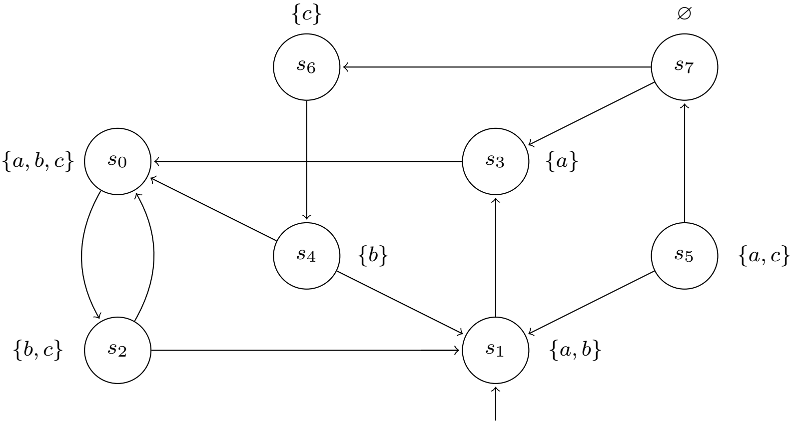

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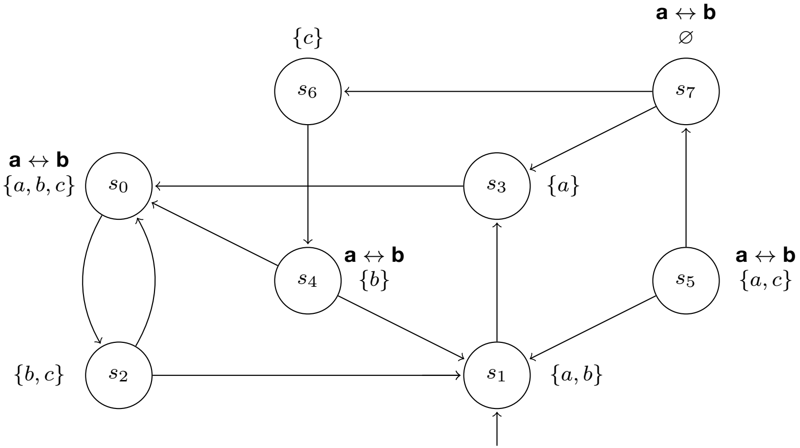

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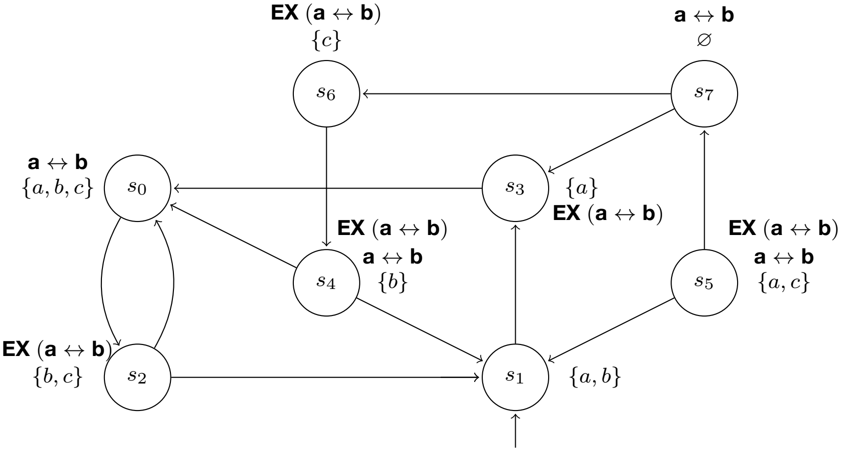

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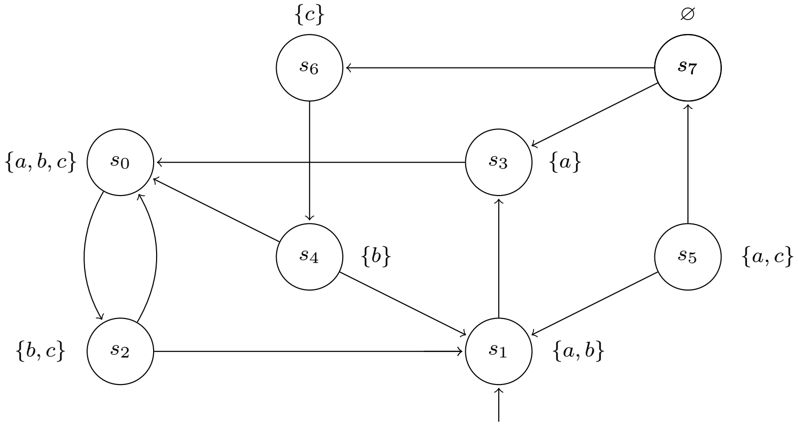
Consider the formula **E** $((b \rightarrow a)$ **U** $c)$



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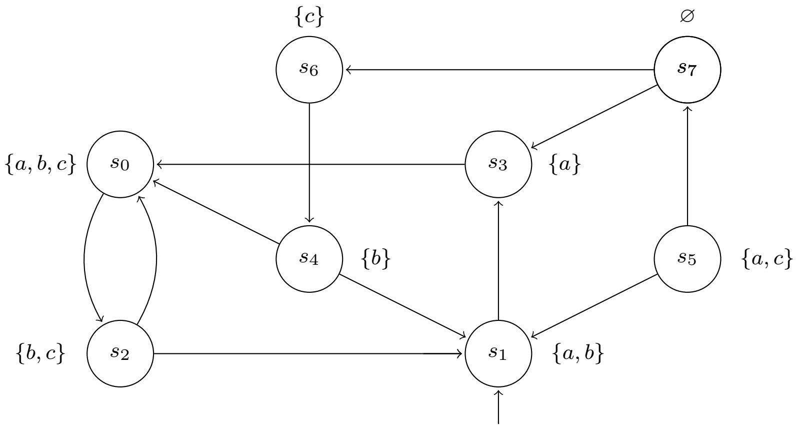

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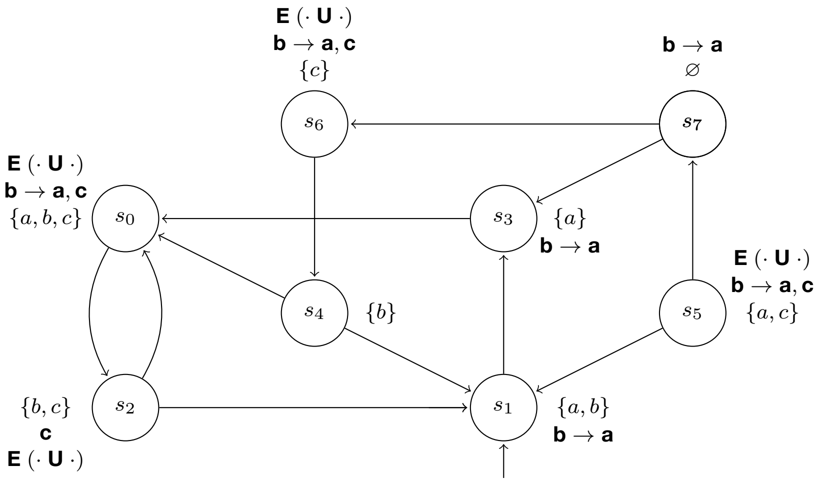

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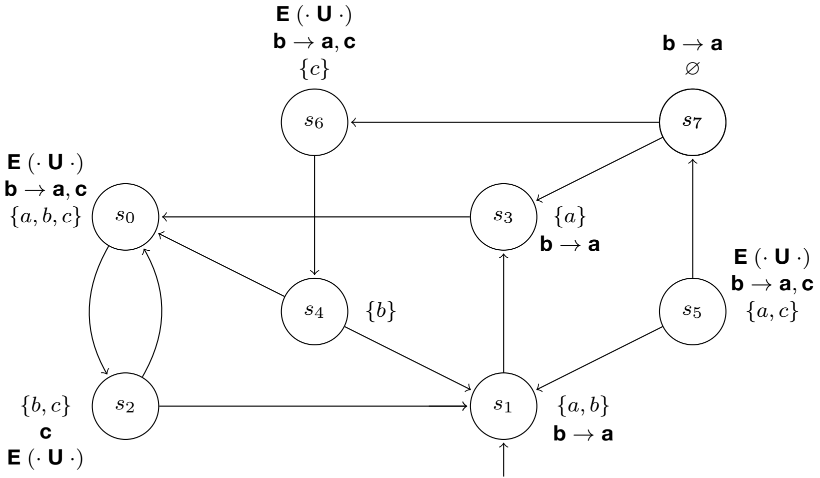
Then work backwards from states labeled **E** $((b \to a)$ **U** $c)$



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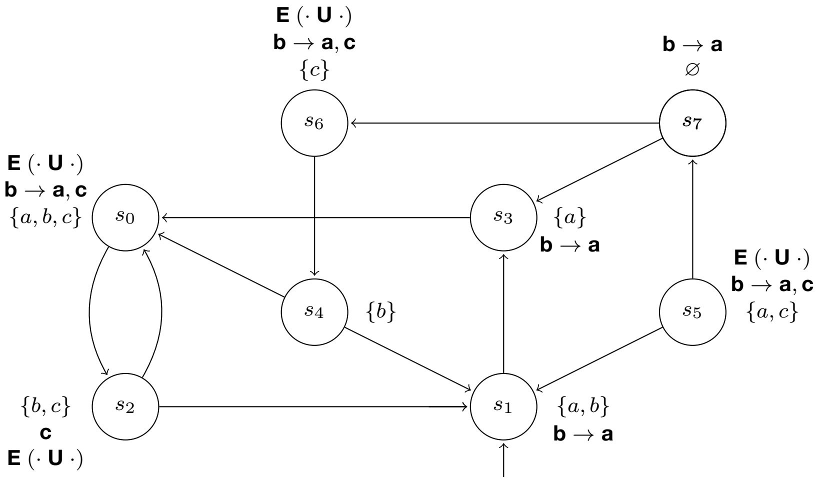
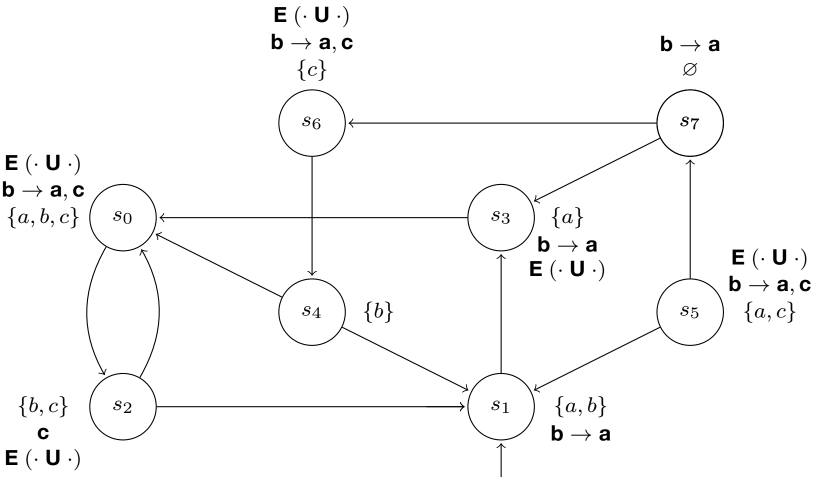
Start with s_0



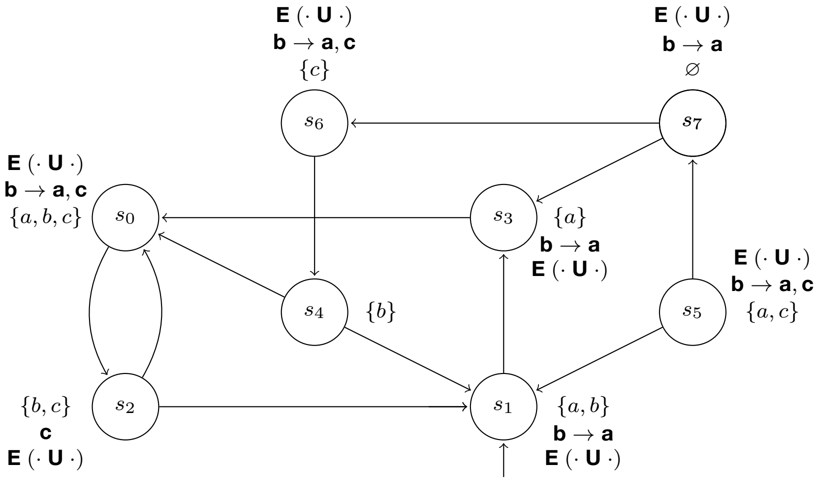
Example

Start with s_0 Then s_3



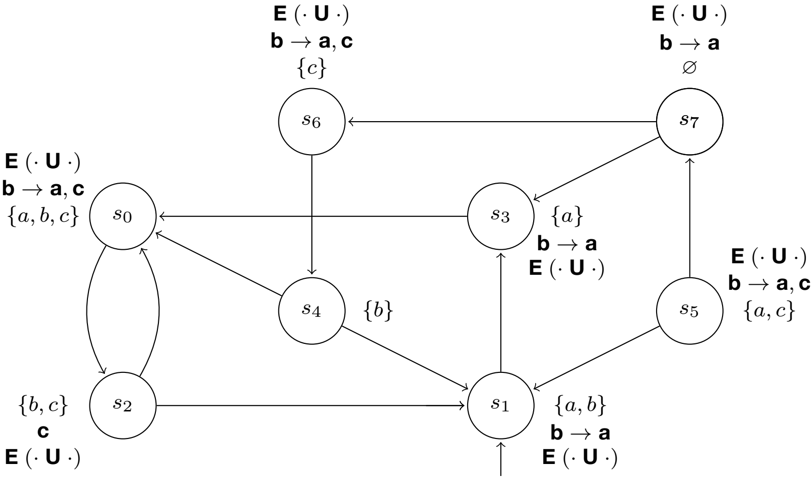
Example

Then s_3



Example

Nothing left to label but s_4; it isn't in *Sat*



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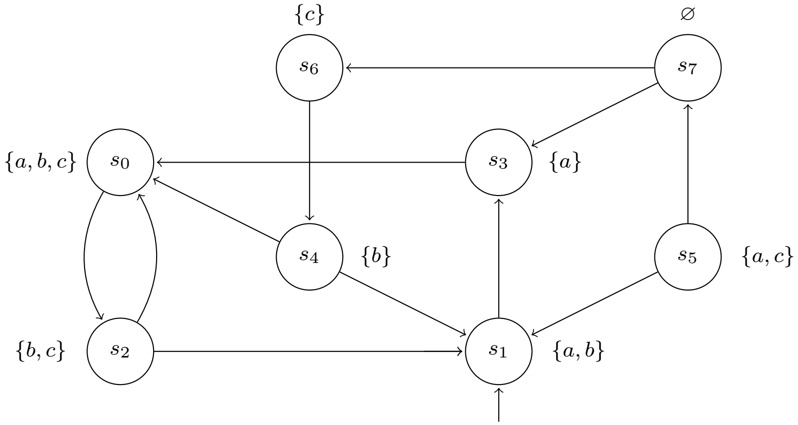

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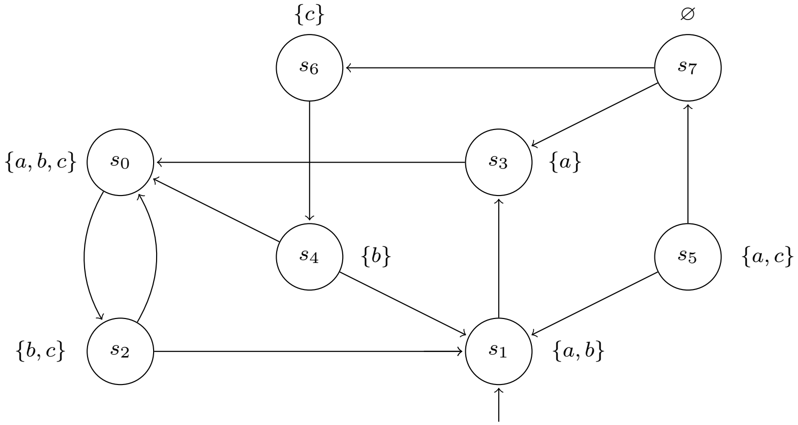

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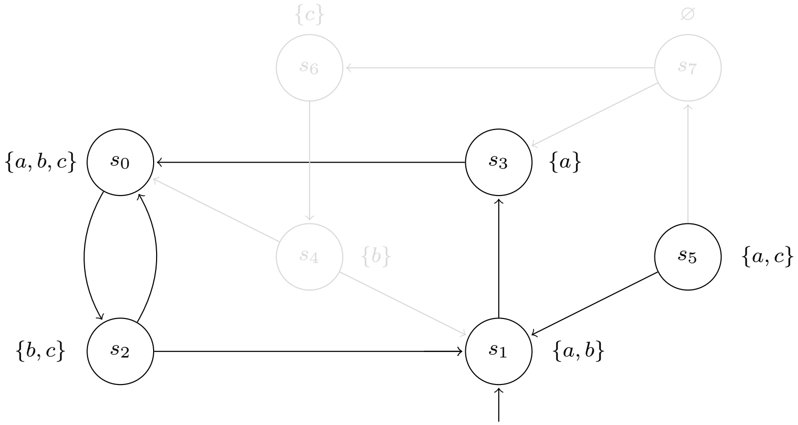

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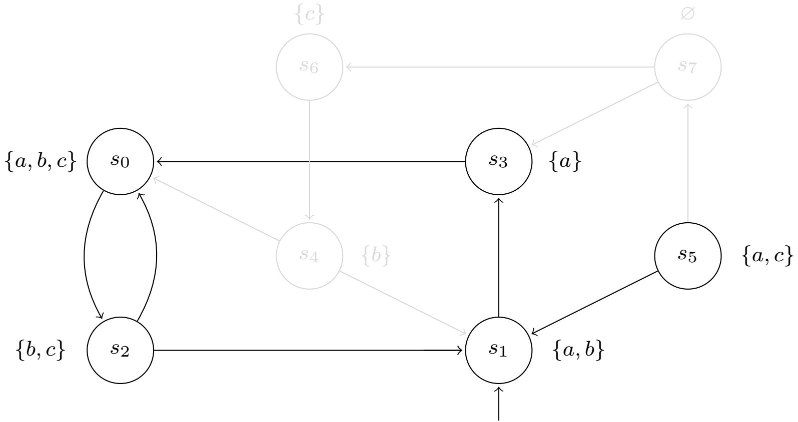

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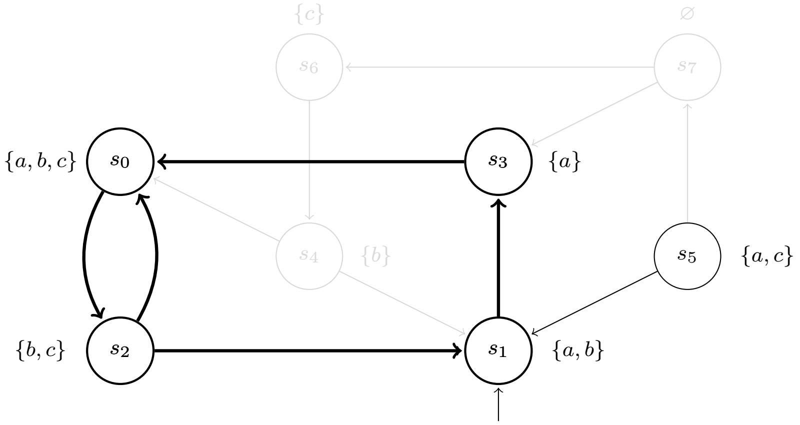

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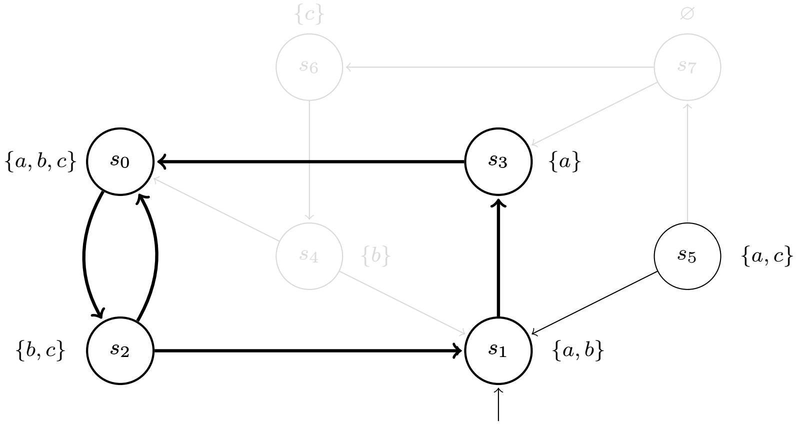

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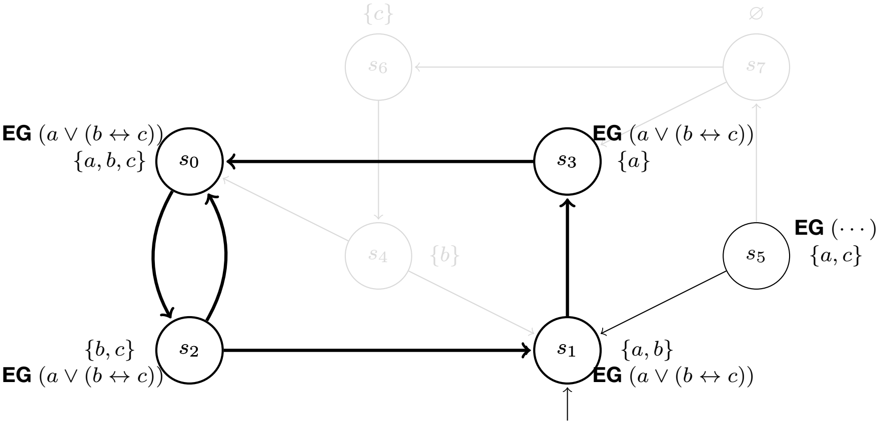

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Therefore, checking CTL is done in time $O(|\phi| \cdot (|S| + |R|))$

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- In this case, the answer "no" might suffice

Counterexamples in CTL

When $M, s \not\models \phi$, we want a **counterexample**

In general, these are path prefixes that refute ϕ

For example, a counterexample of **AF** ϕ is a path π:

- Beginning with a sequence of $\neg\phi$ states
- Ending in a single cycle traversal (with $\neg\phi$)

What is a counterexample for a formula beginning in **E** ?

- In this case, the answer "no" might suffice
- When **E** ϕ holds, we can provide a **witness**

Counterexamples and Witnesses: **X** operator

This is the simplest case

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Counterexamples and Witnesses: **U** operator

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Witnesses are found by looking for cycles that satisfy ϕ, backward search

Next Lecture

- Continue discussing model checking
- Infinite automata
- LTL model checking
- Dealing with state explosion