

# Automated Program Verification and Testing

## 15414/15614 Fall 2016

### Lecture 14:

### Deductive Verification, Part 2

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October 18, 2016

# VC Generation (Review)

Given an assertion  $Q$  and program  $c$ , we'll describe a function:

- ▶ That is a **predicate transformer**: produces another assertion
- ▶ Assertion for the corresponding precondition  $P$  for  $c$
- ▶  $P$  guaranteed to be the **weakest** such assertion

This is the **weakest precondition** predicate transformer  $\text{wp}(c, Q)$

The weakest precondition satisfies the following conditions:

1. The triple  $[\text{wp}(c, Q)] \ c \ [Q]$  is valid
2. For any  $P$  where  $[P] \ c \ [Q]$  is valid,  $P \Rightarrow \text{wp}(c, Q)$

For partial correctness, use **weakest liberal precondition**  $\text{wlp}(c, Q)$

# Weakest Liberal Precondition (Review)

$$\begin{aligned}\mathbf{wlp}(x := a, Q) &= Q[a/x] \\ \mathbf{wlp}(x[a_1] := a_2, Q) &= Q[a \langle a_1 \triangleleft a_2 \rangle / x] \\ \mathbf{wlp}(c_1; c_2, Q) &= \mathbf{wlp}(c_1, \mathbf{wlp}(c_2, Q)) \\ \mathbf{wlp}(\mathbf{if } b \mathbf{ then } c_1 \mathbf{ else } c_2, Q) &= (b \rightarrow \mathbf{wlp}(c_1, Q)) \wedge (\neg b \rightarrow \mathbf{wlp}(c_2, Q))\end{aligned}$$

# Approximate Weakest Precondition (Review)

In general, we can't always compute wlp for loops

Instead, we'll **approximate** it with help from annotations

Now we'll assume loops have the syntax:

**while**  $b$  **do**  $\{I\}$   $c$

$I$  is a loop invariant provided by the programmer

The approximate wlp for **while** will still be a valid precondition

But it may not be the weakest precondition: even if

$\{P\}$  **while**  $b$  **do**  $c$   $\{Q\}$

is valid, it might not be that:

$P \Rightarrow \text{wlp}(\text{while } \{I\} b \text{ do } c, Q)$

# Verification Conditions: While Loop (Review)

If we define

$$\text{wlp}(\mathbf{while } \{I\} b \mathbf{do } c, Q) = I$$

Then we still need to show that

- $I \wedge \neg b$  establishes  $Q$
- $I$  is a loop invariant

Defining the set of verification conditions,

$$\text{vc}(\mathbf{while } \{I\} b \mathbf{do } c, Q) = \left\{ \begin{array}{l} I \wedge \neg b \Rightarrow Q \\ I \wedge b \Rightarrow \text{wlp}(c, Q) \end{array} \right\}$$

To summarize, for  $Q$  to hold after executing a loop:

1. Each formula in  $\text{vc}(\mathbf{while } \{I\} b \mathbf{do } c, Q)$  must be valid
2.  $\text{wlp}(\mathbf{while } \{I\} b \mathbf{do } c, Q) = I$  must be valid

# Propagating Verification Conditions (Review)

**while** is the only command that introduces new conditions

But other statements might contain loops

Need to define  $vc$  for them as well:

- ▶  $vc(x := a, Q) = \emptyset$
- ▶  $vc(c_1; c_2, Q) = vc(c_1, \text{wlp}(c_2, Q)) \cup vc(c_2, Q)$
- ▶  $vc(\text{if } b \text{ then } c_1 \text{ else } c_2, Q) = vc(c_1, Q) \cup vc(c_2, Q)$

In short, compound statements collect conditions from constituents

# Verification Using wlp

Bringing all of this together, we can verify

$$\{P\} \ c \ \{Q\}$$

for an annotated program  $c$

1. Compute  $P' = \text{wlp}(c, Q)$
2. Compute  $\text{vc}(c, Q)$
3. Check validity of  $P \rightarrow P'$
4. Check validity of each  $F \in \text{vc}(c, Q)$

If (3) and (4) pass, then  $\{P\} \ c \ \{Q\}$  is valid

If  $\{P\} \ c \ \{Q\}$  is valid, then will (3) and (4) pass?

**No.** Loop invariants might be too weak!

# Example

Let's verify the example from last lecture:

```
{true}  
r := x; q := 0;  
while y ≤ r do  
    r := r - y; q := q + 1  
    {r < y ∧ x = r + (q × y)}
```

Recall our loop invariant:

```
{true}  
r := x; q := 0;  
while y ≤ r do  
    {x = r + (q × y)}  
    r := r - y; q := q + 1  
    {r < y ∧ x = r + (q × y)}
```

# Example

Define the following shorthand:

► $c_1 : r := x$	$\{true\}$
► $c_2 : q := 0$	$c_1; c_2;$
► $c_3 : r := r - y$	<b>while</b> $y \leq r$ <b>do</b>
► $c_4 : q := q + 1$	$\{x = r + (q \times y)\}$
► $c_5 : \mathbf{while} \ y \leq r \ \mathbf{do} \ c_3; c_4$	$c_3; c_4$ $\{r < y \wedge x = r + (q \times y)\}$

We need to show these are valid:

$$true \Rightarrow \text{wlp}(c_1; c_2; c_5, r < y \wedge x = r + (q \times y))$$

$$\text{vc}(c_1; c_2; c_5, r < y \wedge x = r + (q \times y))$$

We'll start with  $true \Rightarrow \text{wlp}(c_1; c_2; c_5, r < y \wedge x = r + (q \times y))$

# Example

$$\mathbf{true} \Rightarrow \text{wlp}(c_1; c_2; c_5, r < y \wedge x = r + (q \times y))$$

Let's use  $Q : r < y \wedge x = r + (q \times y)$ ,  $I : x = r + (q \times y)$

We begin by applying the rule for composition twice:

$$\text{wlp}(c_1; c_2; c_5, Q) = \text{wlp}(c_1, \text{wlp}(c_2, \text{wlp}(c_5, Q)))$$

This brings us to  $\text{wlp}(c_5, Q)$ :

$$\text{wlp}(\mathbf{while } y \leq r \mathbf{ do } \{I\} c_3; c_4, Q) = I$$

We also have verification conditions:

$$\text{vc}(c_5, Q) = \{I \wedge \neg b \Rightarrow Q, I \wedge b \Rightarrow \text{wlp}(c_3; c_4, Q)\}$$

# Example

Let's work out the VC  $I \wedge b \Rightarrow \text{wlp}(c_3; c_4, Q)$

We have that:

$$\begin{aligned}\text{wlp}(r := r - y; q := q + 1, Q) &= \text{wlp}(r := r - y, \text{wlp}(q := q + 1, Q)) \\ &= \text{wlp}(r := r - y, Q[q/q + 1]) \\ &= \text{wlp}(r := r - y, r < y \wedge x = r + ((q + 1) \times y)) \\ &= (x = (r - y) + ((q + 1) \times y))\end{aligned}$$

So, we have:

$$\text{vc}(c_5, Q) = \{I \wedge \neg b \Rightarrow Q, I \wedge b \Rightarrow (x = (r - y) + ((q + 1) \times y))\}$$

# Example

Recalling that  $\text{wlp}(c_5, Q) = I$ , we now need  $\text{wlp}(c_2, I)$ :

$$\begin{aligned}\text{wlp}(q := 0, x = r + (q \times y)) &= (x = r + (0 \times y)) \\ &= x = r\end{aligned}$$

Moving on, our final step is  $\text{wlp}(c_1, x = r)$ :

$$\text{wlp}(r := x, x = r) = (x = x)$$

Popping back to our top-level procedure:

1. Compute  $P' = \text{wlp}(c, Q)$

$$P' = (x = x)$$

2. Compute  $\text{vc}(c, Q)$

$$\text{vc}(c, Q) = \{I \wedge \neg b \Rightarrow Q, I \wedge b \Rightarrow (x = (r - y) + ((q + 1) \times y))\}$$

3. Check validity of  $P \rightarrow P'$

Clearly, *true*  $\Rightarrow (x = x)$

4. Check validity of each  $F \in \text{vc}(c, Q)$

# Example

Check validity of each  $F \in \text{vc}(c, Q)$ :

$$\text{vc}(c, Q) = \left\{ \begin{array}{l} x = r + (q \times y) \wedge \neg(y \leq r) \Rightarrow r < y \wedge x = r + (q \times y) \\ x = r + (q \times y) \wedge y \leq r \Rightarrow (x = (r - y) + ((q + 1) \times y)) \end{array} \right\}$$

The first is true because  $\neg(y \leq r) \Leftrightarrow r < y$

The second we get by algebraic calculation

Therefore, the triple is valid

```
{true}  
r := x; q := 0;  
while y ≤ r do  
    r := r - y; q := q + 1  
{r < y ∧ x = r + (q × y)}
```

# Imp: New Features

Now we'll add two new features to our language:

- ▶ Assertion annotations
- ▶ Procedure

Assertion annotations take the form:

$$\{P\}$$

Semantically, treat them like runtime assertions

Execution halts if the expression isn't true in current environment

Think of assertions as **formal comments**

# Imp: Procedures

```
proc LinearSearch(a : array, l : int, u : int, e : int)
  requires  $0 \leq l \wedge u < |a|$ 
  ensures (rv = 1)  $\leftrightarrow (\exists i. l \leq i \leq u \wedge a[i] = e)$ 
```

We'll consider programs with annotated procedures

- ▶ **Precondition** annotated with **requires**
- ▶ **Postcondition** annotated with **ensures**
- ▶ Free variables in the pre- and postconditions can be formal parameters
- ▶ Postcondition can also include special variable *rv*
- ▶ *rv* stands for the return value

# Proving Partial Correctness

Now, a program is partially correct if for each procedure  $P$ :

1. Whenever  $P$ 's preconditions are satisfied on entry
2.  $P$ 's postconditions are satisfied on exit

We'll extend the approach we've talked about so far

1. Reduce the annotated program to a set of verification conditions
2. If all VCs are valid, then the program is correct

Our approach will be different:

- ▶ Use annotations to decompose the program into simpler parts
- ▶ Generate VC for each part in isolation, assuming each annotation holds
- ▶ Make sure that correctness of the whole follows from correctness of each part

# Basic Paths

A **basic path** is a sequence of instructions that:

- ▶ Begins at procedure precondition or loop invariant
- ▶ Ends at a loop invariant, assertion, or procedure postcondition
- ▶ Doesn't cross loops: invariants only at beginning or end of path

Basic paths correspond to straight-line segments of code

Think of a Hoare triple over a sequence command:

$$\{P\} \ c_1; c_2; \dots; c_n \ \{Q\}$$

$P, Q$  are pre-/postconditions, loop invariants, or assertion guards

# Basic Paths: Example

```
proc LinearSearch(a : array, l, u, e)
  pre  $0 \leq l \wedge u < |a|$ 
  post  $(rv = 1) \leftrightarrow (\exists i. l \leq i \leq u \wedge a[i] = e)$ 
{
  i := l;
  while(i ≤ u)
    { $l \leq i \wedge \forall j. l \leq j < i \rightarrow a[j] \neq e$ }
    {
      if(a[i] = e) return 1;
      i := i + 1;
    }
  return 0; }
```

First basic path:

$$\begin{aligned} & \{0 \leq l \wedge u < |a|\} \\ & i := l \\ & \{l \leq i \wedge \forall j. l \leq j < i \rightarrow a[j] \neq e\} \end{aligned}$$

Second basic path:

$$\begin{aligned} & \{l \leq i \wedge \forall j. l \leq j < i \rightarrow a[j] \neq e\} \\ & \text{while} (i \leq u); \\ & \text{if} (a[i] = e); \\ & rv := 1; \\ & \{(rv = 1) \leftrightarrow (\exists i. l \leq i \leq u \wedge a[i] = e)\} \end{aligned}$$

# assume statement

$$\{l \leq i \wedge \forall j. l \leq j < i \rightarrow a[j] \neq e\}$$

**while**( $i \leq u$ );  
**if**( $a[i] = e$ );  
 $rv := 1$ ;  
 $\{(rv = 1) \leftrightarrow (\exists i. l \leq i \leq u \wedge a[i] = e)\}$

$$\{l \leq i \wedge \forall j. l \leq j < i \rightarrow a[j] \neq e\}$$

**assume**  $i \leq u$ ;  
**assume**  $a[i] = e$ ;  
 $rv := 1; \{(rv = 1) \leftrightarrow (\exists i. l \leq i \leq u \wedge a[i] = e)\}$

Guarded statements introduce **assumptions** about environment

We write these in basic paths using an **assume** statement

**assume**  $b$  means:

1. Rest of path executed only if  $b$  is true in current environment
2. When reasoning about rest of path, we can assume  $b$  holds

# assume: Path Splitting

Each guarded statement introduces two assumptions

One where **assume**  $b$  holds, one where **assume**  $\neg b$  does

Continuing with our previous basic path, this gives us the next:

$$\begin{array}{ll} \{l \leq i \wedge \forall j. l \leq j < i \rightarrow a[j] \neq e\} & \{l \leq i \wedge \forall j. l \leq j < i \rightarrow a[j] \neq e\} \\ \mathbf{assume} \ i \leq u; & \mathbf{assume} \ i \leq u; \\ \mathbf{assume} \ a[i] = e; & \mathbf{assume} \ a[i] \neq e; \\ \mathit{rv} := 1; & i := i + 1; \\ \{(\mathit{rv} = 1) \leftrightarrow (\exists i. l \leq i \leq u \wedge a[i] = e)\} & \{l \leq i \wedge \forall j. l \leq j < i \rightarrow a[j] \neq e\} \end{array}$$

And one final path:

$$\begin{array}{l} \{l \leq i \wedge \forall j. l \leq j < i \rightarrow a[j] \neq e\} \\ \mathbf{assume} \ i > u; \\ \mathit{rv} := 0; \\ \{(\mathit{rv} = 1) \leftrightarrow (\exists i. l \leq i \leq u \wedge a[i] = e)\} \end{array}$$

# Enumerating Basic Paths

When we enumerate basic paths, we'll follow a convention

Proceed depth-first through the program

When we encounter a guarded command:

1. Assume that it holds first, then generate the ensuing paths
2. Then assume it doesn't hold, proceed as before

# Basic Paths: Procedure Calls

Recall the postcondition summarizes the relationships between:

- ▶ The procedure's formal parameters
- ▶ The return value (special variable *rv*)

We replace procedure calls with postcondition assertions

But postcondition only holds when precondition is satisfied on entry

Introduce another basic path to ensure that the precondition holds

Replace formals in pre/postconditions with actuals appearing in call

# Example: Procedure Calls

```
proc BinarySearch(a : array, l, u, e)
  pre  $0 \leq l \wedge u < |a| \wedge \text{sorted}(a, l, u)$ 
  post (rv = 1)  $\leftrightarrow (\exists i. l \leq i \leq u \wedge a[i] = e)$ 
{
  if(l > u) return 0;
  else {
    m := (l + u)/2;
    if(a[m] = e) return 1;
    else if(a[m] < e) {
      return BinarySearch(a, m + 1, u, e);
    } else {
      return BinarySearch(a, l, m - 1, e);
    }
  }
}
```

# Example: Procedure Calls

## First basic path:

$$\{0 \leq l \wedge u < |a| \wedge \text{sorted}(a, l, u)\}$$

**assume**  $l > u$ ;

$rv := 0$ ;

$$\{(rv = 1) \leftrightarrow (\exists i. l \leq i \leq u \wedge a[i] = e)\}$$

## Second basic path:

$$\{0 \leq l \wedge u < |a| \wedge \text{sorted}(a, l, u)\}$$

**assume**  $l \leq u$ ;

$m := (l + u)/2$ ;

**assume**  $a[m] = e$ ;

$rv := 1$ ;

$$\{(rv = 1) \leftrightarrow (\exists i. l \leq i \leq u \wedge a[i] = e)\}$$

## Third basic path:

$$\{0 \leq l \wedge u < |a| \wedge \text{sorted}(a, l, u)\}$$

**assume**  $l \leq u$ ;

$m := (l + u)/2$ ;

**assume**  $a[m] \neq e$ ;

**assume**  $a[m] < e$ ;

$$\{0 \leq m + 1 \wedge u < |a| \wedge \text{sorted}(a, m + 1, u)\}$$

# Example: Procedure Calls

Fourth basic path:

$\{0 \leq l \wedge u < |a| \wedge \text{sorted}(a, l, u)\}$

**assume**  $l \leq u$ ;

$m := (l + u)/2$ ;

**assume**  $a[m] \neq e$ ;

**assume**  $a[m] < e$ ;

**assume**  $(rv = 1) \leftrightarrow (\exists i. m + 1 \leq i \leq u \wedge a[i] = e)$ ;

$rv := v_1$ ;

$\{(rv = 1) \leftrightarrow (\exists i. l \leq i \leq u \wedge a[i] = e)\}$

Fifth basic path:

$\{0 \leq l \wedge u < |a| \wedge \text{sorted}(a, l, u)\}$

**assume**  $l \leq u$ ;

$m := (l + u)/2$ ;

**assume**  $a[m] \neq e$ ;

**assume**  $a[m] \geq e$ ;

$\{0 \leq l \wedge m - 1 < |a| \wedge \text{sorted}(a, l, m - 1)\}$

# Example: Procedure Calls

Sixth basic path:

$\{0 \leq l \wedge u < |a| \wedge \text{sorted}(a, l, u)\}$

**assume**  $l \leq u$ ;

$m := (l + u)/2$ ;

**assume**  $a[m] \neq e$ ;

**assume**  $a[m] \geq e$ ;

**assume**  $(v_2 = 1) \leftrightarrow (\exists i. l \leq i \leq m - 1 \wedge a[i] = e)$ ;

$rv := v_2$ ;

$\{(rv = 1) \leftrightarrow (\exists i. l \leq i \leq u \wedge a[i] = e)\}$

# Summary: Procedure Calls

Given a procedure  $f$  with prototype

```
proc f( $x_1, \dots, x_n$ )
  pre  $P[x_1, \dots, x_n]$ 
  post  $Q[x_1, \dots, x_n, rv]$ 
```

When  $f$  is called in context  $w := f(e_1, \dots, e_n);$

Augment the calling context with an assertion:

$$\{P[e_1, \dots, e_n]\};$$
$$w := f(e_1/x_1, \dots, e_n/x_n);$$

In paths that pass through the call,

1. Create fresh variable  $v$  to hold the return value
2. Replace call with an assumption of the postcondition:

**assume**  $G[e_1/x_1, \dots, e_n/x_n, v/rv]$

As we enumerate basic paths, we generate verification conditions

**Notice:** We only need to generate VCs for three command types

1. Assignment: we do this exactly as we did before
2. Sequence: same as before
3. **assume.** Recall, with **assume** we said that only paths satisfying the expression proceed past the command.

$$\text{wlp}(\mathbf{assume} \ b, Q) = b \rightarrow Q$$

If  $b \rightarrow Q$  holds before, then satisfying  $b$  ensures that  $Q$  holds afterward

What about the “side conditions” we had for loops?

# VC Generation: Loops

```
proc f( $x_1, \dots, x_n$ )
  pre  $P$ 
  post  $Q$ 
  {
    while( $b$ )  $\{I\}$  {
       $c$ ;
    }
  }
```

$\{P\}$   
**skip**;  
 $\{I\}$

$\{I\}$   
**assume**  $b$ ;  
 $c$ ;  
 $\{I\}$

$\{I\}$   
**assume**  $\neg b$ ;  
 $\{Q\}$

# VC Generation: Loops

$\{P\}$   
**skip**;

$\{I\}$

$P \rightarrow I$

$\{I\}$   
**assume**  $b$ ;  
 $\{Q\}$

$\{I\}$   
**assume**  $b$ ;  
**c**;  
 $\{I\}$

$I \rightarrow \text{wlp}(\text{assume } \neg b, Q)$   
 $\Leftrightarrow$   
 $(I \wedge \neg b) \rightarrow Q$

$I \rightarrow \text{wlp}(\text{assume } b, \text{wlp}(c, I))$   
 $\Leftrightarrow$   
 $(I \wedge b) \rightarrow \text{wlp}(c, I)$

These are same conditions as before!

# Example: Linear Search (1)

Recall the first basic path:

$$\begin{aligned} & \{0 \leq l \wedge u < |a|\} \\ & i := l \\ & \{l \leq i \wedge \forall j. l \leq j < i \rightarrow a[j] \neq e\} \end{aligned}$$

The VC for this is:

$$0 \leq l \wedge u < |a| \rightarrow \text{wlp}(i := l, l \leq i \wedge \forall j. l \leq j < i \rightarrow a[j] \neq e)$$

We have that:

$$\begin{aligned} & \text{wlp}(i := l, l \leq i \wedge \forall j. l \leq j < i \rightarrow a[j] \neq e) \\ \Leftrightarrow & l \leq l \wedge \forall j. l \leq j < l \rightarrow a[j] \neq e \\ \Leftrightarrow & \text{true} \wedge \forall j. \text{false} \rightarrow a[j] \neq e \\ \Leftrightarrow & \text{true} \end{aligned}$$

Our final condition is valid:

$$0 \leq l \wedge u < |a| \Rightarrow \text{true}$$

## Example: Linear Search (2)

Recall the second basic path:

$$\begin{aligned} & \{P : l \leq i \wedge \forall j. l \leq j < i \rightarrow a[j] \neq e\} \\ & c_1 : \mathbf{assume} \ i \leq u; \\ & c_2 : \mathbf{assume} \ a[i] = e; \\ & c_3 : \mathbf{rv} := 1; \\ & \{Q : (\mathbf{rv} = 1) \leftrightarrow \exists j. l \leq j \leq u \wedge a[j] = e\} \end{aligned}$$

The VC for this is:

$$P \rightarrow \mathbf{wlp}(c_1; c_2; c_3, Q) \Leftrightarrow P \rightarrow \mathbf{wlp}(c_1, \mathbf{wlp}(c_2, \mathbf{wlp}(c_3, Q)))$$

We have that:

$$\begin{aligned} & \mathbf{wlp}(\mathbf{rv} := 1, (\mathbf{rv} = 1) \leftrightarrow \exists j. l \leq j \leq u \wedge a[j] = e) \\ \Leftrightarrow & (1 = 1) \leftrightarrow \exists j. l \leq j \leq u \wedge a[j] = e \\ \Leftrightarrow & \exists j. l \leq j \leq u \wedge a[j] = e \end{aligned}$$

Our final condition is:

$$0 \leq l \wedge u < |a| \Rightarrow \mathbf{true}$$

## Example: Linear Search (3)

Recall the second basic path:

$$\{P : l \leq i \wedge \forall j. l \leq j < i \rightarrow a[j] \neq e\}$$

$$c_1 : \mathbf{assume} \ i \leq u;$$

$$c_2 : \mathbf{assume} \ a[i] = e;$$

$$c_3 : \mathbf{rv} := 1;$$

$$\{Q : (\mathbf{rv} = 1) \leftrightarrow \exists j. l \leq j \leq u \wedge a[j] = e\}$$

The VC for this is:

$$P \rightarrow \mathbf{wlp}(c_1; c_2; c_3, Q) \Leftrightarrow P \rightarrow \mathbf{wlp}(c_1, \mathbf{wlp}(c_2, \mathbf{wlp}(c_3, Q)))$$

Moving on,

$$\begin{aligned} & \mathbf{wlp}(c_1, \mathbf{wlp}(\mathbf{assume} \ a[i] = e, \exists j. l \leq j \leq u \wedge a[j] = e)) \\ \Leftrightarrow & \mathbf{wlp}(c_1, a[i] = e \rightarrow \exists j. l \leq j \leq u \wedge a[j] = e) \\ \Leftrightarrow & \mathbf{wlp}(\mathbf{assume} \ i \leq u, a[i] = e \rightarrow \exists j. l \leq j \leq u \wedge a[j] = e) \\ \Leftrightarrow & i \leq u \rightarrow (a[i] = e \rightarrow \exists j. l \leq j \leq u \wedge a[j] = e) \end{aligned}$$

Our final condition is:

$$0 \leq l \wedge u < |a| \Rightarrow \mathbf{true}$$

## Example: Linear Search (4)

Our final condition is:

$$\begin{aligned} & (l \leq i \leq u \wedge \forall j. l \leq j < i \rightarrow a[j] \neq e) \rightarrow (a[i] = e \rightarrow \exists j. l \leq j \leq u \wedge a[j] = e) \\ \Leftrightarrow & (l \leq i \leq u \wedge a[i] = e \wedge \forall j. l \leq j < i \rightarrow a[j] \neq e) \rightarrow \exists j. l \leq j \leq u \wedge a[j] = e \end{aligned}$$

Notice that:

$$l \leq i \leq u \wedge a[i] = e \rightarrow \exists j. l \leq j \leq u \wedge a[j] = e$$

is valid

So, the condition is valid, and the corresponding triple is as well

# Next Class

- ▶ Mid-term on Thursday
- ▶ Check out mid-term guide posted on Blackboard
- ▶ Come to office hours with questions