Regret Minimization and the Price of Total Anarchy

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Overview

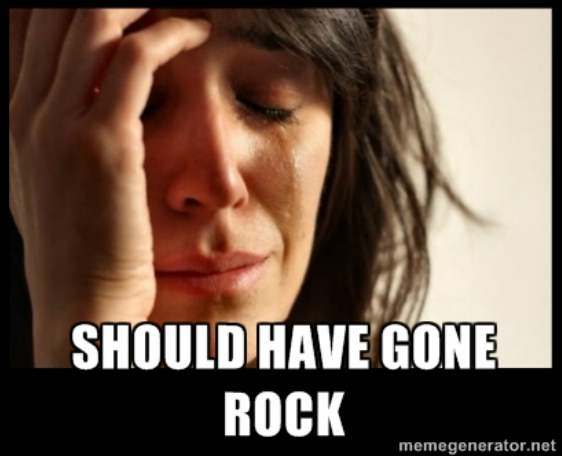
- Basic Terms
- The Cost of Disorganization
- Traditional Approach
- New Approach
- Results and Conclusions

Basic Terms

- Game
  - Players
  - Available moves
  - Payoff specifications
- Disorganized system
  - Limited information
  - Net social cost?

Basic Terms

- Nash Equilibrium
  - State in which no player wants to switch strategies
- Regret
  - Difference between a player's total payoff and the optimal t[illegible]spect



Example: Prisoner's Dilemma

	Confess	Don't Confess
Confess	**5 years - 5 years**	**0 years - 10 years**
Don't Confess	**10 years - 0 years**	**2 years - 2 years**

The Cost of Disorganization

- Social welfare: objective function
- What can we say about the net social cost when players act selfishly and with limited information?

Traditional Approach

- Traditional assumption:
  - Selfish players play according to Nash equilibria
  - All players act selfishly
- Price of Anarchy:
  - Ratio between optimum social value and the worst Nash equilibrium social value:

$$\frac{OPT}{NASH}$$

Traditional Approach

- Disadvantages:
  - Calculating Nash equilibria is expensive
  - Unclear that selfish players should play according to Nash equilbria

A New Approach

- Instead, assume that selfish players minimize their **regret**
- Do *not* assume that all players act selfishly
- Advantages:
  - Fast calculation
  - Reasonable for systems where a single player's decisions have little effect on others

New Approach: Motivation

- What makes for a good strategy?
  - Adversary can't counter
  - Extracts as much payoff as possible
- Winning as much as possible means adapting to opponent strategies.

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Regret minimization does this!

… But it has limitations

Regret Minimization

- Idea:
  - Start with a set of *experts* and weight each one's advice.
  - Adjust weights during each round
  - converge toward optimal payoff over time

Conclusions

- Under appropriately constrained systems, *regret minimization* is a reasonable prescription for self-interested behavior.
- For some games, can show results as strong as traditional approach.
- Can prove results that apply even in the presence of arbitrary or adversarial players

References

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