Perfect Matchings in *O(n* log *n)* Time in a Regular Bipartite Graph

By William Ross Macrae

Perfect Matchings in *O(n* log *n)* Time in a Regular Bipartite Graph

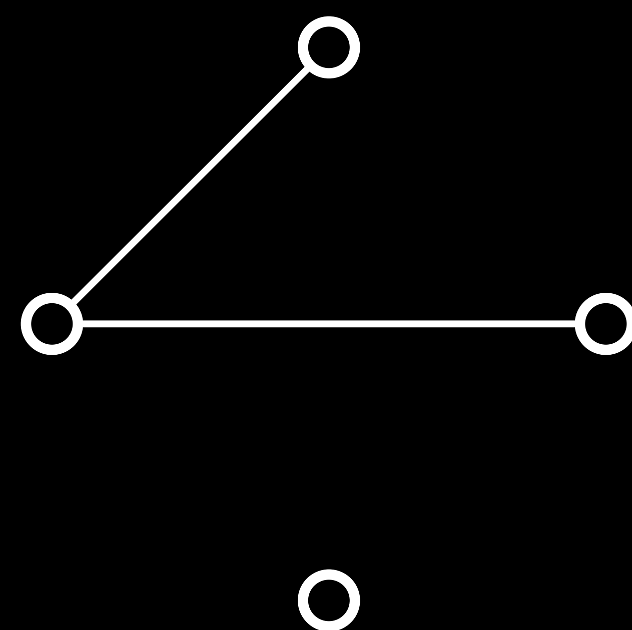
What this means
Deterministic algorithm
How we find this
Why we care

Perfect Matchings in *O(n* log *n)* Time in a Regular Bipartite Graph

$$G = (V, E)$$

$$V \neq \emptyset$$

$$E \subset \binom{V}{2}$$



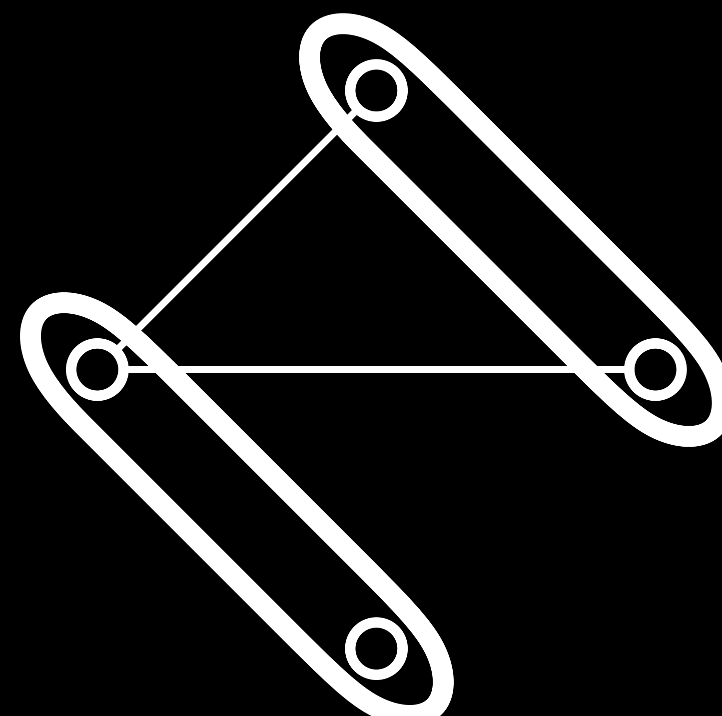
Definitions - Techniques - Hopcraft & Karp - Main Result

Perfect Matchings in $O(n \log n)$ Time in a Regular Bipartite Graph

$$G = (U, W, E)$$

$$[V = U \mathbin{\dot\cup} W \neq \varnothing]$$

$$E \subset \{\{u, w\} : u \in U, w \in W\}$$



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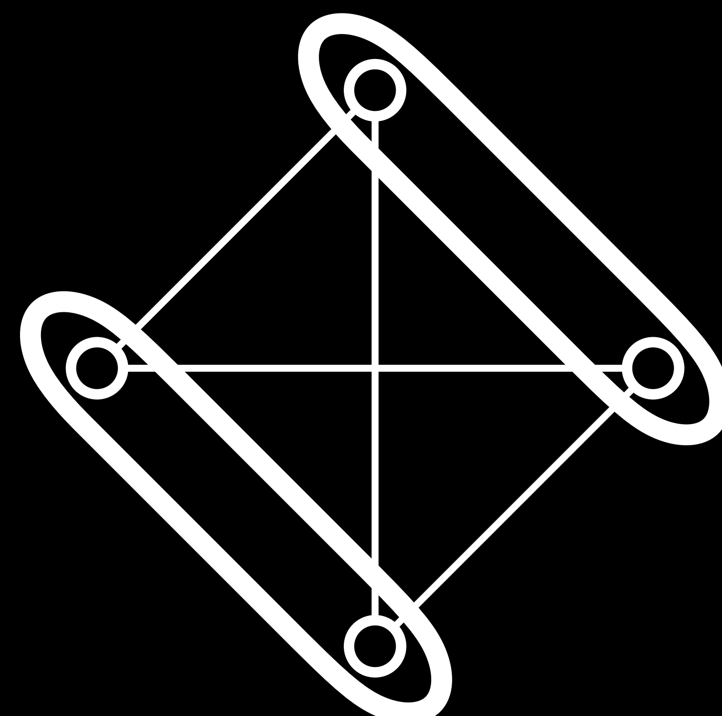

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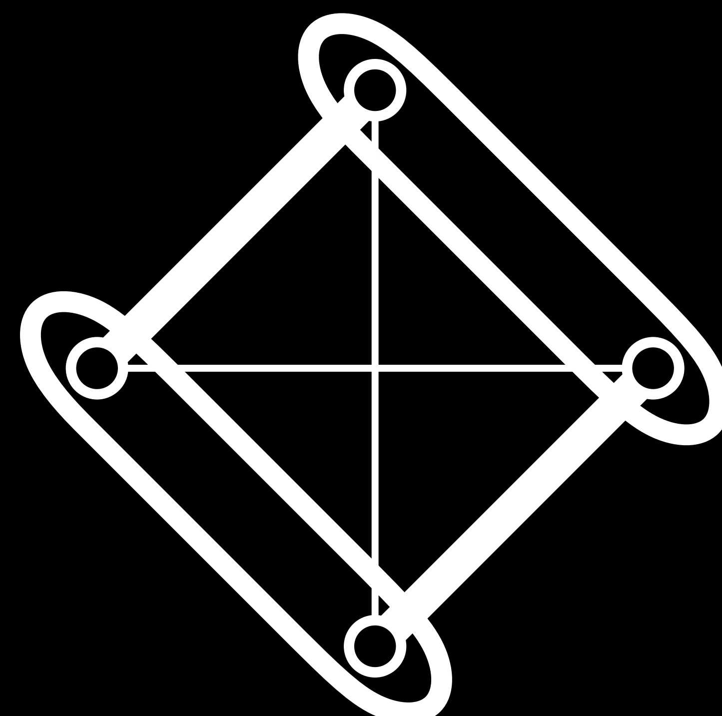

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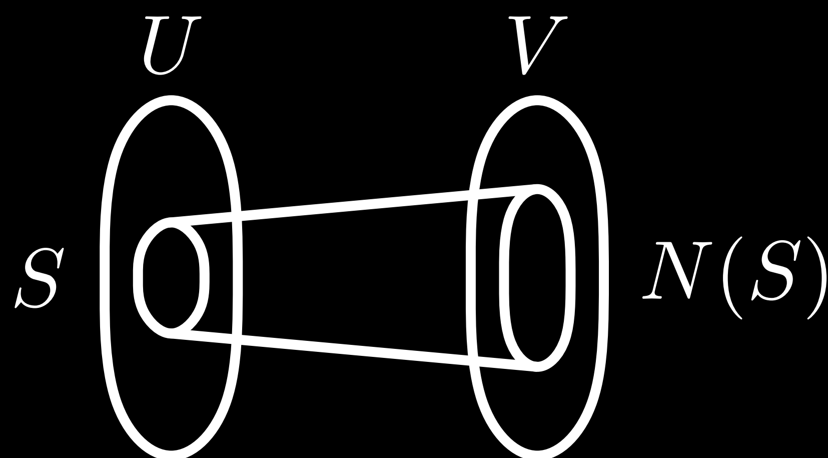
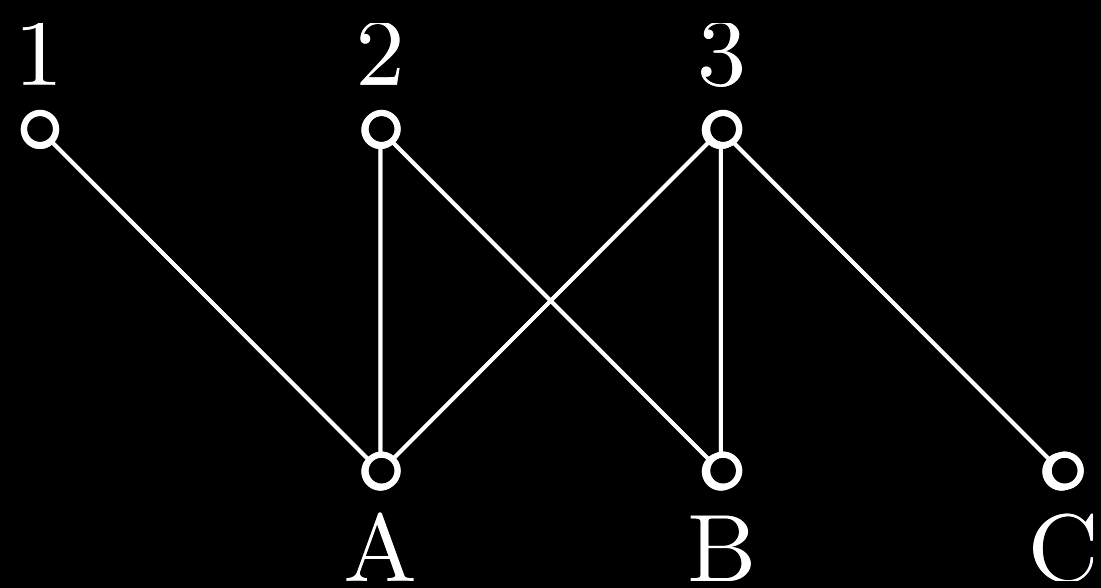
Marriage Theorem
(Why there is a perfect matching)

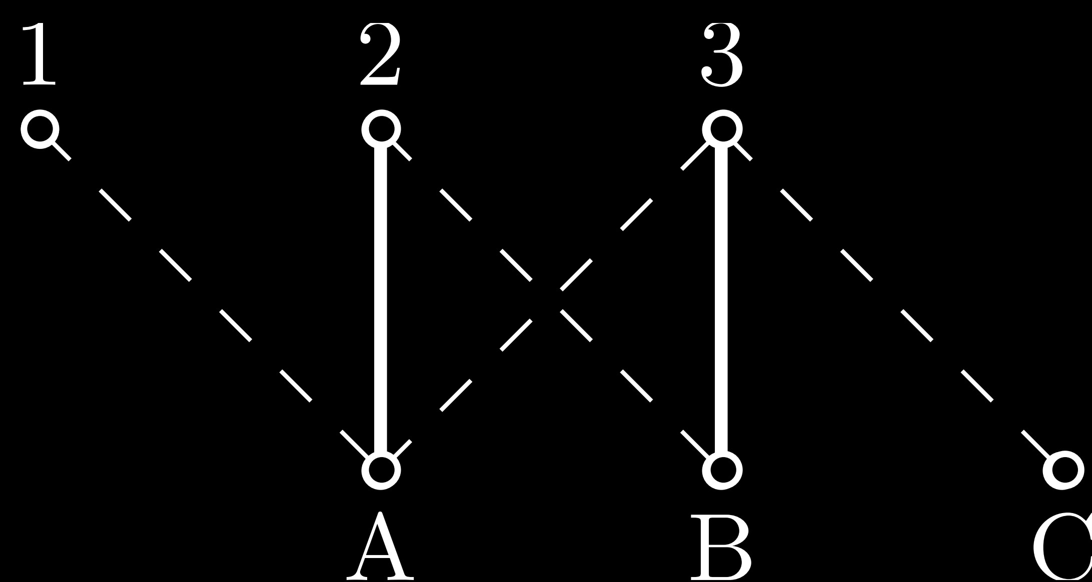
$$\forall S \subset V \quad |S| \leq |N(S)|$$



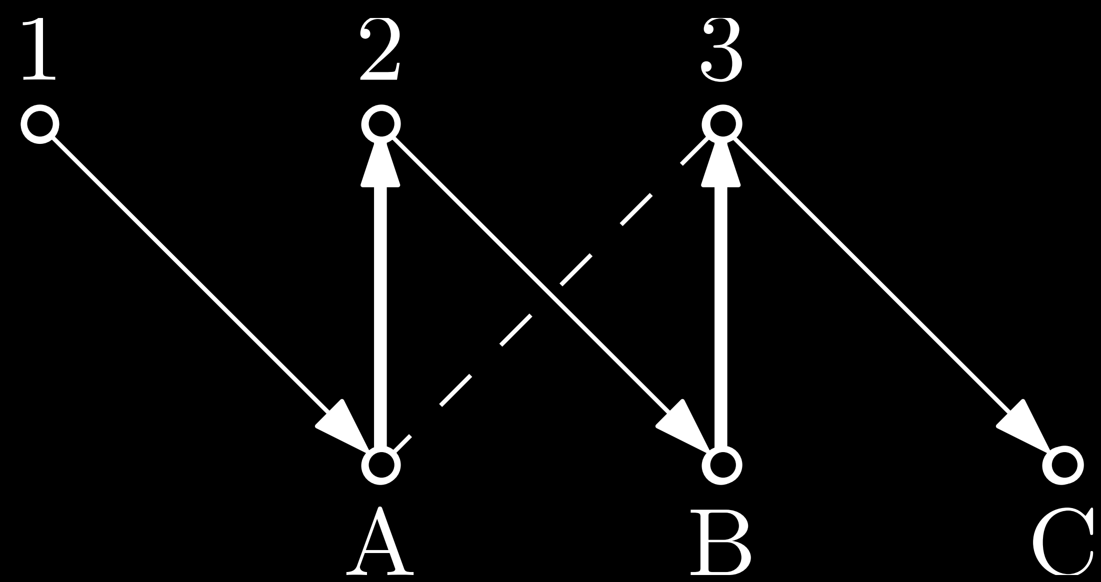
Perfect Matchings by Augmenting Paths



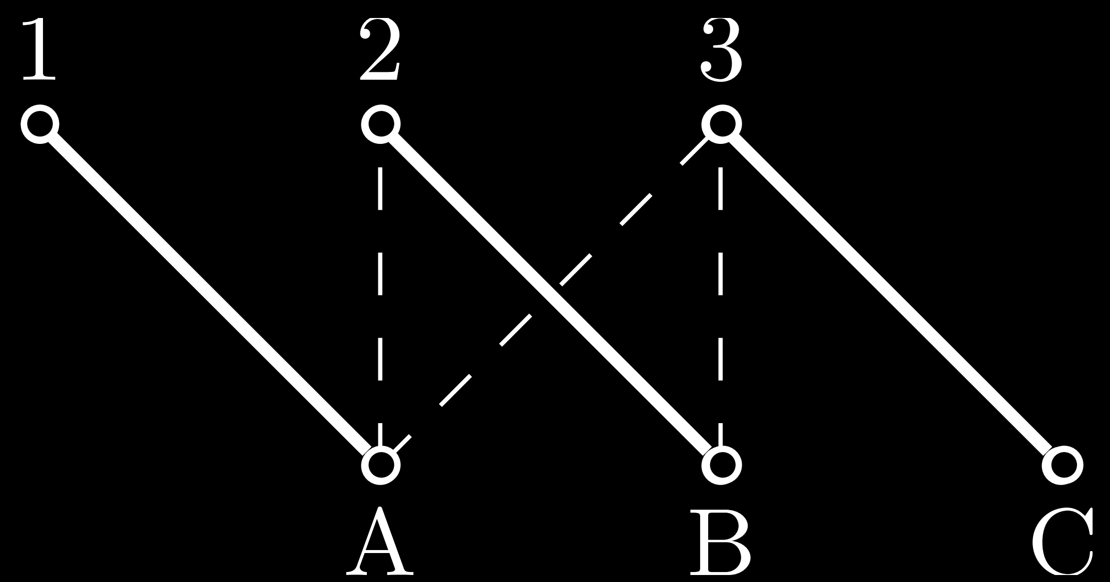
Perfect Matchings by Augmenting Paths



Perfect Matchings by Augmenting Paths



Perfect Matchings by Augmenting Paths



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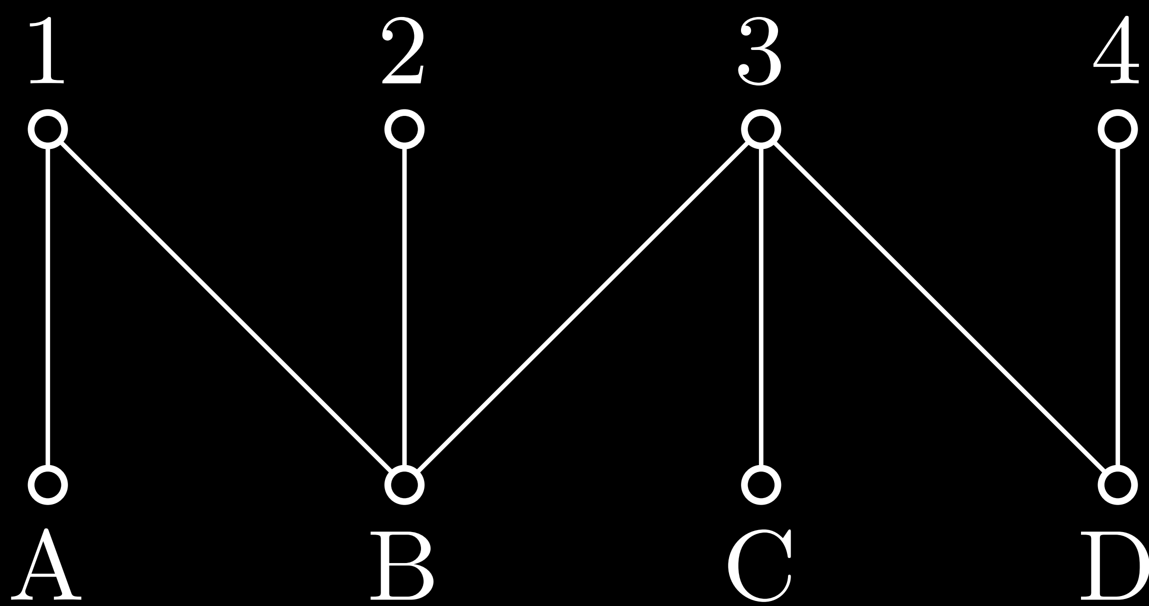
Hopcraft & Karp

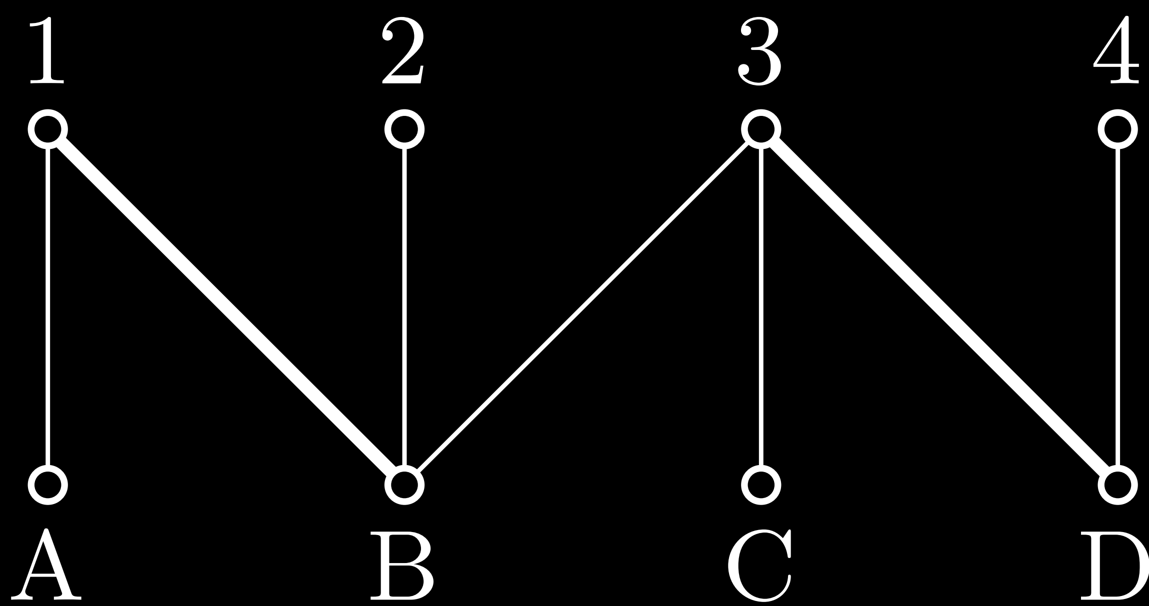
Finds a maximum matching in a bipartite graph
Takes time $O(m\sqrt{n})$
Deterministic

Breadth-First Searches from unmatched vertices of U to find augmenting paths of length k

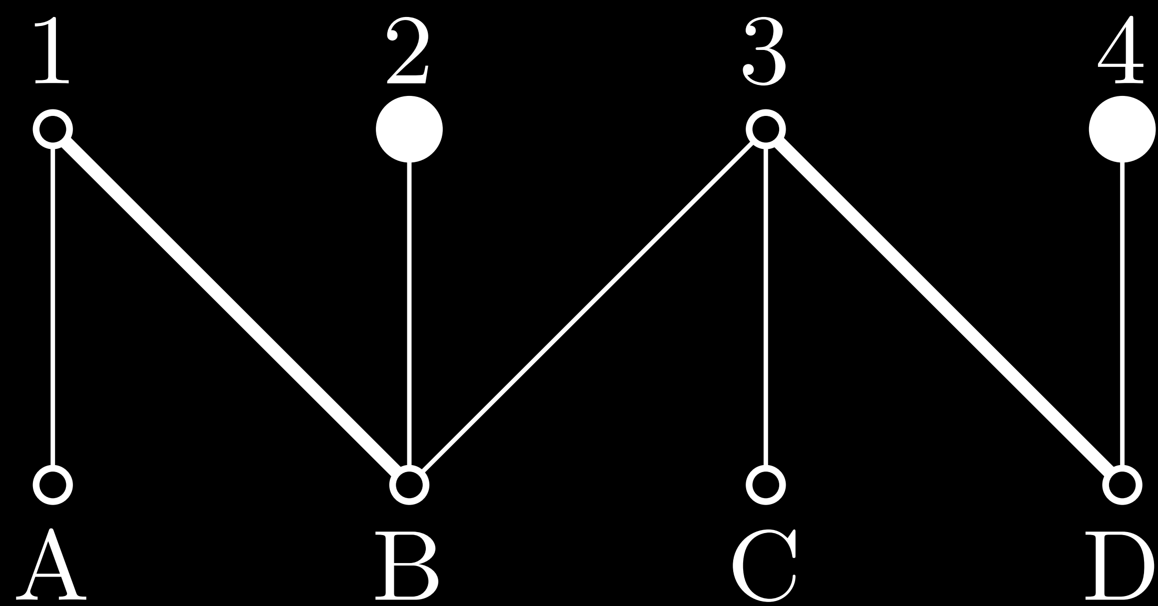
Hopcraft & Karp



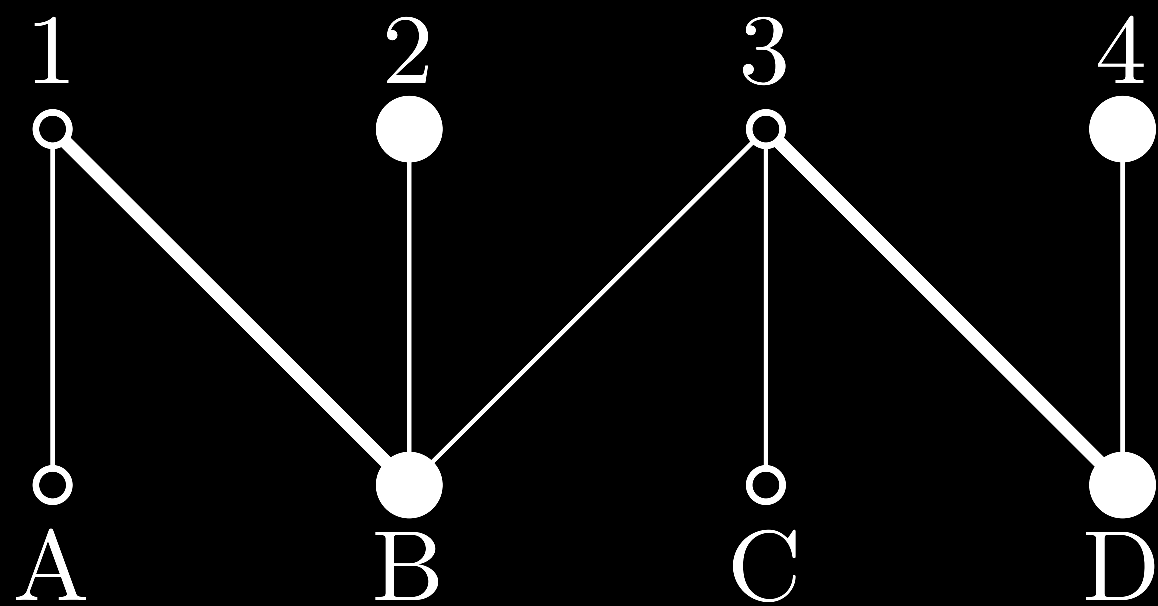
Hopcraft & Karp



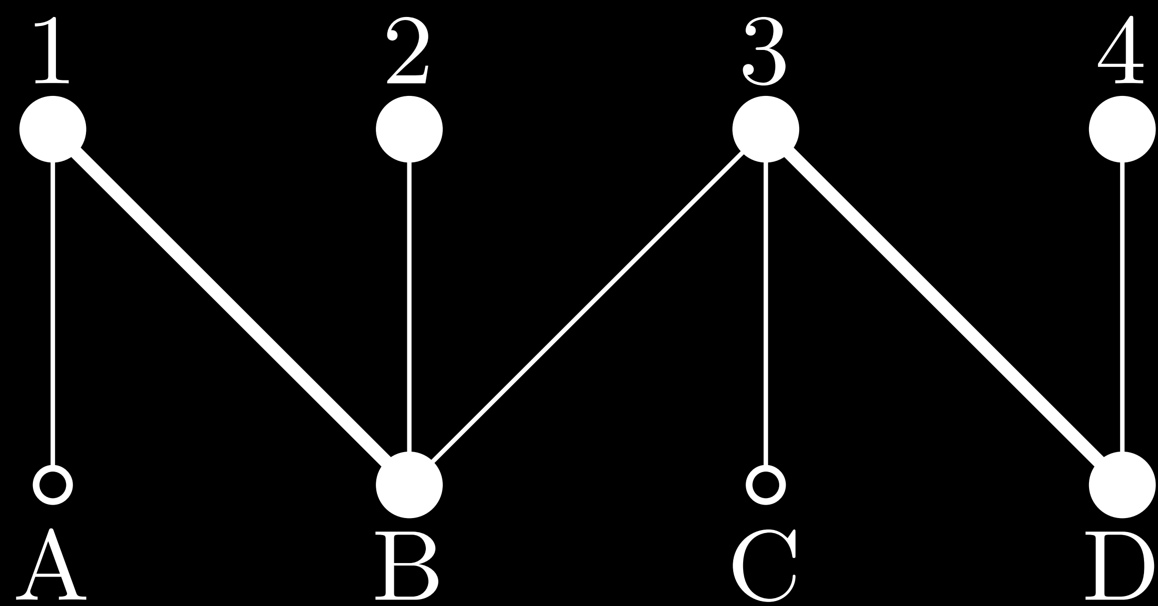
Hopcraft & Karp



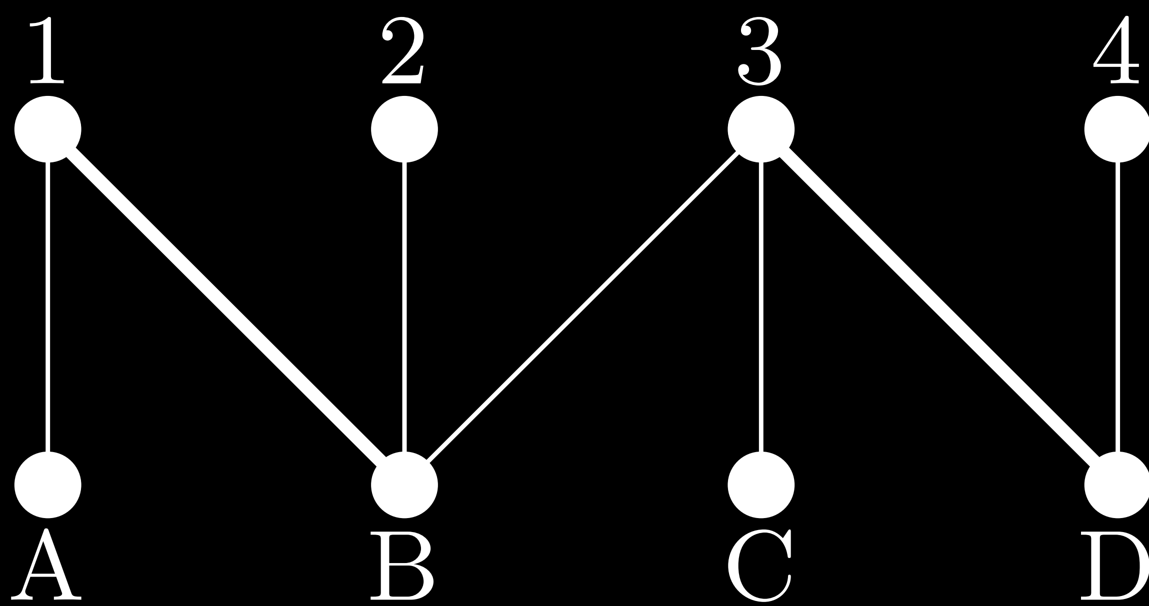
Hopcraft & Karp



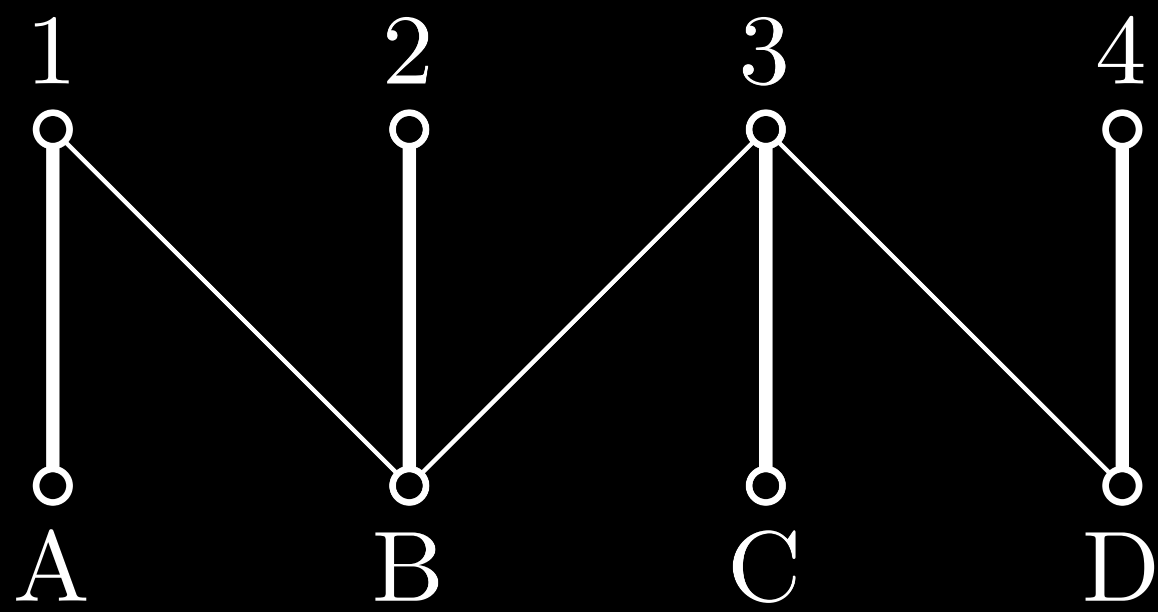
Hopcraft & Karp



Hopcraft & Karp



Hopcraft & Karp



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Main Result

Goel, Kapralov, and Khanna

Finds a maximum matching in a bipartite graph
Takes time $O(n \log n)$
Non-Deterministic

Try random walks of a bounded length k
until an augmenting path is found.

$$k = 2\left(2 + \frac{n}{n-j}\right)$$

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Sources / References

1. http://arxiv.org/pdf/0909.3346v3.pdf [4/25]
2. http://arxiv.org/pdf/0902.1617v2.pdf [4/25]
3. The Design and Analysis of Algorithms, Dexter Kozen
4. Introduction to Algorithms, Cormen, Leiserson, Rivest, and Stein
5. Randomized Algorithms, Motwani and Raghavan
6. http://www.stanford.edu/class/msande337/notes/erdos.pdf [4/25]
7. http://www.andrew.cmu.edu/user/jnir/Combinotes.html [4/25]

1. Main Paper
2. Related Paper
3. Hopcraft & Karp
4. Bounds, Randomized Algorithms
5. Randomized Algorithms
6. Random Walks
7. Theoretical Background