

WARNING: This is complicated. Suppose our formula looks like this.

We'll start at the top node a.

We make a gadget that has these little diamonds for every variable in the formula.

First, if the left node is chosen, that will correspond to setting the variable TRUE, and if the right node is chosen, that's like setting it to FALSE.

Player 1 mimics the optimal strategy for Player E and Player 2 mimics the optimal strategy for player A in assigning values to variables.

Player 1 goes first, and picks left or right. (ie Player 1 picks a value for x1 that mimics Player E.) Then Player 2's move is forced, and Player 1's move is forced.

Now Player 2 picks left or right. (Picks a value for x2 according to player A.)

Then Player 1's move is forced, etc.

Now, at the bottom node, the players have selected a variable assignment, and it's Player 1's move. This move is forced, to a node c.

Now from this gadget we connect it to a node c, which points to nodes representing the clauses of the CNF. Now each literal has an edge to the node that satisfies it.

FINALLY,

Suppose the formula is true. Then no matter what clause Player 2 picks, Player 1 can pick a literal that is satisfied.

So Player 2 can't move since the node it connects to has already been taken! (The literal is true.) Suppose the formula is false. Then there is a clause c_i that is falsified by the assignment chosen.

Suppose Player 2 picks this clause c_i.

Then no matter what literal Player 1 picks, every literal is "false", so Player 2 can follow the edge and win.