Mining Large Dynamic Graphs and Tensors: Thesis Proposal

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Abstract

Graphs are everywhere from online social networks to bipartite review graphs. Many of them are large and dynamic. Moreover, they are with extra information and thus naturally modeled as tensors. Given large dynamic graphs and tensors, how can we analyze their structure? How can we detect interesting anomalies? Lastly, how can we model the behavior of the individuals in the data?

My thesis focuses on these closely related questions, all of which are fundamental to understand large growing data on user behavior. For structure analysis, we develop streaming algorithms that incrementally update several connectivity measures in dynamic graphs. We also improve the scalability of Tucker Decomposition, which summarizes the structure of tensors. For anomaly detection, we develop several approximation algorithms that detect anomalously dense subgraphs and subtensors. The algorithms are designed for various settings, including distributed and streaming settings. Lastly, regarding behavior modeling, we build a stage model for progression of users on social media, and a game-theoretic model for purchasing behavior of individuals in social networks.

As future work, we will further explore the relation between the structure of social networks and the behavior of individuals in them by building a model for polarization in social networks. We will also advance our algorithms for larger and more dynamic data (e.g., fully dynamic graph streams where edges are not only added but also deleted).

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Contents

Chapter 1

Introduction

Graphs are everywhere from online social networks to bipartite review graphs. Many of them are large (e.g., Facebook announced that they had more than two billion monthly active users in 2017) and dynamic (i.e., nodes and edges can be added or removed). Moreover, they are with extra information (e.g., each review on Amazon has a timestamp, a rating, and text) and thus naturally modeled as tensors (i.e., multi-dimensional arrays).

My thesis focuses on the following questions, all of which are fundamental to understand large growing data on user behavior:

Given large dynamic graphs and tensors,

Q1. Structure Analysis: How can we analyze their structure?

Q2. Anomaly Detection: How can we detect interesting anomalies worthy of further investigation?

Q3. Behavior Modeling: How can we model the behavior of the individuals in the data?

These questions are closely related, and answering one helps answering the others. For example, the structure of a social network affects the behavior of the people in it and vice versa. In addition, anomalies show abnormal behavior, not explainable by behavior models, and often form unlikely substructures. Thus, structure analysis and behavior modeling help identifying anomalies.

For structure analysis, we develop streaming algorithms that incrementally update several connectivity measures in dynamic graphs. Specifically, within a limited memory budget, our algorithms estimate the degeneracy [SERF18, SERF16] and the counts of global and local triangles [Shi17, SHL+18, SLO+18]. For accuracy, our algorithms exploit structural [SERF18, SERF16] and temporal [Shi17] patterns in real graph streams, or utilize computational and storage resources distributed across multiple machines [SHL+18, SLO+18]. Moreover, in [OSP+17], we improve the scalability of Tucker Decomposition, which summarizes the structure of tensors. Specifically, our algorithm avoids the explosion of intermediate data, which previous algorithms suffer from.

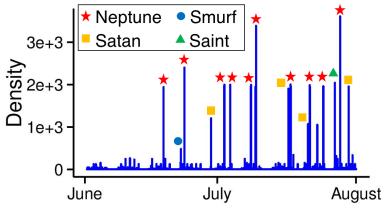
For anomaly detection, we develop several approximation algorithms that detect anomalously dense subgraphs and subtensors. In [SERF18, SERF16], based on the observation that natural dense subgraphs are formed by high-degree nodes, we propose algorithms for identifying dense subgraphs consisting of low-degree nodes, which indicate anomalies such as a 'follower-boosting' service on Twitter and 'copy-and-paste' bibliography in a citation graph. In [SHF18, SHF16], we propose algorithms for detecting dense subtensors (e.g., dense subgraphs formed within a short time), which signal anomalies such as 'edit wars' in Wikipedia and network attacks in computer networks. We extend these algorithms for disk-resident or distributed tensors [SHKF18, SHKF17a] and dynamic tensors [SHKF17b]. Figure 1.1 shows some anomalies detected by our algorithms.

Lastly, regrading behavior modeling, we build a game-theoretic model to examine incentives to pur-

(a) Social Networks

User	Review	Date
Ti*	type in *** and you will get ...	Mar-4
Fo*	type in for the bonus code: ...	Mar-4
dj*	typed in the code: *** ...	Mar-4
Di*	enter this code to start with ...	Mar-4
Fe*	enter code: *** to win even ...	Mar-4

(b) E-commerce



(c) Network Security

Figure 1.1: Applications of our anomaly-detection algorithms in various fields. **(a) Social Networks:** CORE-A (§ 4.1) detects a follower-boosting service on Twitter. **(b) E-commerce:** D-CUBE (§ 4.2.3) detects spam reviews on an online software market. **(c) Network Security:** DENSEALERT (§ 4.2.3) detects various types of network attacks.

chase a good sharable with neighbors in a social network [SLEP17]. Using our models, we analyze the effects of rental fees and structures of social networks on social inefficiency. In addition, to summarize progression of users on social media, we devise a stage model with an optimization algorithm that fits our model to terabyte-scale behavior logs [SSK+18].

As future work, we will further explore the relation between the structure of social networks and the behavior of individuals in them. To this end, we will build behavior models for influence between the individuals, which we expect to explain polarization in social networks. In addition, we will advance our algorithms for larger and more dynamic data. Specifically, we will extend our streaming algorithms for fully dynamic graphs, where edges are not only added but also deleted, and improve the speed of Tucker Decomposition by exploiting structural patterns in real-world data, such as skewed degree distributions.

1.1 Impact and Reproducibility

We made most of the algorithms produced throughout this thesis open source for reproducibility and the benefit of the community. In Table 1.1, we list the open source algorithms and their impacts in terms of the number of downloads and the number of institutions and countries where each algorithm was downloaded. In addition, we achieved the following results:

- Our work [SERF16] was selected as one of the best papers of ICDM 2016 and invited to a special issue of the Knowledge and Information Systems journal [SERF18].
- Our work [SLEP17] was featured in an article in New Scientist titled "Game theory says you should charge your friends to borrow things."
- The model and optimization algorithm proposed in our work [SSK+18] were implemented and used at LinkedIn Corporation.

1.2 Thesis Organization

The rest of the thesis proposal is organized as follows. In Chapter 2, we give preliminaries on graphs and tensors. In Chapter 3, we present our completed and proposed work on structure analysis. In Chapter 4, we present our completed and proposed work on anomaly detection. In Chapter 5, we present our completed and proposed work on behavior modeling. After discussing the timeline in Chapter 6, we draw conclusions in Chapter 7. In Appendix A, we review related work. In Appendix B, we describe

the datasets used throughout the thesis. The detailed organization of completed and proposed work is provided in Table 1.2.

Table 1.1: Impacts of the open source algorithms produced throughout this thesis. The source code is available at `http://www.cs.cmu.edu/~kijungs/proposal/`. The statistics were collected from August 2017 to February 2018.

Algorithms	Related Sections	Number of Downloads*	Number of Institutions	Number of Countries
WRS [LINK]	§ 3.1.1	25	15	4
CORE-D & CORE-A [LINK]	§ 3.2 & § 4.1	23	16	7
M-ZOOM [LINK]	§ 4.2.1	23	12	7
D-CUBE [LINK]	§ 4.2.2	26	12	5
DENSESTREAM [LINK]	§ 4.2.3	26	17	7
TRI-FLY [LINK]	§ 3.1.2	0 (new)	0 (new)	0 (new)
S-HOT [LINK]	§ 3.3.1	unavailable	unavailable	unavailable

* or the number of clicks on links to the corresponding Github repository.

Table 1.2: Overview of the thesis proposal. For the reader's convenience, we provide hyperlinks to published or submitted papers.

Structure Analysis (§ 3)	S1: Triangle Counting in Graph Streams (§ 3.1) S1-1: Exploiting Temporal Patterns (§ 3.1.1) [Shi17][PDF] S1-2: Utilizing Distributed-memory Machines [SHL+18] (§ 3.1.2)[PDF] [SLO+18][PDF] S1-3: Handling Deletions (§ 3.1.3) [Proposed] S2: Degeneracy Estimation in Graph Streams (§ 3.2) [SERF18][PDF] [SERF16][PDF] S3: Decomposing High-order Tensors (§ 3.3) S3-1: Avoiding Intermediate Data Explosion (§ 3.3.1) [OSP+17][PDF] S3-2: Exploiting Skewed Degree Distributions (§ 3.3.2) [Proposed]
Anomaly Detection (§ 4)	A1: Anomalous Subgraphs (§ 4.1) [SERF18][PDF] [SERF16][PDF] A2: Dense Subtensors (§ 4.2) A2-1: in Static Tensors (§ 4.2.1) [SHF18][PDF] [SHF16][PDF] A2-2: in Web-scale Static Tensors (§ 4.2.2) [SHKF18][PDF] [SHKF17a][PDF] A2-3: in Dynamic Tensors (§ 4.2.3) [SHKF17b][PDF]
Behavior Modeling (§ 5)	B1: Progressive Behavior on Social Media (§ 5.1) [SSK+18][PDF] B2: Purchasing Behavior in Social Networks (§ 5.2) [SLEP17][PDF] B3: Polarization in Social Networks (§ 5.3) [Proposed]

Chapter 2

Preliminaries: Graphs and Tensors

In this chapter, we introduce concepts useful for understanding the thesis.

2.1 Graphs

Graph: A *graph* $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is a pair of two sets $\mathcal{V}$ and $\mathcal{E}$ where every element of $\mathcal{E}$ is an unordered pair of elements of $\mathcal{V}$. Each element of $\mathcal{V}$ is called a *node*, and each element $e = \{u, v\}$ of $\mathcal{E}$ is called the *edge* between nodes u and v. A graph naturally represents a set of objects where some pairs of the objects are related in certain ways. Examples are as follows:

- **[Social network]**
  - The node set $\mathcal{V}$ is people.
  - The edge set $\mathcal{E}$ is the pairs of friends.
- **[E-commerce]**
  - The node set $\mathcal{V}$ is people and products.
  - The edge set $\mathcal{E}$ is the pairs of a user and a product where the user bought the product.
- **[Network security]**
  - The node set $\mathcal{V}$ is IPs.
  - The edge set $\mathcal{E}$ is the pairs of IPs where connections were made between them.

Neighbor and Degree: We say a node u is a *neighbor* of a node v (or u is *adjacent* to v) when the edge between u and v exists. The *degree* of a node v is defined as the number of neighbors of v.

Subgraph, Clique, and Triangle: We say a graph $\mathcal{G}' = (\mathcal{V}', \mathcal{E}')$ is a *subgraph* of a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ when $\mathcal{V}' \subset \mathcal{V}$ and $\mathcal{E}' \subset \mathcal{E}$. We say a subset $\mathcal{V}'$ of $\mathcal{V}$ is a *clique* if every pair of nodes in $\mathcal{V}'$ is connected by an edge in $\mathcal{E}$. A clique of three nodes is called a *triangle*.

k-Core, Coreness, Degeneracy, k-Truss, and Trussness: The *k-core* of a graph is the maximal subgraph where every node has degree at least k within the subgraph. The *coreness* of a node is the largest k such that it belongs to the k-core. The *degeneracy* of a graph is the largest k such that the k-core exists. The *k-truss* of a graph is the maximal subgraph where every edge is contained in at least $(k-2)$ triangles within the subgraph. The *trussness* of a node is the largest k such that it belongs to the k-truss.

Graph Stream: A *graph stream* is an ordered sequence of edges. In the thesis, we assume a streaming model where the edges in a graph stream can be accessed once in the given order unless they are explicitly stored in memory.

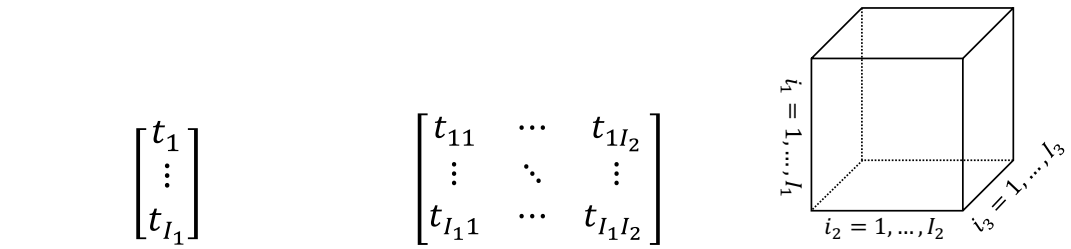
2.2 Tensors

Tensor: A *tensor* is a multi-dimensional array of entries. The *order* of a tensor is the number of dimensions, also known as *modes*. Consider an N-order tensor $\mathcal{T}$ of size $I_1 \times ... \times I_N$. We denote each $(i_1, ..., i_N)$-th entry of $\mathcal{T}$ as $t_{i_1 \cdots i_N}$ where each *n-th mode index* i_n ranges from 1 to I_n. For each n-th mode, we call I_n the *dimensionality* of the mode. Figures 2.1(a)-2.1(c) show illustrations of tensors. Tensors have more expressive power than graphs, as the following examples show:

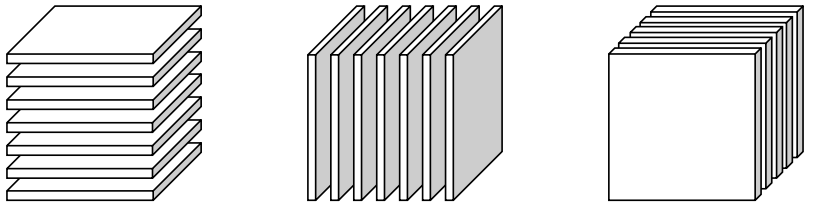
- **[Social network]**
  - Consider a 3-order tensor $\mathcal{T}$ whose modes are people, people, and dates, respectively.
  - An entry $t_{i_1 i_2 i_3}$ is 1 if the i_1-th person and the i_2-th person became friends on the i_3-th date Otherwise, $t_{i_1 i_2 i_3}$ is 0.
- **[E-commerce]**
  - Consider a 3-order tensor $\mathcal{T}$ whose modes are products, users, and dates, respectively.
  - An entry $t_{i_1 i_2 i_3}$ is the number of the i_1-th products bought by the i_2-th user on the i_3-th date.
- **[Network security]**
  - Consider a 4-order tensor $\mathcal{T}$ whose modes are IPs, IPs, protocols, and dates, respectively.
  - An entry $t_{i_1 i_2 i_3 i_4}$ is the number of connections made from the i_1-th IP to the i_2-th IP using the i_3-th protocol on the i_4-th date.

Slice and Degree: The *slices* of an N-order tensor are the $(N-1)$-order tensors obtained by fixing an mode index. Among the slices, the *n-mode slices* are those obtained by fixing the n-th mode index. For example, the entries whose n-mode index is i_n form an n-mode slice. Figures 2.1(d)-2.1(f) show illustrations of the slices of a 3-order tensor. We define the *degree* of a slice as the number of non-zero entries of the slice. We define the *weighted degree* of a slice as the sum of the entries of the slice.

Subtensor: We say an N-order tensor $\mathcal{T}'$ is a subtensor of an N-order tensor $\mathcal{T}$ when $\mathcal{T}'$ can be obtained by removing some slices from $\mathcal{T}$.



(a) 1-order tensor (i.e., vector) (b) 2-order tensor (i.e., matrix) (c) 3-order tensor $\mathcal{T}$



(d) 1-mode slices of $\mathcal{T}$ (e) 2-mode slices of $\mathcal{T}$ (f) 3-mode slices of $\mathcal{T}$

Figure 2.1: Illustration of tensors and slices.

Chapter 3

Structure Analysis

We present our completed and proposed work on structure analysis. For each completed task, we briefly summarize our goal, approach, and results. References for detailed discussions are given at the beginning of each section. For each proposed task, we give a brief description of our goal and plan.

3.1 Counting Triangles in Graph Streams

"Given a dynamic graph that is too large to fit in memory, how can we estimate the count of *triangles* (i.e., cliques of three nodes), which is one of the most basic measures in graph analysis?" To answer this question, we consider streaming algorithms that incrementally process each edge as it arrives and maintain only a small summary of the input graph instead of the entire graph. Specifically, we consider streaming algorithms for three realistic variants of Problem 3.1. Our common goal of the variants is to accurately estimate the counts of *global triangles* (i.e., all triangles) and *local triangles* (i.e., the triangles incident to each node) in a graph stream.

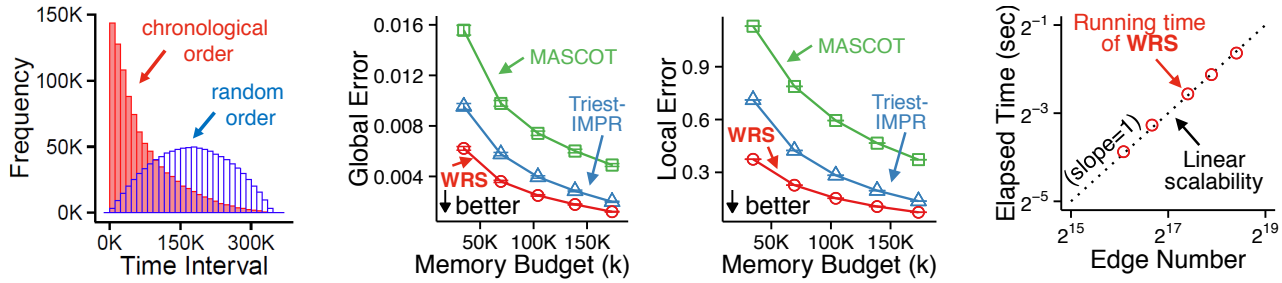
Problem 3.1 (Triangle Counting in a Graph Stream)**.**
(1) **Given:** a graph stream and a memory budget
(2) **Maintain & Update:** estimates of the global triangle count and the local triangle counts
(3) **to Minimize:** the estimation error

3.1.1 Counting Triangles in Graph Streams with Temporal Patterns

Section based on work that appeared at ICDM17 [Shi17][PDF].

Goal: "If we cannot store all edges in a graph stream, which edges should we store to estimate the triangle count accurately?" Graph stream algorithms typically assume that the order of the edges in a graph stream is random or adversarial. However, a more realistic order is the chronological order since it is natural that new edges in a dynamic graph are streamed as they are created. We consider triangle counting in a graph stream where edges are in the chronological order, as described in Problem 3.2. Specifically, we explore a temporal pattern in real-world graphs and show how to exploit the pattern for accurate triangle counting.

Problem 3.2 (Triangle Counting in a Graph Stream in the Chronological Order)**.**
(1) **Given:** a graph stream where edges are in the chronological order, and a memory budget
(2) **Maintain & Update:** estimates of the global triangle count and the local triangle counts
(3) **to Minimize:** the estimation error.



(a) Pattern: Temporal Locality (b) Accuaracy of WRS (c) Scalability of WRS

Figure 3.1: (a) WRS, devised for Problem 3.2, exploits temporal locality, described in the figure. (b) As a result, WRS is more accurate than its best competitors. (c) Moreover, the time complexity of WRS is linear to the number of edges in the input stream.

Approach: We observe in real-world graphs that if edges are streamed in the chronological order, new edges are more likely to form triangles with recent edges than older edges. We call this tendency *temporal locality*. Let the time interval of a triangle be the difference between the arrival orders of its first and last edges. Temporal locality is clearly shown in Figure 3.1(a), where we show how the arrival orders of the edges in a graph stream affect the time intervals of the triangles.

This temporal locality is exploited by Waiting-Room Sampling (WRS), our streaming algorithm for global and local triangle counting. WRS prioritizes recent edges, which new edges are more likely to form triangles with, over older edges when it samples edges to be stored in memory. Specifically, WRS divides the memory space into the waiting room, where the latest edges are stored with probability one, and the reservoir, where the remaining edges are uniformly sampled.

Results: As seen in Figure 3.1(b), WRS produces up to 47% **smaller estimation error** than its best competitors, which sample edges without prioritization. Moreover, WRS scales linearly with the number of edges in the input graph stream, as seen in Figure 3.1(c). We also prove that WRS gives **unbiased estimates**, whose expected values are equal to the true values, as formalized in Theorem 3.3.

Theorem 3.3 (Unbiasedness of WRS [Shi17][PDF]). *Let $\mathcal{G}^{(t)} = (\mathcal{V}^{(t)}, \mathcal{E}^{(t)})$ be the graph composed of the edges whose arrival order is less than or equal to t. Then, let $\Delta^{(t)}$ and $\{\Delta_v^{(t)}\}_{v\in\mathcal{V}^{(t)}}$ be the global and local triangle counts, respectively, in $\mathcal{G}^{(t)}$.* WRS *gives unbiased estimates $\hat{\Delta}^{(t)}$ and $\{\hat{\Delta}_v^{(t)}\}_{v\in\mathcal{V}^{(t)}}$. That is,*

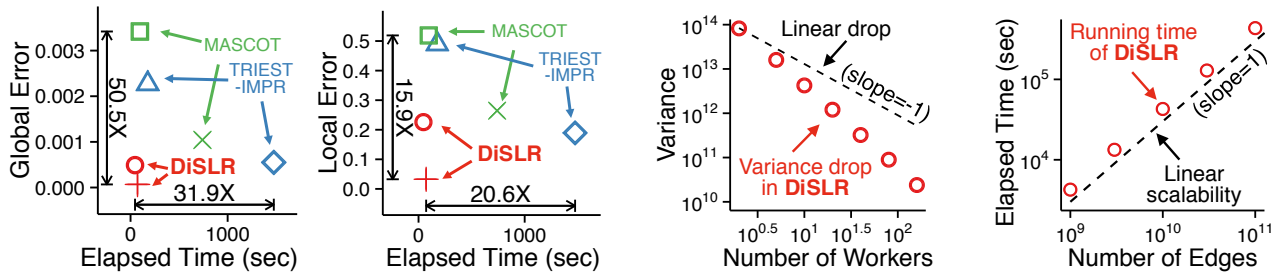
$$\mathbb{E}[\hat{\Delta}^{(t)}] = \Delta^{(t)}, \forall t \geq 1 \quad \textbf{and} \quad \mathbb{E}[\hat{\Delta}_v^{(t)}] = \Delta_v^{(t)}, \forall t \geq 1, \forall v \in \mathcal{V}^{(t)}.$$

3.1.2 Counting Triangles in Graphs Streams on Distributed-memory Machines

Section based on work that will appear at PAKDD18 [SHL+18][PDF] and work under review [SLO+18][PDF].

Goal: "Given a graph stream, how can we estimate the number of triangles in it using multiple machines with limited memory?" As formalized in Problem 3.4, we consider counting the global and local triangles in a graph stream using multiple machines with limited memory. Our goal is to devise a fast and accurate algorithm for the problem.

Problem 3.4 (Triangle Counting in a Graph Stream on Distributed-memory Machines).
(1) **Given:** a graph stream and a memory budget for each of multiple machines
(2) **Maintain & Update:** estimates of the global triangle count and the local triangle counts
(3) **to Minimize:** the estimation error.



(a) Speed & Accuracy of DiSLR (b) Variance drop in DiSLR (c) Scalability of DiSLR

Figure 3.2: (a) DiSLR, devised for Problem 3.4, provides a significantly better trade-off between speed and accuracy than the single-machine baselines. (b) In DiSLR, the variances of estimates decrease super linearly with the number of machines. (c) Moreover, the time complexity of DiSLR is linear to the number of edges in the input stream.

Approach: Our distributed streaming algorithm DiSLR (**Di**stributed **S**ampling with **L**imited **R**edundancy) [SLO+18][PDF], which outperforms our previous algorithm TRI-FLY [SHL+18][PDF], minimizes the redundant use of distributed resources. Specifically, in DiSLR, nodes are colored by a hash function. Then, depending on the colors, masters selectively send edges to workers, which also selectively sample edges. As a result, the following desirable conditions are satisfied:

- **Limited redundancy in stroage**: Each edge is stored in at most two workers.
- **No redundancy in computation:** Each triangle is counted by at most one worker.
- **No definitely missing triangles:** Each triangle is counted with non-zero probability.

The aggregators simply sum the estimates of the workers to give final estimates.

Results: As seen in Figure 3.2(a), DiSLR with 30 workers gives a significantly better trade-off between speed and accuracy than the best single-machine baselines, without suffering from "distributed-algorithm paradox." Specifically, DiSLR is up to $\mathbf{132}\times$ **more accurate** than the baselines with similar speeds and up to $\mathbf{32}\times$ **faster** than the baselines with similar accuracy. Moreover, DiSLR scales linearly with the number of edges in the input graph stream, as seen in Figure 3.2(c). We also prove that DiSLR gives **unbiased estimates** (as WRS does in Theorem 3.3) whose variance drops inversely proportional to the number of workers or faster, as described in Theorem 3.5 and Figure 3.2(b).

Theorem 3.5 (Variance Drop in DiSLR [SLO+18][PDF])**.** *Let* $\hat{\Delta}^{(t)}$ *and* $\{\hat{\Delta}_v^{(t)}\}_{v\in\mathcal{V}^{(t)}}$ *be the estimates of the global and local triangle counts, respectively, that* DiSLR *with* n *workers gives after processing* t *edges in the input graph stream. Than, their variances drop inversely proportional to* n *or faster. That is,*

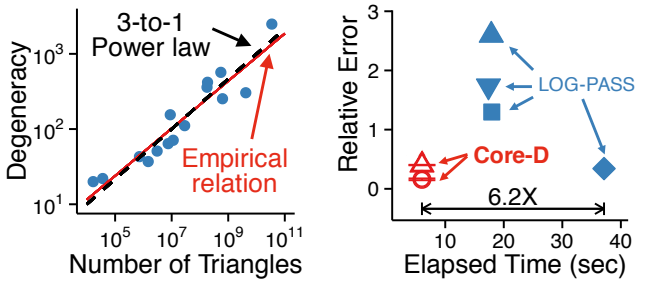
$$Var[\hat{\Delta}^{(t)}] = O(1/n), \forall t \geq 1 \quad \textbf{and} \quad Var[\hat{\Delta}_v^{(t)}] = O(1/n), \forall t \geq 1, \forall v \in \mathcal{V}^{(t)}.$$

3.1.3 [Proposed] Counting Triangles in Graphs Streams with Deletions

"Given *a fully dynamic graph stream*, where edges are both added and deleted, how can we estimate the number of triangles?" In many real graphs, edges are not only added but also deleted. For cases where such graphs are also too large to fit in memory, we consider counting the global and local triangles in a fully dynamic graph stream, as formalized in Problem 3.6. Specifically, we aim to devise a streaming algorithm that gives unbiased estimates with small variances in such a stream. This goal has not been achieved by existing algorithms, as summarized in Table 3.1.

Table 3.1: Motivation for a new streaming algorithm for Problem 3.6. No existing algorithm gives unbiased estimates with small variances in a graph stream with deletions.

Algorithms	Variance	Unbiased w/ deletions?
WRS [Shi17]	smallest	no
TRIEST-IMPR [SERU17]	small	no
MASCOT [LK15]	large	no
TRIEST-FD [SERU17]	largest	yes



(a) Triangle-core Pattern (b) Performance of CORE-D

Figure 3.3: (a) CORE-D, devised for Problem 3.7, exploits the triangle-core pattern, shown in the figure. (b) CORE-D outperforms its semi-streaming competitor.

Problem 3.6 (Triangle Counting in a Graph Stream with Deletions)**.**
(1) **Given:** a graph stream with edge deletions, and a memory budget
(2) **Maintain & Update:** estimates of the global triangle count and the local triangle counts
(3) **to Minimize:** the estimation error.

3.2 Estimating Degeneracy in Graph Streams

Section based on work that appeared at ICDM16 [SERF16][PDF] and KAIS18 [SERF18][PDF].

Goal: "Given a dynamic graph that is too large to fit in memory, how can we estimate the degeneracy, a widely-used connectivity measure?" The *degeneracy* of a graph is the largest k such that a subgraph where every node has degree at least k (within the subgraph) exists. For estimating the degeneracy of large dynamic graphs, we consider streaming algorithms that incrementally process each edge as it arrives and maintain only a small summary of the input graph instead of the entire graph. Specifically, as stated in Problem 3.7, our goal is to devise an accurate streaming algorithm for degeneracy.

Problem 3.7 (Estimating Degeneracy in a Graph Stream)**.**
(1) **Given:** a graph stream and a memory budget
(2) **Maintain & Update:** an estimate of the degeneracy
(3) **to Minimize:** the estimation error

Approach: We observe in real-world graphs that a 3-to-1 power law exists between the degeneracy and the number of triangles, as shown in Figure 3.3(a). This pattern, which we call *triangle-core pattern*, is exploited by CORE-D, our streaming algorithm for degeneracy. Specifically, CORE-D estimates the number of triangles in a graph stream (as explained in the previous sections) and estimates the degeneracy from the estimated number of triangles using the triangle-core pattern.

Results: CORE-D estimates the degeneracy accurately by making a single pass over the input graph stream. Specifically, as shown in Figure 3.3(b), CORE-D is up to **12× faster** than its semi-streaming competitor with similar accuracy.

3.3 Decomposing High-order Tensors

"Given a high-order tensor that is too large to fit in memory, how can we summarize it using Tucker Decomposition?" *Tucker Decomposition* (a.k.a., N-mode factor analysis and N-mode PCA) is a widely-used summarization technique. It decomposes the input tensor into a small core tensor and low-rank matrices that describe the original tensor best, as defined in Problem 3.8. We first discuss improving the scalability of Tucker Decomposition by avoiding intermediate data explosion. Then, we propose accelerating Tucker Decomposition by exploiting a structural property of real-world tensors.

Problem 3.8 (Tucker Decomposition [KB09])**.**
(1) **Given:** an N-order tensor $\mathcal{T} \in \mathbb{R}^{I_1 \times \ldots \times I_N}$ and ranks $J_1, ..., J_N$
(2) **Find:** an N-order core tensor $\mathcal{G} \in \mathbb{R}^{J_1 \times \ldots \times J_N}$
and N orthogonal matrices $\mathbf{A}^{(1)} \in \mathbb{R}^{I_1 \times J_1}, ..., \mathbf{A}^{(N)} \in \mathbb{R}^{I_N \times J_N}$
(3) **to Minimize:** the reconstruction error

$$||\mathcal{T} - [\mathcal{G}; \mathbf{A}^{(1)}, \cdots, \mathbf{A}^{(N)}]||_F^2,$$

where $[\mathcal{G}; \mathbf{A}^{(1)}, \cdots, \mathbf{A}^{(N)}]_{i_1 \cdots i_N} = \sum_{j_1=1}^{J_1} \cdots \sum_{j_N=1}^{J_N} g_{j_1 \cdots j_N} a^{(1)}_{i_1 j_1} ... a^{(N)}_{i_N j_N}$.

3.3.1 Decomposing High-order Tensors without Intermediate Data Explosion

Section based on work that appeared at WSDM17 [OSP+17][PDF].

Goal: "Given a high-order tensor that is too large to fit in memory, how can we perform Tucker Decomposition without intermediate data explosion?" Developing a scalable Tucker Decomposition algorithm has been a challenge due to the *intermediate data explosion* [KS08]. Briefly speaking, Alternating Least Square (ALS), the most widely-used Tucker Decomposition algorithm, repeats the following two steps:

1. compute an intermediate tensor, denoted by $\mathcal{Y}$
2. compute the singular-value decomposition (SVD) of $\mathbf{Y}$, which is the matricized $\mathcal{Y}$.

However, as shown in Table 3.2, existing algorithms require substantial space for the intermediate data generated while computing or materializing (matricized) $\mathcal{Y}$. Our goal is to devise an algorithm that performs Tucker Decomposition, avoiding the intermediate data explosion. Moreover, we assume that the input tensor is disk-resident and too large to fit in memory, as described in Problem 3.9.

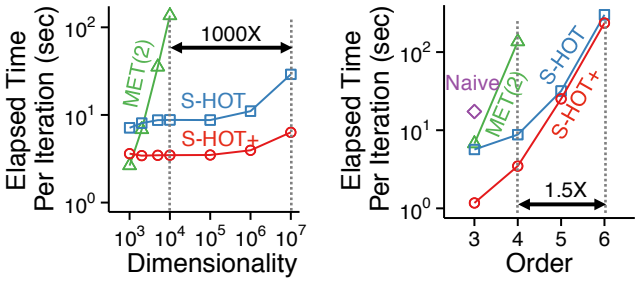
Problem 3.9 (High-order Tucker Decomposition without Intermediate Data Explosion)**.**
(1) **Given:** a High-order tensor that is too large to fit in memory
(2) **Perform:** the Tucker Decomposition of the tensor
(3) **to Minimize:** the space required for intermediate data.

Approach: Our algorithm S-HOT computes the SVD of $\mathbf{Y}$ *on-the-fly* without materializing $\mathbf{Y}$ (or even $\mathcal{Y}$) by utilizing the *reverse communication interface* of an eigensolver called *Implicitly Restart Arnoldi Method* [LSY98]. While performing the SVD of $\mathbf{Y}$, the interface repeatedly requires the product of $\mathbf{Y}$ and a vector that it gives but does not require $\mathbf{Y}$ itself. S-HOT performs this multiplication on-the-fly by sequentially reading the non-zero entries of the input tensor, which is stored on disk, at most twice.

Results: S-HOT requires significantly smaller space than its best competitors, as shown in Table 3.2. As a result, as seen in Figure 3.4, S-HOT decomposes larger (e.g., **1000× larger** in terms of dimensionality) and higher-order tensors than its best competitors.

Table 3.2: S-HOT, which we devise for Problem 3.9, does not suffer from the explosion of intermediate data. Below, we assume that a 5-order input tensor whose dimensionality is 10 million has 1 billion non-zero entries.

Algorithms	Space required for intermediate data
NAIVETUCKER	$\sim$ 40TB
HATEN2 [JPF+16]	$\sim$ 1.4TB
MET(2) [KS08]	$\sim$ 40GB
S-HOT	$\sim$ 4MB
S-HOT+	$\sim$ 40KB

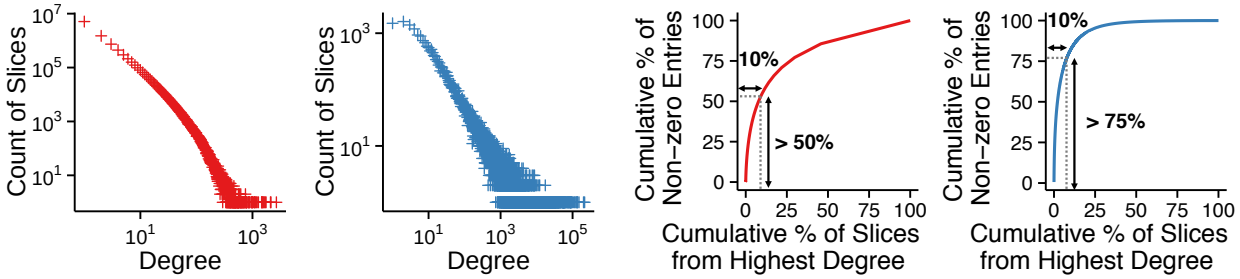


(a) w.r.t. dimensionality (b) w.r.t. order

Figure 3.4: S-HOT, devised for Problem 3.9, decomposes larger and higher-order tensors than its best competitors.

3.3.2 [Proposed] Decomposing High-order Tensors with Skewed Distributions

"If the input tensor has realistic skewed degree distributions, how can we exploit this fact to accelerate Tucker Decomposition?" As an extension of our previous efforts to improve the scalability of Tucker Decomposition, we aim to devise a faster algorithm for Problem 3.9. To this end, we plan to exploit skewed degree distributions (e.g., power-law distributions in Figure 3.5(a)) that many real-world tensors have. Specifically, we plan to selectively materialize a small part of the intermediate tensor $\mathcal{Y}$ that corresponds to high-degree slices (instead of computing entire $\mathcal{Y}$ on-the-fly as in the previous section) to maximize savings in computation. As shown in Figure 3.5(b), it is expected that materializing a small part of $\mathcal{Y}$ saves considerable computation.



(a) Power-law Degree Distributions (b) Distributions of Non-zero Entries

Figure 3.5: We propose exploiting skewed degree distributions, which many real-world tensors have, to devise a faster algorithm for Problem 3.9. (a) We give examples of such skewed degree distributions in a real-world tensor. (b) We show what percentage ($y\%$) of non-zero entries belong to top $x\%$ highest-degree slices. This implies that materializing intermediate results for a small number of slices can save considerable computation.

Chapter 4

Anomaly Detection

We present our completed on anomaly detection. For each task, we briefly summarize our goal, approach, and results. References for detailed discussions are given at the beginning of each section.

4.1 Detecting Anomalous Subgraphs

Section based on work that appeared at ICDM16 [SERF16][PDF] and KAIS18 [SERF18][PDF].

Goal: "Given a graph, how can we detect anomalous subgraphs that are distinguished from typical subgraphs?" *Coreness*[1] roughly measures the maximum connectivity of the subgraphs that each node belongs to. We observe that a strong positive correlation exists between coreness and degree in real-world graphs, as shown in Figure 4.1(a). This pattern, which we call the *mirror pattern*, suggests that high-degree nodes tend to belong to a subgraph with higher connectivity. As described in Problem 4.1, our goal is to detect nodes and subgraphs that deviate from the mirror pattern, which turn out to be interesting anomalies.

Problem 4.1 (Anomalous Subgraph Detection)**.**
(1) **Given:** a graph
(2) **Find:** anomalous nodes and subgraphs
(3) **to Maximize:** deviation from "mirror pattern."

Approach: Our algorithm CORE-A first computes the *anomaly score* of each node, which measures the degree of deviation of the node from the mirror pattern as follows:

$$anomaly\_score(v) := |\log(rank_{degree}(v)) - \log(rank_{coreness}(v))|,$$

where $rank_{degree}(v)$ is the rank of node v in terms of degree and $rank_{coreness}(v)$ is that in terms of coreness. Note that computing the anomaly score of every node takes linear time because each of the following steps takes linear time:

1. computing the degree and coreness of every node using [BZ03]
2. computing the ranks of each node using a linear-time sorting algorithm (e.g., radix sort).

Then, CORE-A detects a subgraph composed of nodes deviating from the mirror pattern. Specifically, CORE-A aims to find the subgraph that maximizes a weighted sum of (a) the density of the subgraph (i.e., the number of edges divided by the number of nodes) and (b) the average anomaly score

[1] The *coreness* of a node is the largest k such that it belongs to the k-*core*, which is the maximal subgraph where every node has degree at least k within the subgraph.

(a) Pattern: Mirror pattern (b) Effectiveness of CORE-A: follower booster in Twitter (c) Accuracy of CORE-A

Figure 4.1: (a) CORE-A, devised for Problem 4.1, detects anomalies deviating from the structural pattern shown in the figure. (b) CORE-A successfully spots interesting anomalies in real-world graphs, including a follower booster on Twitter. (c) CORE-A accurately identifies small injected anomalies that are overlooked by its competitor.

of the nodes in the subgraph. To this end, CORE-A uses a greedy search that gives a 2-approximation in near-linear time.

CORE-A+ is a slower but more accurate variant of CORE-A. The only difference is that CORE-A+ uses trussness instead of coreness when measuring the anomaly score of nodes.

Results: CORE-A successfully spots interesting anomalies in real-world graphs, including a following booster on Twitter (shown in Figure 4.1(b)), copy-and-paste bibliographies in patents, and a helicopter-shape subgraph on the web. Moreover, as shown in Figure 4.1(c), CORE-A accurately identifies up to **26× smaller injected subgraphs** than its competitor that aims to find the densest subgraph.

4.2 Detecting Dense Subtensors

"Given behavior logs with rich side information, how can we spot fraudulent lockstep behavior?"

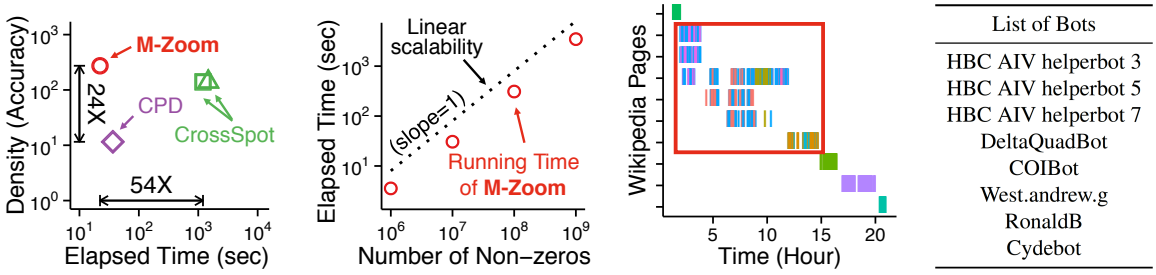
Imagine that we manage a review site (e.g., Yelp) and have the records of which accounts wrote reviews for which restaurants. How can we find suspicious *lockstep behavior*: for example, a set of accounts that give fake reviews to the same set of restaurants? This problem has been extensively studied from the perspective of dense-subgraph detection. Intuitively, in the above example, highly synchronized behavior induces dense subgraphs in the review graph of accounts and restaurants.

How can we utilize additional information to identify suspicious lockstep behavior more accurately? In the above example, the fact that reviews forming a dense subgraph were also written at about the same time, with the same keywords and number of stars, makes the reviews even more suspicious. A natural and effective way to incorporate such extra information is to model data as a tensor and find dense subtensors in it. In the following subsections, we discuss the three algorithms that we have developed for fast and accurate dense-subtensor detection in large dynamic tensors.

4.2.1 Detecting Dense Subtensors in Static Tensors

Section based on work that appeared at PKDD16 [SHF16][PDF] and TKDD18 [SHF18][PDF].

Goal: "Given a tensor, how can we find dense subtensors in near-linear time with approximation guarantees?" From the hardness of densest-subgraph detection, it follows that fining the exact densest subtensors for a general density measure, as described in Problem 4.2, is NP-hard. Our goal is to devise algorithms for Problem 4.2 that (a) have near-linear scalability, (b) provide approximation guarantees at least for some reasonable density measures, and (c) empirically outperform state-of-the-art algorithms.



(a) Fast & Accurate (b) Scalable (c) Effective: Edit Wars (d) Effective: Bots

Figure 4.2: (a) M-ZOOM, devised for Problem 4.2, is faster with similar or better accuracy than its best competitors. (b) M-ZOOM scales linearly with the size of the input tensor. (c)-(d) M-ZOOM spots interesting anomalies in real-world tensors, including edit wars and bots in Wikipedia.

Problem 4.2 (Detecting Dense Subtensors in a Static Tensor)**.**

(1) **Given:**
- a static tensor
- the number of subtensors that we aim to find: k
- a density measure: ρ

(2) **Find:** k subtensors

(3) **to Maximize:** the density in terms of ρ

Approach: Our algorithm M-ZOOM (**M**ulti-dimensional **Zoom**) repeats the following steps k times for detecting k dense subtensors:

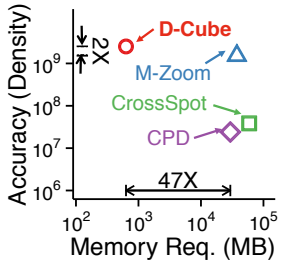
1. **Greedy search**: Starting from the input tensor, remove the slices one at a time in a greedy manner so that the density is maximized. Once all the slices are removed, return the densest subtensor among those encountered during the search.
2. **Local search** (optional): Starting from the subtensor obtained in the previous step, repeatedly add or remove a slice in a greedy manner so that the density is maximized. Once a local optimum is reached, return the current subtensor.
3. **Deflation**: Remove the entries belonging to the subtensor obtained in the previous step from the input tensor to prevent the same subtensor from being found again.

Results: As seen in Figure 4.2(a), M-ZOOM is up to $\mathbf{114}\times$ **faster** with similar or better accuracy than its best competitor in our experiments with four reasonable density measures. As shown in Theorem 4.3 and Figure 4.2(b), M-ZOOM has near-linear scalability with an approximation guarantee. Subtensors spotted by M-ZOOM in real-world tensors indicate interesting anomalies including edit wars and bot activity in Wikipedia (see Figure 4.2(c)-(d)) and network intrusion.

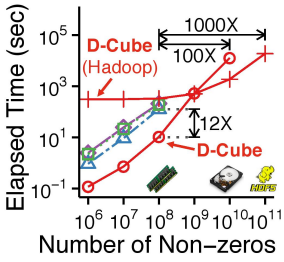
Theorem 4.3 ($\frac{1}{N}$-Approximation Guarantee of M-ZOOM [SHF18][PDF])**.** *Assume an N-order input tensor $\mathcal{T} \in \mathbb{R}^{I_1 \times \cdots \times I_N}$ with M non-zero entries. Let $\mathcal{T}^*$ be its densest subtensor and $I = \max(\{I_n\}_{n=1}^N)$ be the largest dimensionality. Then, the greedy search step of* M-ZOOM *takes $O(NM\log(I))$ returning a subtensor $\mathcal{T}'$ satisfying the following condition:*

$$\rho_{avg}(\mathcal{T}') \geq \frac{1}{N}\rho_{avg}(\mathcal{T}^*), \tag{4.1}$$

where ρ_{avg} measures the average weighted degree of the slices of a given subtensor.



(a) Efficient & Accurate



(b) Fast & Scalable

Spam Reviews
type in *** and you will get ...
type in for the bonus code: ...
typed in the code: *** ...
enter this code to start with ...
enter code: *** to win even ...

(c) Effective: Spam Reviews

Volume	Mass	Attack Type (Ratio)
1	1.93M	Smurf (**100%**)
8	2.53M	Smurf (**100%**)
6,160	897K	Neptune (**100%**)
63.5K	1.02M	Neptune (**94.7%**)
930K	1.00M	Neptune (**94.7%**)

(d) Effective: Network Attacks

Figure 4.3: (a)-(b) D-CUBE, devised for Problem 4.2 with web-scale input tensors, is more memory-efficient and scalable with similar or better accuracy than its best competitors. (c)-(d) D-CUBE spots interesting anomalies in real-world tensors, including spam reviews and network attacks.

4.2.2 Detecting Dense Subtensors in Web-scale Static Tensors

Section based on work that appeared at WSDM17 [SHKF17a][PDF] and its extension [SHKF18][PDF].

Goal: "Given a tensor that is too large to fit in memory or even on a disk, how can we find dense subtensors without sacrificing speed and accuracy provided by in-memory algorithms?" M-ZOOM, explained in the previous section, is inefficient for tensors stored on disk or distributed across multiple machines due to its highly iterative nature. For dense-subtensor detection (described in Problem 4.2) in web-scale tensors, we devise out-of-core and distributed algorithms. Specifically, our goal is to design algorithms that require few scans of data while providing the same accuracy guarantee of M-ZOOM.

Approach: Our algorithm D-CUBE (**D**isk-based **D**ense-block **D**etection) repeats the following steps k times for detecting k dense subtensors:

1. **Aggressive greedy search**: Starting from the input tensor, repeats the following substeps until all the slices are removed:
   1-1. choose a mode n with largest dimensionality (or choose a mode n greedily)
   1-2. remove the n-mode slices whose weighted degree is less than or equal to θ (≥ 1) times the average weighted degree of the n-mode slices.

   Return the densest subtensor among those encountered during the search.
2. **Deflation**: Remove the entries belonging to the subtensor obtained in the previous step from the input tensor to prevent the same subtensor from being found again.

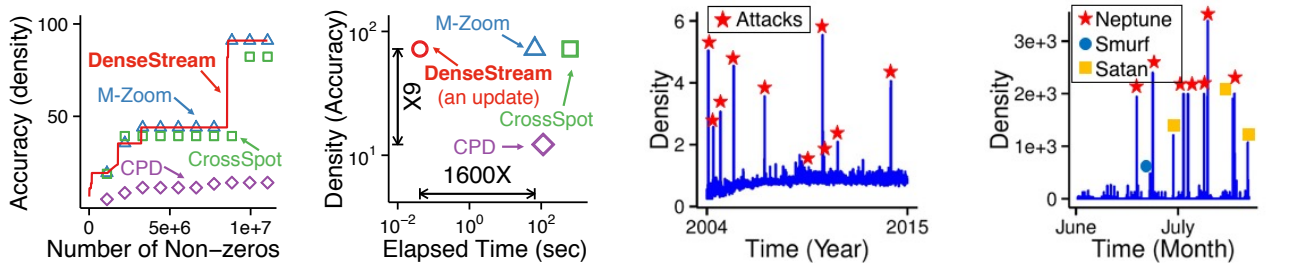
Note that the greedy search step of D-CUBE removes slices more aggressively requiring fewer scans of the input tensor than that of M-ZOOM.

Results: As shown in Figure 4.3(a) and Theorem 4.4, D-CUBE requires up to **1600× less memory** and fewer scans of the input tensor than M-ZOOM, while achieving theoretically and empirically similar accuracy. As a result, as seen in Figure 4.3(b), D-CUBE, especially its Hadoop implementation, successfully handles terabyte-scale tensors. Subtensors spotted by D-CUBE in real-world tensors indicate interesting anomalies, including spam reviews and network intrusion (see Figures 4.3(c)-(d)).

Theorem 4.4 ($\frac{1}{\theta N}$-Approximation Guarantee of D-CUBE [SHKF18][PDF]). *Assume an N-order input tensor $\mathcal{T} \in \mathbb{R}^{I_1 \times \cdots \times I_N}$ with M non-zero entries. Let $\mathcal{T}^*$ be its densest subtensor and $I = \max(\{I_n\}_{n=1}^N)$ be the largest dimensionality. Then, the greedy search step of* D-CUBE *requires $O(N \min(\log_\theta I, I))$ scans of the input tensor, returning a subtensor $\mathcal{T}'$ satisfying the following condition:*

$$\rho_{avg}(\mathcal{T}') \geq \frac{1}{\theta N} \rho_{avg}(\mathcal{T}^*),$$

where ρ_{avg} measures the average weighted degree of the slices of a given subtensor.



(a) 'Any Time' & Accurate (b) Fast & Accuracy (c) Effective: Rating Attacks (d) Effective: Network Attacks

Figure 4.4: (a)-(b) DENSESTREAM, devised for Problem 4.5, maintains and updates a dense subtensor rapidly as accurate as the best batch algorithms. (c)-(d) In real-world dynamic tensors, DENSESTREAM spots interesting anomalies, including rating manipulations and network attacks.

4.2.3 Detecting Dense Subtensors in Dynamic Tensors

Section based on work that appeared at KDD17 [SHKF17b][PDF].

Goal: "Given a dynamic tensor that changes over time, how can we maintain and update a dense subtensor with approximation guarantees?" For real-time detection of fraudulent lock-step behavior, we develop an incremental algorithm that detects a dense subtensor in a dynamic tensor whose entries can both increase and decrease over time. Our goal is to maintain and update a dense subtensor significantly faster than running batch algorithms while providing the same accuracy guarantee. Different from the previous task, we use one specific density measure for this task, as described in Problem 4.5.

Problem 4.5 (Detecting a Dense Subtensor in a Dynamic Tensor)**.**
(1) **Given:** a dynamic tensor that changes over time
(2) **Maintain & Update:** a subtensor
(3) **to Maximize:** the density in terms of ρ_{avg} (i.e., the average weighted degree of the slices).

Approach: In addition to the maintained subtensor, our algorithm DENSESTREAM maintains a summary of the N-order input tensor that is sufficient to (a) decide whether the maintained subtensor needs to be updated and (b) find a dense subtensor with a $\frac{1}{N}$-approximation guarantee. The summary consists of a *D-order* of the slices, which is an order by which M-ZOOM (explained in Section 4.2.1) may remove the slices, and two real numbers per slice.

Whenever there is a change in the input tensor, DENSESTREAM performs the following steps:

1. **Find Range**: Find a range of the maintained D-order that needs to be reordered.
2. **Reorder**: Reorder the slices in the range from the previous step and update their statistics.
3. **Update Subtensor** (performed only when necessary): Find a new dense subtensor rapidly using the D-order from the previous step, and replace the old subtensor with the new one.

Empirically, the region in the first step is usually narrow and the last step is rarely performed.

Results: Our algorithm DENSESTREAM maintains a dense subtensor as accurate as the best batch algorithms, as seen in Figure 4.4(a). Moreover, DENSESTREAM gives a $\frac{1}{N}$-approximation guarantee (i.e., Eq. (4.1) in Theorem 4.3). As shown in Figure 4.4(b), an update by DENSESTREAM is up to **a million times faster** than running the batch algorithms. In dynamic tensors where only recent entries are maintained (while old entries are discarded), DENSESTREAM detects interesting anomalies, including rating manipulations and network attacks, shown in Figure 4.4(c)-(d).

Chapter 5

Behavior Modeling

We present our completed and proposed work on behavior modeling. For each completed task, we briefly summarize our goal, approach, and results. References for detailed discussions are given at the beginning of each section. For the proposed task, we give a brief description of our goal and plan.

5.1 Modeling Progressive Behavior on Social Media

Section based on work that will appear at WWW18 [SSK+18][PDF].

Goal: "How do the behaviors of users in a web service progress over time?" Given behavior logs, which are modeled as a 3-order tensor whose modes are users, features, and timestamps, our goal is to discover progression stages that many users go through. Specifically, through the stages, we aim to capture the followings:

- changes in frequently-used features,
- changes in the frequency of use,
- different directions of progression (unless every user follows the same direction).

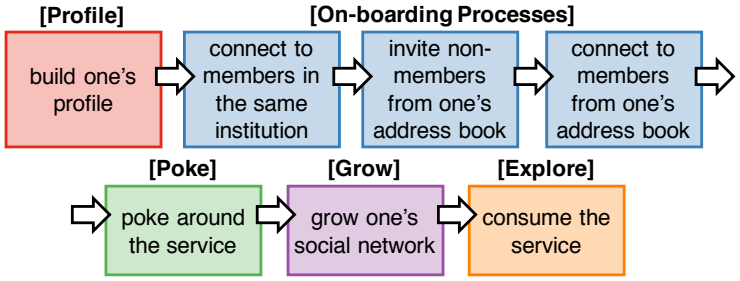
We also aim to address scalability issues in handling web-scale behavior logs. An informal definition of progressive behavior modeling is provided in Problem 5.1.

Problem 5.1 (Progressive Behavior Modeling)**.**
(1) **Given:**
- a behavior log (i.e., a 3-order tensor whose modes are users, features, and timestamps)
- the number of desired stages: k

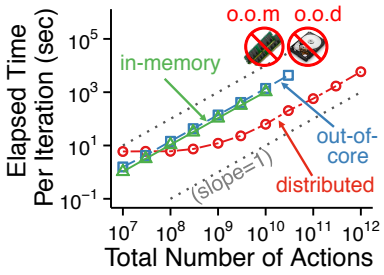
(2) **Segment:** the records in the log into k stages
(3) **to Best Describe:** the progression of the users in the log

Approach: We build a behavior model where types of actions (or types of used features), time gaps between consecutive actions, and transitions to next stages depend on the current stage that each user lies in. In our model, to capture progression (instead of cyclic patterns), we assume that there is an order among the stages, and we allow the users to progress to next stages but not to return to previous stages. This constraint is exploited also when we devise parallel, out-of-core, and distributed optimization algorithms to fit our model to log data. Our algorithms repeat the following two steps until convergence:

- **Assignment:** assign the actions to stages while fixing the probability distributions in the stages,
- **Update:** update the probability distributions in the stages while fixing the stage assignment.



(a) Effectiveness: prgressive behavior of members of a social networking service



(b) Speed & Scalability

Figure 5.1: (a) Summary of the latent progression stages that our model discovers in a dataset from a social networking service. The second, third, and fourth stages correspond to the onboarding steps that the service provides to new users. (b) Linear scalability of our optimization algorithms (o.o.m: out of memory & o.o.d: out of disk). The distributed version successfully handles a billion-scale dataset.

Results: Our model discovers meaningful stages in real-world datasets from a social networking service, as seen in Figure 5.1(a). We also show that our trained models are useful for downstream tasks such as prediction of future actions. As seen in Figure 5.1(b), our algorithms scale linearly with the size of data. Especially, the distributed version successfully handles a dataset with one billion actions.

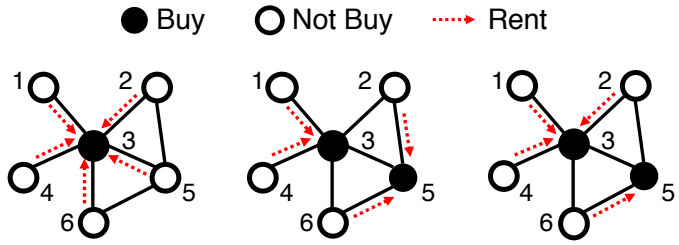
5.2 Modeling Purchasing Behavior in Social Networks

Section based on work that appeared at IJCAI17 [SLEP17][PDF].

Goal: "How does the underlying social network affect the individuals' purchase of sharable goods (e.g., ski gear, hiking equipment, etc.)?" We investigate the incentive to buy a good sharable with k-hop neighbors (i.e., the nodes within k hops from a buyer) to answer the following questions:

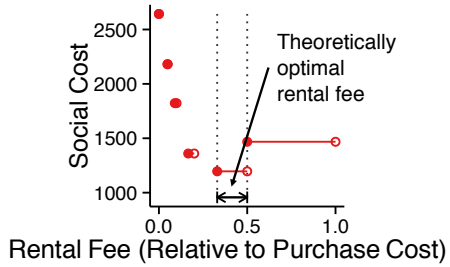
- How does the connectivity of a node relate to its tendency to buy such a sharable good?
- How inefficient can individually optimal choices be compared to socially optimal choices?
- How does the inefficiency depend on the structure of the social network and rental fees?

Approach: We build a game-theoretic model where each node decides whether to (a) buy a good or (b) rent a good from one of its k-hop neighbors who buy a good. In our model, each node prefers to buy a good if at least ξ nodes rent its good ($\xi = \infty$ if no rental fee is charged) or it does not have a k-hop neighbor who buys a good. Otherwise, each node prefers to rent a good. We define the *inefficiency* of a Nash equilibrium (NE) as the the number of buyers in the NE divided by that in socially optimal



(a) NE (cost= 1) (b) NE (cost= 2) (c) Not NE

Figure 5.2: Example outcomes of our game-theoretic model when $k = 1$ and $\xi = 2$: (a) an efficient Nash equilibrium (NE) with the minimum number of buyers, (b) an inefficient NE with more buyers, (c) an outcome that is not an NE (Node 5 prefers renting a good from node 3 to buying a good).



(a) Inefficiency of NEs in our simulation ($k = 1$)

		$\xi = \infty$	$\xi < \infty$
$k = 1$	PoA	$\Theta(n)$	$\Theta(n)$
	PoS		$\Theta(\xi)$ ($= \mathbf{1}$ if $\xi \leq 2$)
$k > 1$	PoA	$\Theta(n/k)$	$\Theta(\max(n/k, n/\xi))$
	PoS		$\Theta(\xi/k)$ ($= \mathbf{1}$ if $\xi \leq 2\lfloor k/2 \rfloor + 1$)

(d) Inefficiency of NEs in our analysis

Figure 5.3: Summary of our simulation and theoretical analysis of the inefficiency of Nash Equilibria (NEs). The inefficiency of NEs tends to decrease as we increase the rental fee (i.e., as we decrease ξ).

outcomes (see Figure 5.2 for examples). We analyze the inefficiency of NEs experimentally using simulations of best-response dynamics and theoretically in terms of the *price of anarchy* (PoA) (i.e., inefficiency of the worst NE) and the *price of stability* (PoS) (i.e., inefficiency of the best NE)

Results: As seen in Figure 5.3, both our simulation and theoretical analysis show that the inefficiency of NEs tends to decrease as we increase the rental fee (i.e., as we decrease ξ). This is because a higher rental fee gives nodes with high connectivity, who can share their goods with many other nodes, more incentives to buy a good. Moreover, our simulation shows that the inefficiency tends to be high in social networks with highly skewed degree distributions (e.g., star graphs).

5.3 [Proposed] Modeling Polarization in Social Networks

"How does individuals' perspective on controversial issues (e.g., marijuana, abortion, etc.) affect the dynamics of the underlying social network and vice versa?" Our goal is to study the (a) effects of individuals' belief on the dynamics of the underlying social network and (b) the effects of the social network on the individuals' belief. For example, one question we have is how each node in a social network chooses between (a) aligning its belief to that of its current neighbors and (b) changing its edges towards nodes with the same belief?

To this end, we will build a game-theoretic model where the utility of each node in a social network depends on factors such as:

- cost of changing one's belief
- the number of neighbors with the same belief
- the number of triangles that each node forms with its neighbors with the same belief.

We plan to fit our model (e.g., by [Kos04]) to real-world datasets (e.g., datasets collected from the NetSense project [SLM+13]) to measure how much each factor contributes to the utility. We also plan to use our model to predict future beliefs of nodes, appearance of edges, and disappearance of edges.

Chapter 6

Timelines

My expected timeline is provided in Table 6.1.

Table 6.1: My expected timeline.

Time	Tasks	Expected outputs
March 2018	thesis proposal	
March–May 2018	S1-3: counting triangles in graph streams with deletions (§ 3.1.3)	a new conference paper
June–August 2018	B3: modeling polarization in social networks (§ 5.3)	a new conference paper
September–October 2018	S2-2: decomposing tensors with skewed degree distributions (§ 3.3.2)	journal extension of [OSP+17]
November 2018–February 2019	thesis writing & job applications	
March–April 2019	interviewing	
May 2019	thesis defense	

Chapter 7

Conclusions

In this thesis, we address the problem of mining large dynamic graphs and tensors. Specifically, we develop models and algorithms for structure analysis, anomaly detection, and behavior modeling.

Structure analysis (Chapter 3): We develop streaming algorithms for three important connectivity measures: global triangle count, local triangle counts, and degeneracy. For accuracy, our algorithms exploit structural and temporal patterns in real graphs and utilize multiple machines. We also improve the scalability of Tucker Decomposition, which is widely used to summarize tensors, by reducing intermediate data. We propose to extend our streaming algorithms to fully dynamic graphs with deletions and exploit structural patterns in real tensors to speed up Tucker Decomposition.

Anomaly detection (Chapter 4): We devise an algorithm that seeks subgraphs deviating from a structural pattern. Such subgraphs signal various anomalies including a follower-booster in Twitter and copy-and-paste bibliographies. We also develop an algorithm that searches densest subtensors in near-linear time with approximation guarantees. The algorithm and its extensions to web-scale and dynamic tensors successfully spot spam reviews, rating manipulations, network attacks, bots, etc.

Behavior modeling (Chapter 5): We model progression of users in a web service as transitions between stages and develop scalable optimization algorithms that fit our stage model to billion-scale behavior logs. In addition, we model the purchase of goods sharable with neighbors in a social network as a game among the nodes. We use our model to analyze the effects of structure of social networks and rental fees on social inefficiency. Then, we propose to design a game-theoretic model that relates beliefs of nodes to the dynamics of the underlying social network, which we aim to predict.

For reproducibility and the benefit of the community, we make most of the algorithms and datasets used throughout this thesis available at `http://www.cs.cmu.edu/~kijungs/proposal/`.

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Appendix A

Related Work

Analyzing Structure of Graphs and Tensors

Triangle Counting (Section 3.1): Counting triangles is a classical graph problem with numerous applications. Examples relevant to structure analysis include dense subgraph mining [WC12], and web analysis [EM02]. Moreover, many important concepts in social network analysis, such as social balance and the clustering coefficients, are based on the count of triangles [WF94]. Due to this importance, many algorithms have been proposed for triangle counting in different settings. For streaming settings, which this thesis focuses on, various sampling techniques have been developed for global triangle counting [TKMF09, JSP13, PTTW13] and local triangle counting [LK15, SERU17].

k-Cores and Degeneracy (Section 3.2): k-cores and degeneracy, defined in Section 2.1, have been used extensively in diverse applications. Examples related to structure analysis include hierarchy analysis [AhBV08], visualization [AHDBV06], and graph clustering [GMTV14]. For the k-cores and degeneracy in a graph fitting in memory, [BZ03] proposed a linear-time algorithm. For larger graphs, [FCT14, GSERF16] proposed semi-streaming algorithms, which require multiple passes over data.

Tensor Decomposition (Section 3.3): Decomposing a tensor into a small core tensor and low-rank factor matrices is a widely-used summarization technique in many fields, including chemometrics [Bro98], neuroscience [DGCW13], and signal processing [SDLF+17]. Among a variety of decomposition methods (see a survey [KB09]), the most widely-used methods in data mining are CP Decomposition and Tucker Decomposition. Numerous techniques, including sampling [PFS15] and distributed computing [KPHF12, SSK17], have been proposed for scalable CP Decomposition. Regarding scalable Tucker Decomposition, previous work [KS08, JPF+16] focused on reducing intermediate data.

Detecting Anomalies in Graphs and Tensors

Anomaly Detection in Graphs (Section 4.1): A number of approaches have been proposed for detecting anomalies or fraud in graphs; see a survey [ATK15]. [PCWF07, RA15] proposed semi-supervised approaches based on belief propagation. [PSS+10, JCB+14, SBGF14] proposed unsupervised approaches based on the eigendecomposition and SVD of the adjacency matrix. Other unsupervised approaches use egonet features [AMF10], clustering [BXG+13], or greedy search [HSS+17]. The aforementioned approaches identify lock-step followers on Weibo [JCB+14], spam reviews on Yelp [RA15], auction fraud on eBay [PCWF07], 'Like' boosting on Facebook [BXG+13], etc.

Anomaly Detection in Tensors (Section 4.2): Tensor decomposition and dense-subtensor detection are two major approaches for identifying anomalies in tensors. See a survey [FTG16] for anomaly detection based on tensor decomposition. Regarding dense-subtensor detection, [JBC+16] proposed a density measure and a heuristic local search algorithm for maximizing the measure. For further analyses of detected dense subtensors, [LHS+17] proposed prioritizing the entities forming the dense subtensors based on their contributions to the subtensors. Our work on dense-subtensor detection is also related to theoretical work on densest-subgraph detection, especially [KS09, BKV12, TBG+13].

Modeling Behavior in Social Networks and Web

Modeling Progressive Behavior (Section 5.1): Our work is related to the large body of literature on clustering event sequences, which has typically been used to obtain the overall view of user behaviors in certain aspects, such as navigation patterns on websites [CHM+00], clusters of user types [WZT+16], and topics of event streams [DFA+15]. Notably, grouping events was also used to present each entity's progression and thus to account for relationships between groups of events [SYS+13], regime shifts [MS16], and evolution of users [ML13]. In particular, [YML+14] modeled the progression of entities as transitions between latent stages to distinguish different patterns of progression.

Modeling Purchasing Behavior (Section 5.2): See [GGJ+10, JZ14] for an introduction to games on networks in general. [BK07, BKD14, BCAZ06] studied a game on a network where each node decides its contribution to a public good. In the game, the utility of each node depends on its contribution and its neighbors'. Extended games with direct edges [EG13] and the combination of public and private goods [All15] were considered. Our work is also related to the huge body of literature on the price of anarchy [KP99] and the price of stability [ADK+08].

Modeling Polarization (Section 5.3): See [SVdBS10] for an introduction to actor-based models, which interpret the dynamics in a network as a stochastic process driven by the nodes. Actor-based models have considered a variety of effects, including triadic effects [HL71], degree-related effects [BA99, New02], and exogenous effects [MSLC01]. Especially, our proposed work is closely related to [WHL13], where it was observed that nodes with similar drinking behaviors are more likely to be connected and nodes tend to align their drinking behaviors to those of their neighbors.

Appendix B

Datasets

The datasets used throughout the thesis are summarized in Tables B.1 and B.2. Most of the datasets are available at `http://www.cs.cmu.edu/~kijungs/proposal/`.

Table B.1: Graphs used throughout thesis thesis. Most of them are available at http://www.cs.cmu.edu/~kijungs/proposal/.

Name	Description	# Nodes	# Edges	Related Figures
KarateClub	Social Network	34	78	
Hamster	Friendship Network	1.86K	12.6K	3.3(a)
Facebook	Friendship Network	63.7K	817K	
Catster	Friendship Network	150K	5.45M	3.3(a), 4.1(a)
YouTube	Friendship Network	1.13M	2.99M	3.3(a), 4.1(c)
Flickr	Friendship Network	1.72M	15.6M	3.3(a), 3.3(b)
Flickr(L)	Friendship Network	2.30M	22.8M	3.2(a)
Orkut	Friendship Network	3.07M	117M	3.3(a)
YouTube(L)	Friendship Network	3.22M	9.38M	
LiveJournal	Friendship Network	4.00M	34.7M	3.3(a)
Friendster	Friendship Network	65.6M	1.81B	3.3(a)
Advogato	Trust Network	5.16K	51.1K	5.3
Epinion	Truest Network	132K	711K	
Twitter	Subscription Network	41.7M	1.20B	1.1(a), 3.3(a), 4.1(b)
Email	Email Network	36.7K	184K	3.3(a)
Email(L)	Email Network	87.0K	297K	
Stanford	Web Graph	282K	1.99M	3.3(a)
NotreDame	Web Graph	326K	1.09M	3.3(a)
BerkStan	Web Graph	685K	6.65M	
Google	Web Graph	876K	5.10M	3.2(b)
Caida	Internet Topology	26.5K	53.4K	3.3(a)
Skitter	Internet Topology	1.70M	11.1M	3.3(a)
HepTh	Citation Network	27.8K	352K	3.3(a)
HepPh	Citation Network	34.5K	422K	3.1(a), 3.1(b), 3.1(c)
Patent	Citation Network	3.77M	16.5M	3.3(a)

Table B.2: Tensors used throughout thesis thesis. Most of them are available at http://www.cs.cmu.edu/~kijungs/proposal/.

Name	Modes	Entries	Dimensionality	# Nonzeros	Related Figures
Youtube	User × User × Date	1 (Friend) or 0	3.22M × 3.22M × 203	18.7M	4.4(b)
SMS	User × User × Timestamp	# Messages	1.25M × 7.00M × 4.39K	103M	
StackOver.	User × Post × Timestamp	1 (Like) or 0	545K × 96.7K × 1.15K	1.30M	
Wiki (Kor)	User × Page × Timestamp	# Revisions	470K × 1.18M × 101K	11.0M	4.2(a), 4.2(c), 4.3(a)
Wiki (Eng)	User × Page × Timestamp	# Revisions	44.1M × 38.5M × 129K	483M	4.2(d)
Yelp	User × Business × Date × Score	# Reviews	552K × 77.1K × 3.80K × 5	2.23M	
AppMarket	User × App × Date × Score	# Reviews	967K × 15.1K × 1.38K × 5	1.13M	1.1(b), 4.3(c)
Android	User × App × Date × Score	# Reviews	1.32M × 61.3K × 1.28K × 5	2.64M	4.4(c)
Netflix	User × Movie × Date × Score	# Reviews	480K × 17.8K × 2.18K × 5	99.1M	
YahooMusic	User × Item × Date × Score ×	# Reviews	1.00M × 625K × 84.4K × 101	253M	
Academic	Author × Venue × Year × Keyword	# Papers	9.38M × 18.9K × 2.02K × 37.0K	35.4M	3.5(a), 3.5(b)
DARPA	Src IP × Dst IP × Timestamp	# Connections	9.48K × 23.4K × 46.6K	522K	1.1(c), 4.4(a), 4.4(d)
AirForce	Protocol × Service × Flag × Src-bytes × Dst-bytes × Counts × Srv-counts	# Connections	3 × 70 × 11 × 7.20K × 21.5K × 512 × 512	648K	4.3(d)
LinkedIn	User × Feature × Timestamp	# Uses	Not Available		5.1(a)