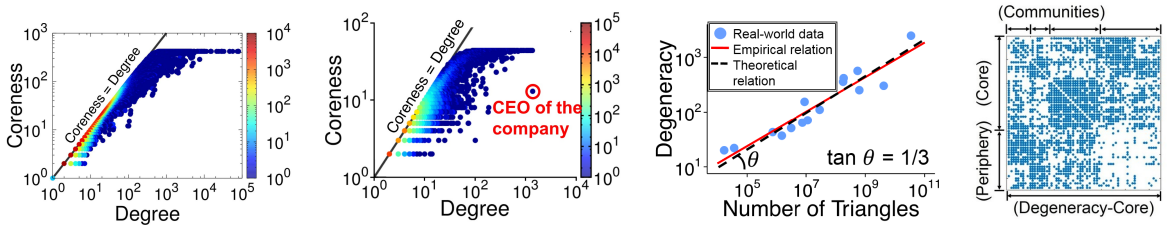
Patterns and Anomalies in k-Cores of Real-world Networks

Kijung Shin,* Tina Eliassi-Rad,† and Christos Faloutsos*

∗ Carnegie Mellon University, Pittsburgh, PA, USA
† Northeastern University, Boston, MA, USA

Keywords: k-cores, degeneracy, coreness, patterns, anomalies.

The k-core of an undirected graph G is the maximal subgraph of G where every node is adjacent to at least k nodes. Introduced by Seidman in 1983 [1], k-cores have been utilized in many applications [2]. Examples include graph topology characterization, hierarchical structure analysis, graph visualization, protein function prediction, and community discovery. From k-cores, we have the following definitions: (1) the *coreness of a node*, which is the highest k value such that the node is in the k-core; (2) the *degeneracy of a graph*, which is the highest value k such that the k-core exists; and (3) the *degeneracy-core of a graph*, which is the k-core with k equal to the degeneracy value. From a recent study on numerous social networks, Web graphs, Internet topology graphs and citation networks, we found the following patterns and anomalies [2]. **Mirror Pattern**: There is a strong correlation between a node's degree and its coreness with a "Boomerang" shape distribution. Nodes that deviate from this pattern fall into two anomaly categories. **Loner Star Anomaly**: These are nodes who rank higher in terms of degree but relatively lower in terms of coreness such as the email account of a company's CEO. **Lockstep Behavior Anomaly**: These are nodes who rank higher in terms of coreness but relatively lower in terms of degree; they are involved in a follower-boosting service, copy-and-paste behavior, or an isolated near-clique. **Core-triangle Pattern**: There is a power-law relationship between degeneracy and triangle count – namely, $degeneracy \propto (\#\ triangles)^{1/3}$. **Structured-core Pattern**: Degeneracy-cores have non-trivial structures. In particular, we observe core-periphery and community structures in degeneracy-cores.



(a) Mirror Pattern (b) Mirror Pattern & Loner Star Anomaly (c) Core-triangle Pattern (d) Structured-core Pattern

Figure 1: (a) Mirror Pattern: Coreness and degree are strongly correlated. (b) Loner Star Anomaly (higer rank in degree and lower rank in coreness) in a Mirror Pattern. (c) Core-triangle Pattern: Degeneracy and the number of triangles in graphs obey a *3-to-1* power law. (d) Structured-core Pattern: Degeneracy-cores have structure, such as core-periphery and communities.

References

[1] Stephen B Seidman. Network structure and minimum degree. *Social networks*, 5(3):269–287, 1983.

[2] Kijung Shin, Tina Eliassi-Rad, and Christos Faloutsos. CoreScope: Graph mining using k-core analysis – patterns, anomalies and algorithms. In *ICDM*, 2016.