Carnegie Mellon
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CVPR 2014 Tutorial on Visual SLAM
Large Scale – Reducing Computational Cost

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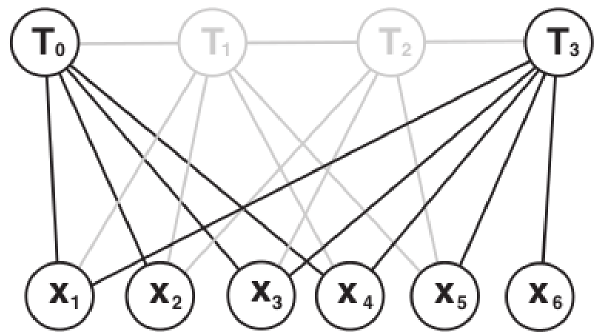
Large-Scale Visual SLAM

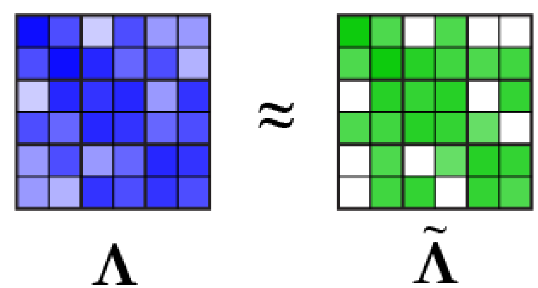
Computational cost grows with time

Two approaches to reduce cost:

- Formulation
  - Keyframes
  - Submaps
  - Reduced pose graph



- Simplification
  - Cut old data
  - Sparsification



Large-Scale Visual SLAM

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- Formulation
  - Keyframes
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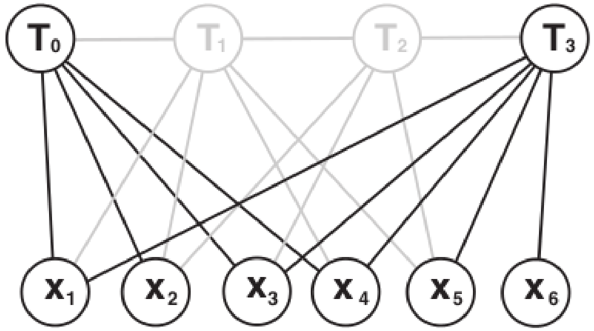
  - **Reduced pose graph**



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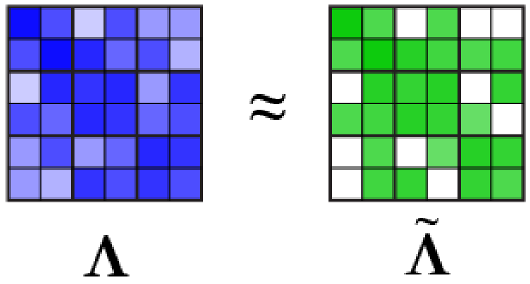
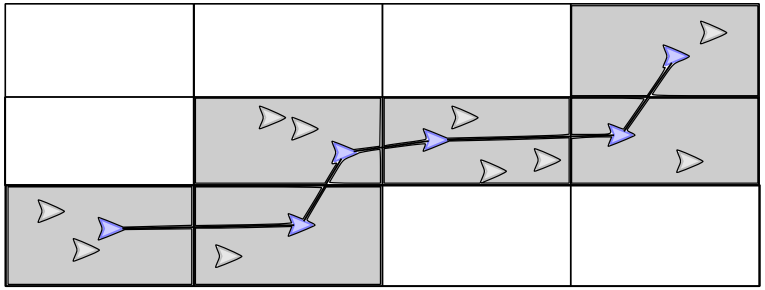


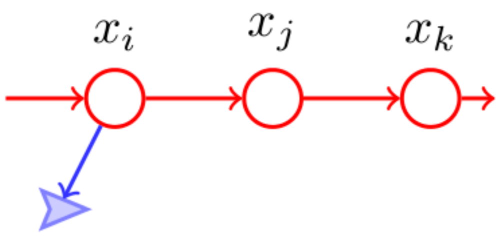
Reduced pose graph

- Key-frame approach
- Reuses existing poses
- Grows with explored space, not time
- Partitions the environment
  - Maintains a set of poses that cover all the partitions

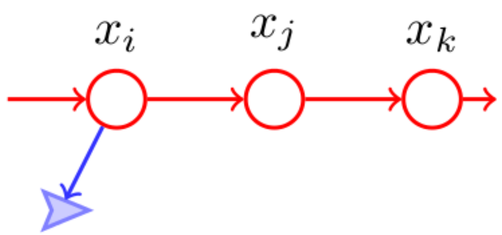


Reduced Pose Graph (step n) - Construction

In general, not revisiting exactly same poses



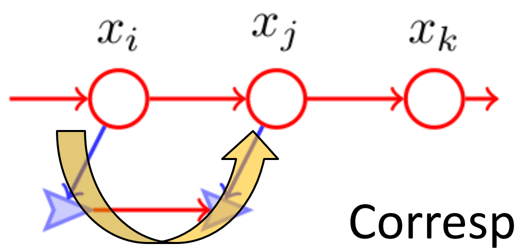
Standard pose graph:



[Johannsson, Kaess, Fallon, Leonard, ICRA 13]

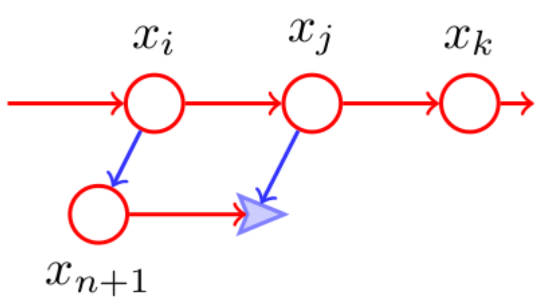
Reduced Pose Graph (step n+1)

In general, not revisiting exactly same poses



Corresponds to a constraint between x_i and x_j

Standard pose graph:

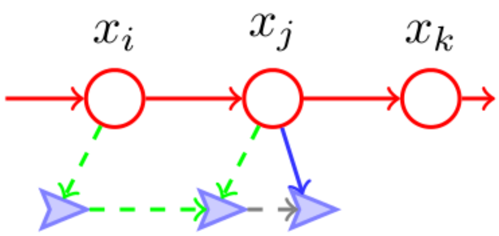


New pose is added

[Johannsson, Kaess, Fallon, Leonard, ICRA 13]

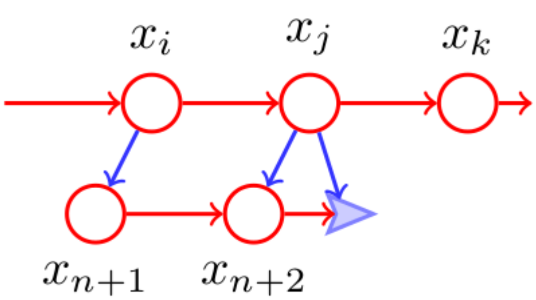
Reduced Pose Graph (step n+2)

Avoiding inconsistency



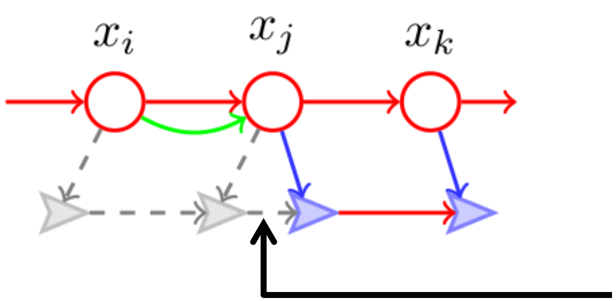
Second loop closure to x_j to avoid double use of constraint

Standard pose graph:



Reduced Pose Graph (step n+3)

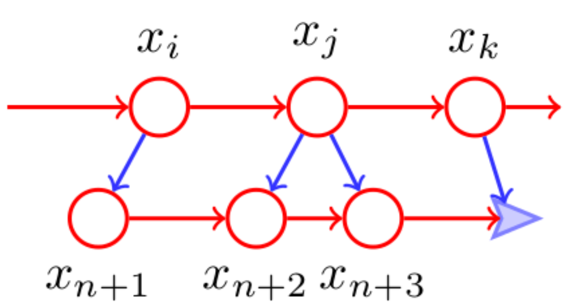
Avoiding inconsistency



Constraint between x_i and x_j added

Omitting short odometry links

Standard pose graph:



Long-term Visual Mapping

MIT Stata Center Dataset (publicly available)

- Duration: 6 months
- Operation time: 9 hours
- Distance travelled: 11 km (about 7 miles)
- VO keyframes: 630K

[Johannsson, Kaess, Fallon, Leonard, ICRA 13]

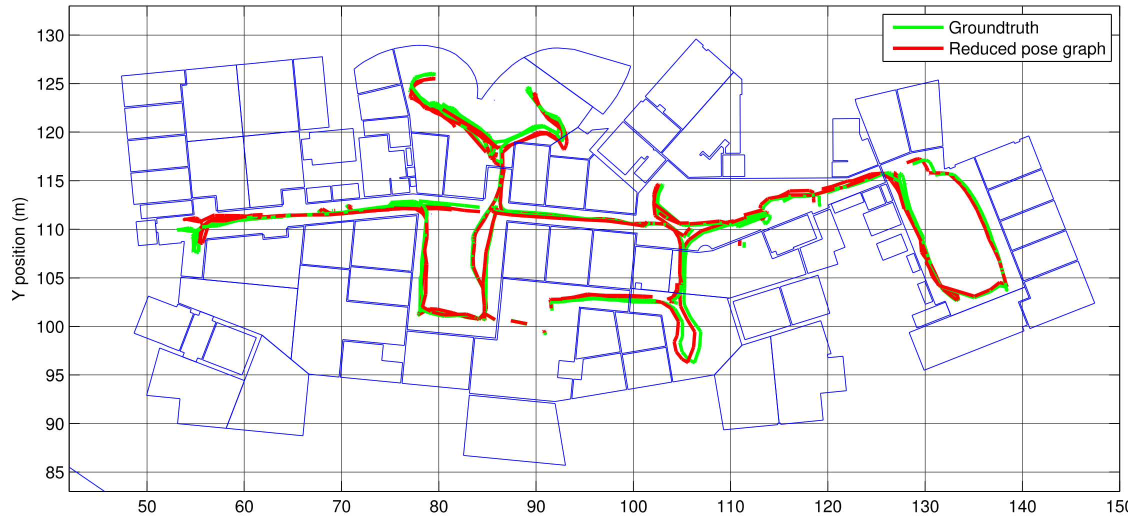
Reduced Pose Graph – Second Floor

iSAM optimizes reduced pose graph

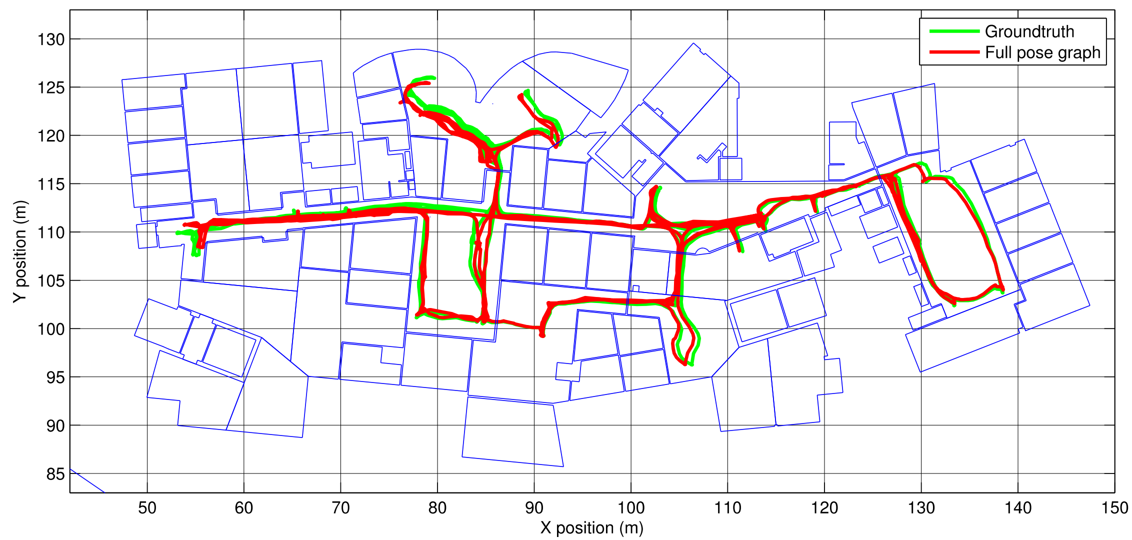
Comparison of full vs reduced pose graph

- 4 Hours of data

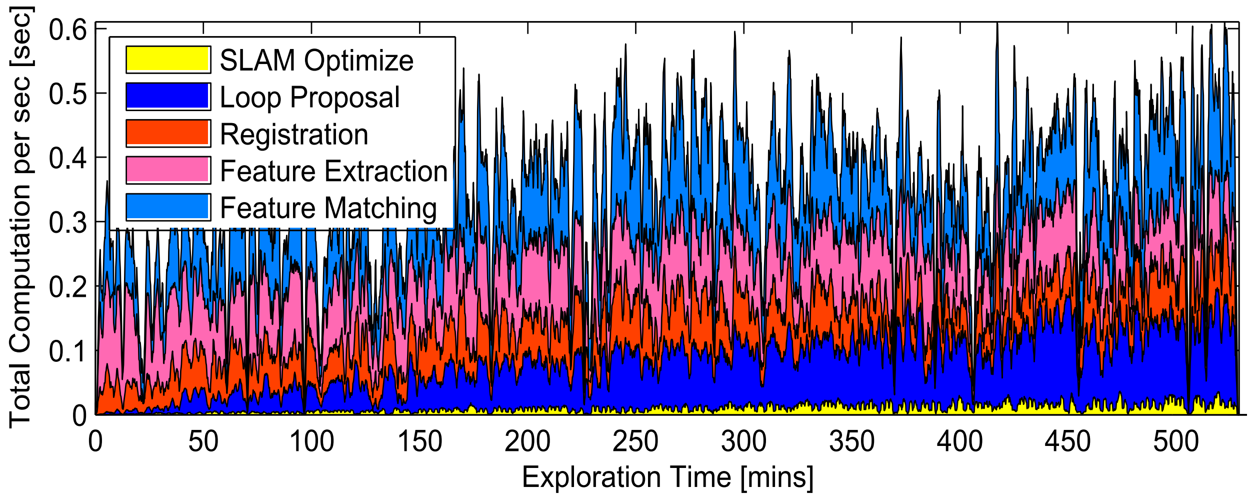
- **Reduced pose graph**
  # Poses 1363
  Mean error 0.44m

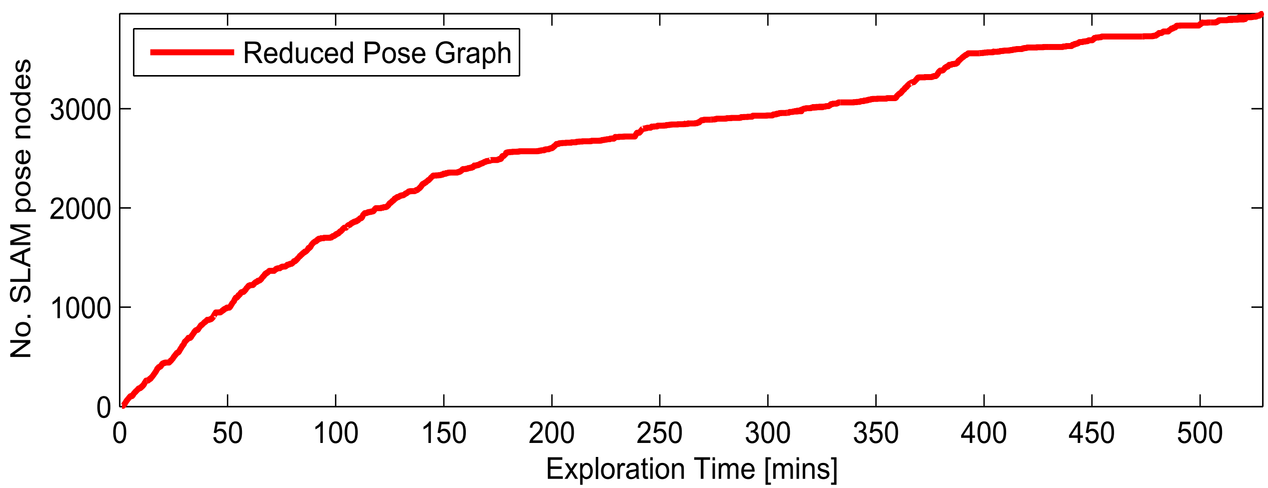


- **Full pose graph**
  # Poses 28520
  Mean error 0.37m



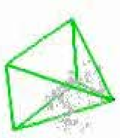
Timing (approx. 9 hours of mission)





[Johannsson, Kaess, Fallon, Leonard, ICRA 13]

Reduced Pose Graph – 10 Floors



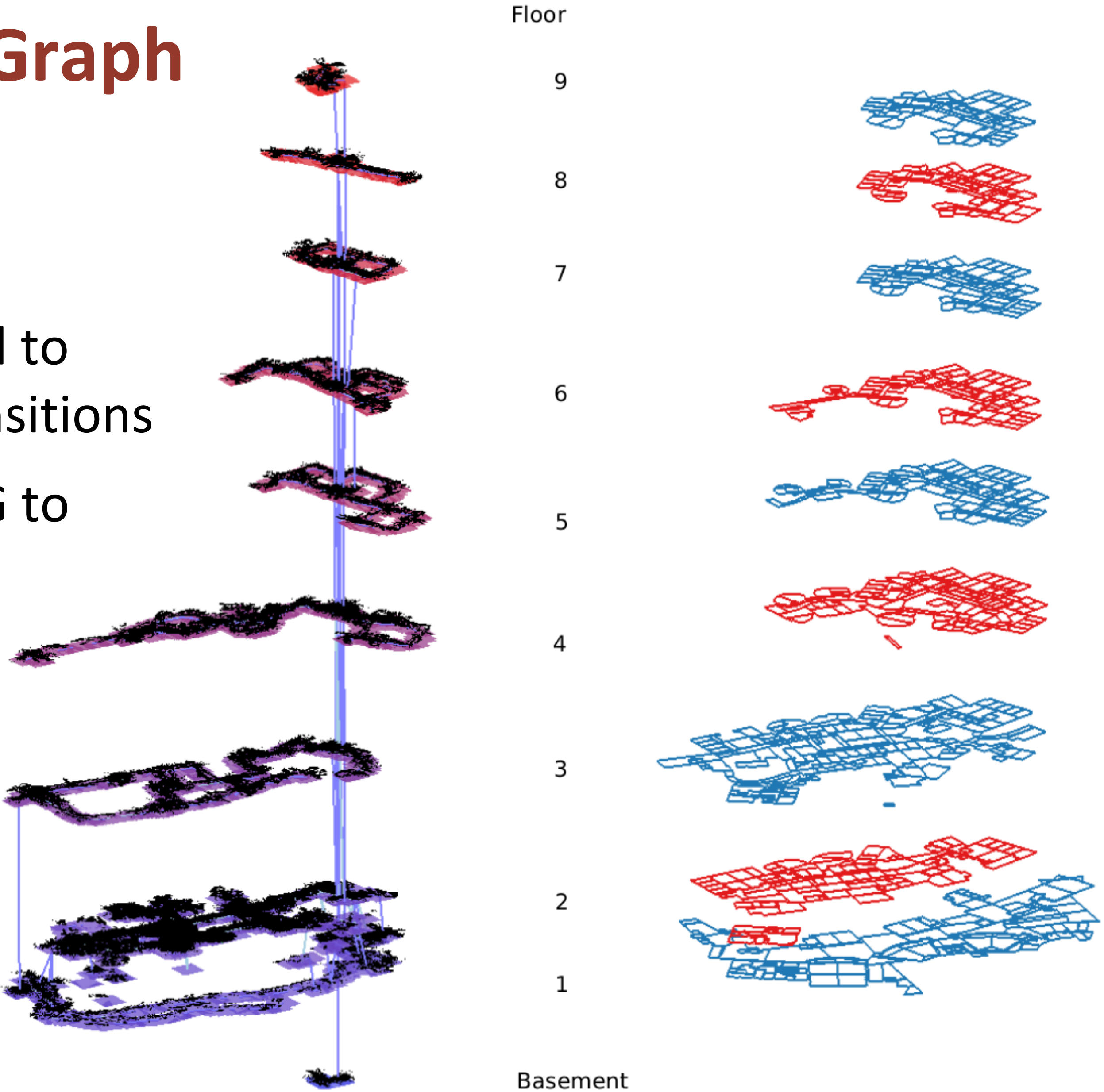
iSAM optimizes reduced pose graph

[Johannsson, Kaess, Fallon, Leonard, ICRA 13]

Reduced Pose Graph

Map of 10 floors

- Accelerometer used to detect elevator transitions
- iSAM optimizes RPG to achieve real-time



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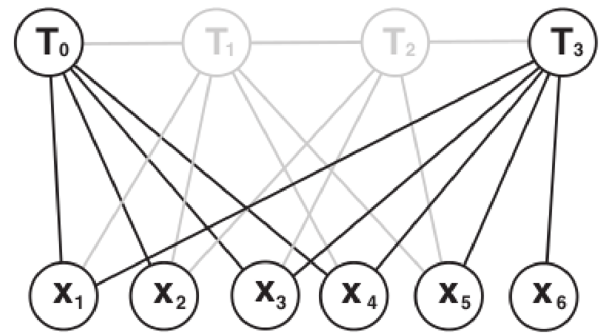
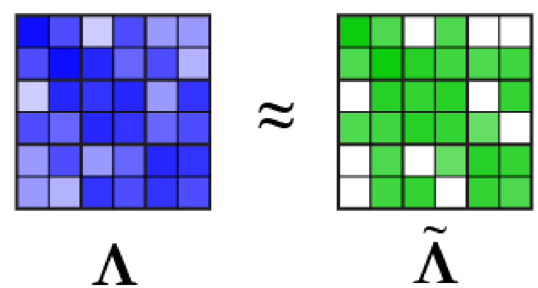
  - Submaps
  - Reduced pose graph



- Simplification
  - Cut old data
  - **Sparsification**



Sparsification: Factor Graph Node Removal

- Control complexity of performing inference in graph
  - Long-term multi-session SLAM
  - Reduces the size of graph
  - Storage and transmission
- Graph maintenance
  - Forgetting old views

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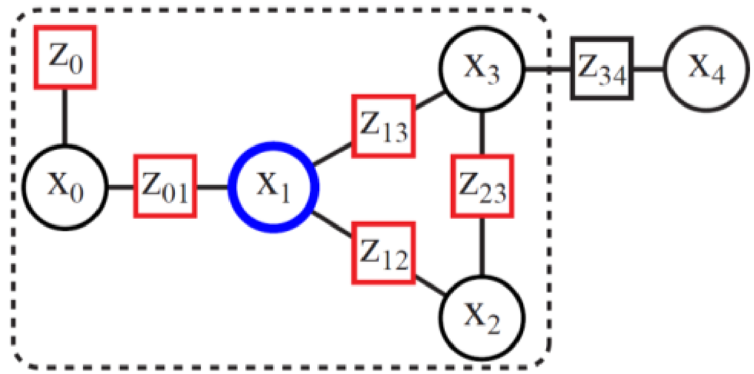
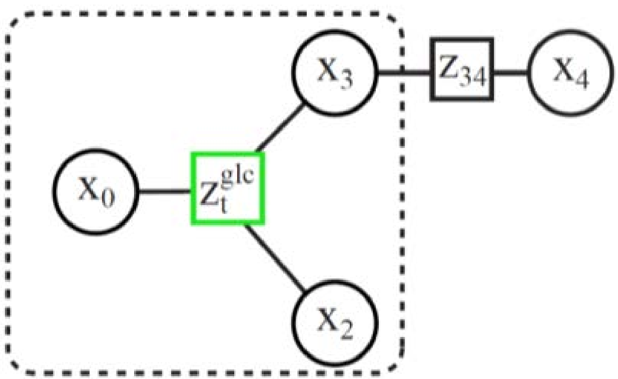
Sparsification: Factor Graph Node Removal

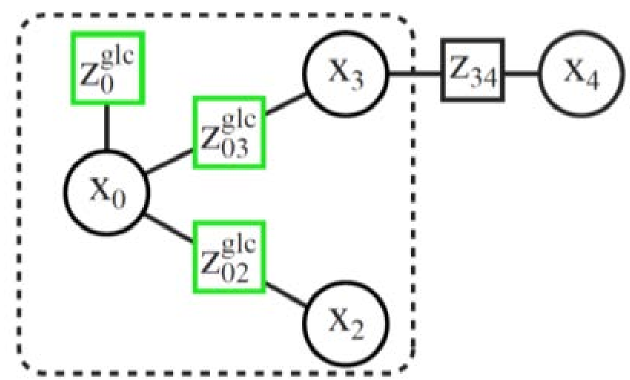
Remove node from graph → marginalize variable from distribution



Original Graph

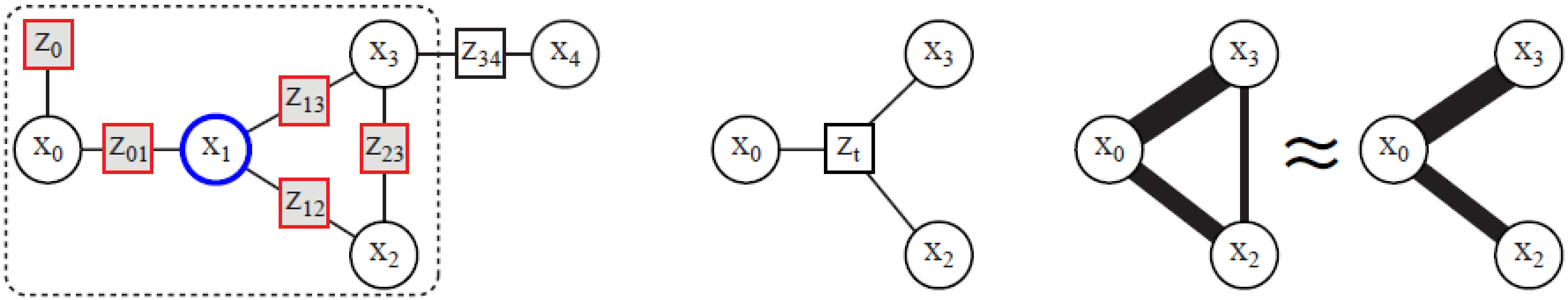


Dense Node Removal (Marginalization)

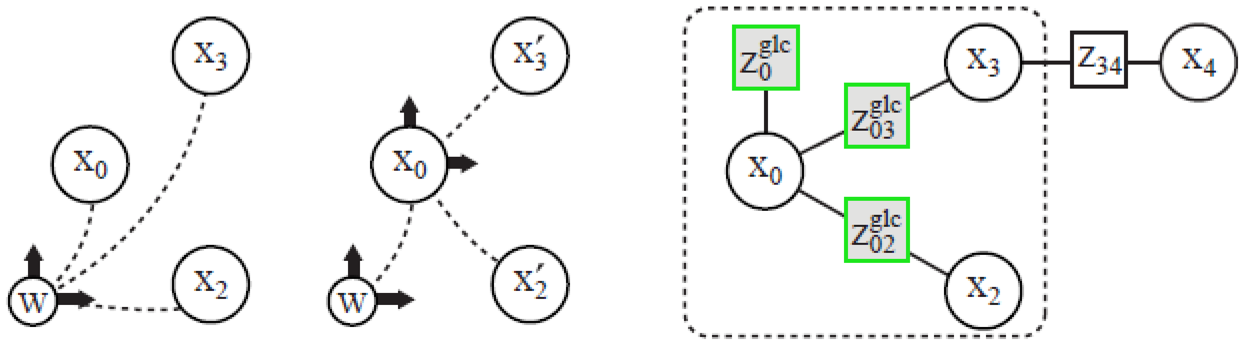


Sparse-Approximate Node Removal

Generic Linear Constraint Node Removal



(a) Original Graph (b) Target Info. (c) Chow-Liu Tree



(d) Root Shift (e) Final Graph

Sparse Approximate GLC: Ensuring Conservative Approximations

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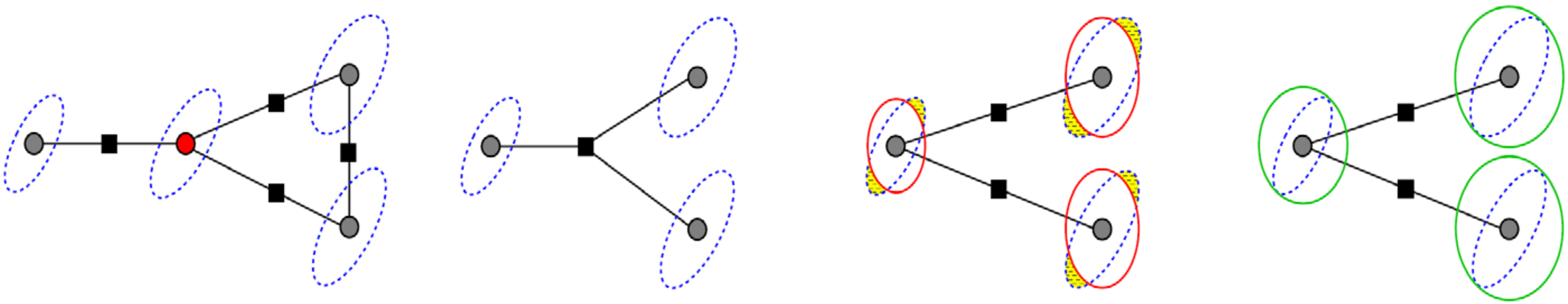
(a) Original Graph (b) Node Marginalization (c) Chow-Liu Tree Approx. (d) Conservative Approx.

- Chow-Liu Tree minimizes KLD

$$\mathcal{D}_{KL}\left(\mathcal{N}^{-1}(\boldsymbol{\eta}_t, \Lambda_t) \| \mathcal{N}^{-1}(\tilde{\boldsymbol{\eta}}_t, \tilde{\Lambda}_t)\right) = \frac{1}{2}\left(\mathrm{tr}(\tilde{\Lambda}_t \Lambda_t^{-1}) + \frac{\ln|\Lambda_t|}{\ln|\tilde{\Lambda}_t|} - \dim(\boldsymbol{\eta}_t)\right)$$

Sparse Approximate GLC: Ensuring Conservative Approximations

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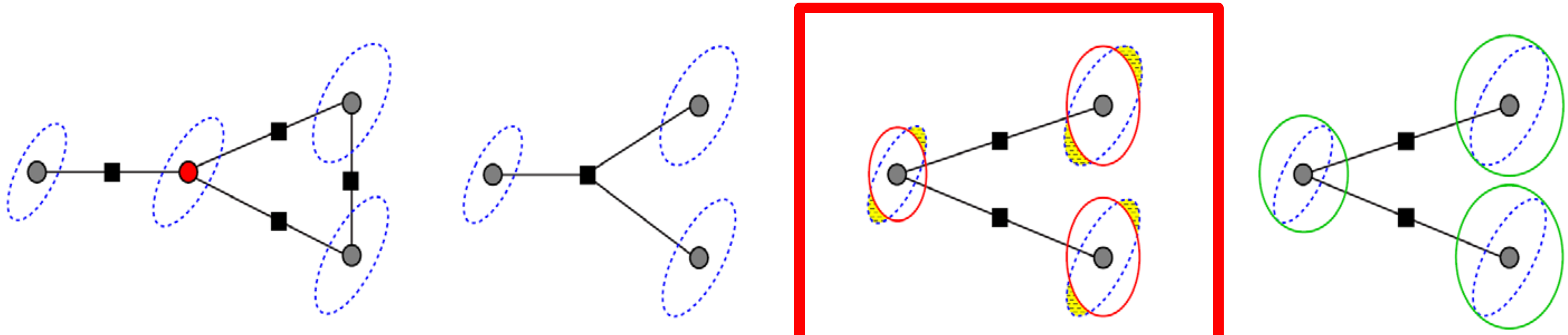
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- Often results in a slightly overconfident estimate

Sparse Approximate GLC: Ensuring Conservative Approximations

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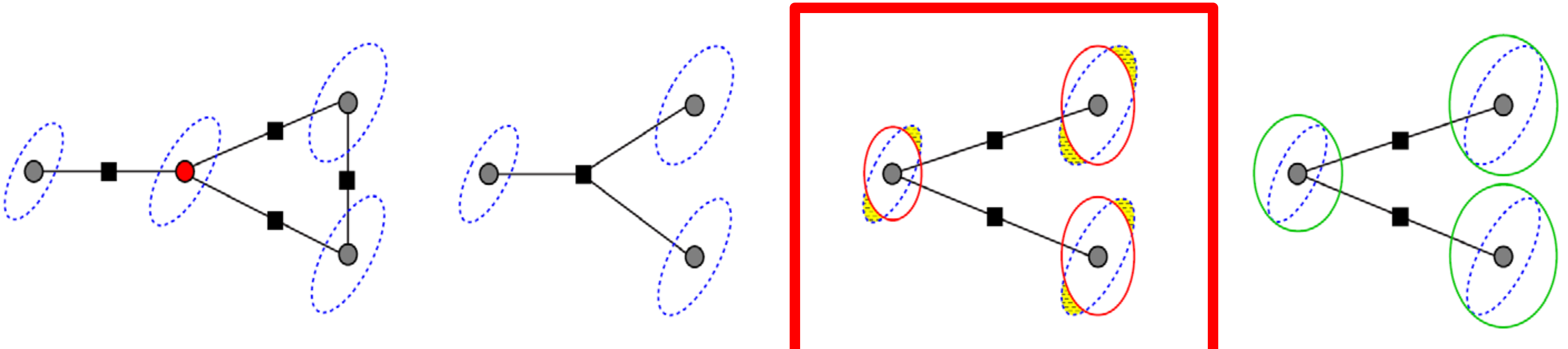
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- Why care about overconfident estimates?
  - Overconfidence in pose or obstacle location → unsafe paths
  - Overconfidence in pose or landmark location → failed data association

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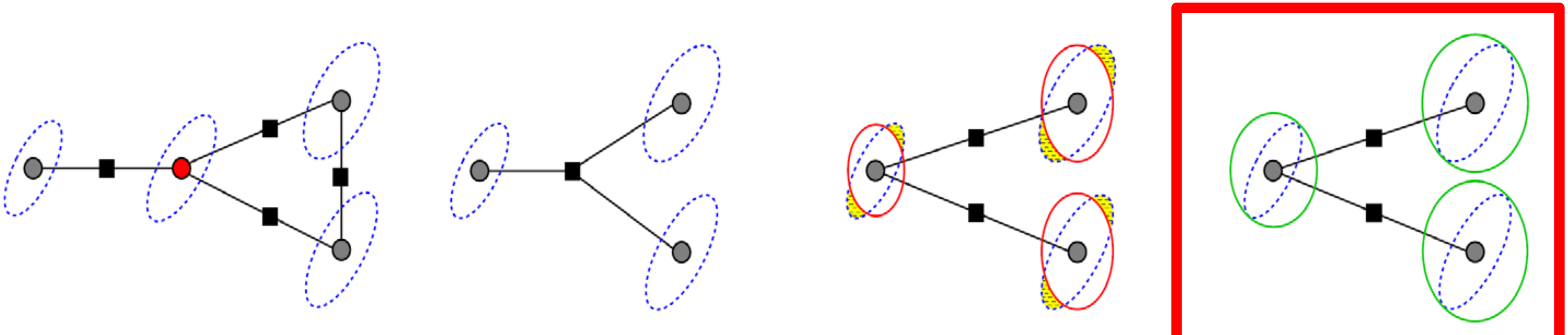

(a) Original Graph (b) Node Marginalization (c) Chow-Liu Tree Approx. (d) Conservative Approx.

- Propose method to ensure conservative approximation
- Start with CLT which minimizes the KLD and then numerically adjust it to produce a conservative estimate

Sparse Approximate GLC: Ensuring Conservative Approximations

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- Constrained convex optimization problem
- Minimize the KLD

$$\mathcal{D}_{KL}\left(\mathcal{N}^{-1}(\boldsymbol{\eta}_t, \Lambda_t) \| \mathcal{N}^{-1}(\tilde{\boldsymbol{\eta}}_t, \tilde{\Lambda}_t)\right) = \frac{1}{2}\left(\operatorname{tr}(\tilde{\Lambda}_t \Lambda_t^{-1}) + \frac{\ln|\Lambda_t|}{\ln|\tilde{\Lambda}_t|} - \dim(\boldsymbol{\eta}_t)\right)$$

$$f_{KL}(\tilde{\Lambda}_t) = \operatorname{tr}(\tilde{\Lambda}_t \Lambda_t^{-1}) - \ln|\tilde{\Lambda}_t|$$

- Subject to conservative constraint (difference is PSD)

$$\tilde{\Sigma} \geq \Sigma \quad \Leftrightarrow \quad \Lambda \geq \tilde{\Lambda}$$

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Sparse Approximate GLC: Ensuring Conservative Approximations

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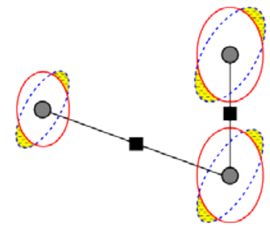
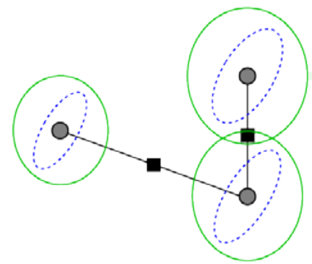
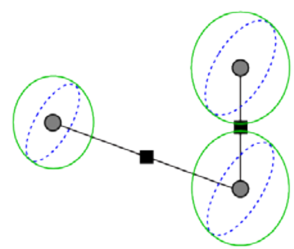
- Start with the Chow-Liu Tree



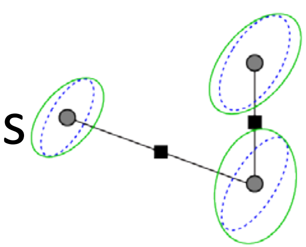
- Consider three methods
  - Covariance Intersection

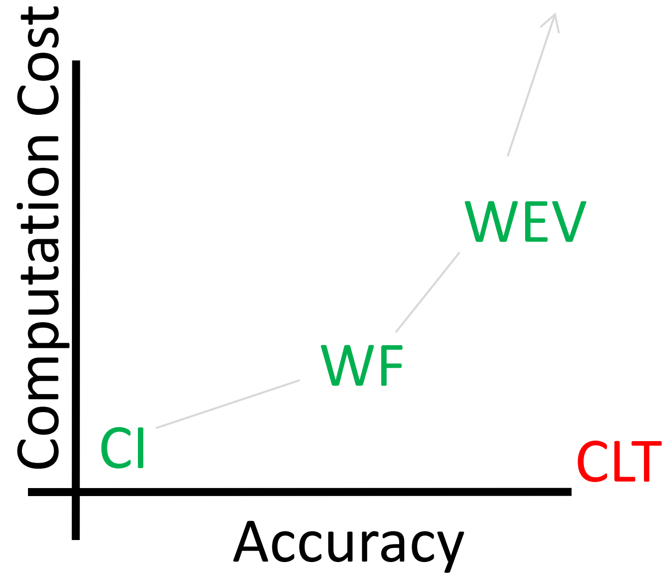


  - Weighted Factors



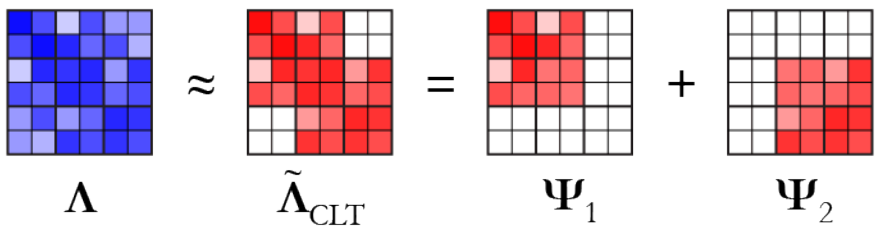
  - Weighted Eigenvalues
- Convex semidefinite programs

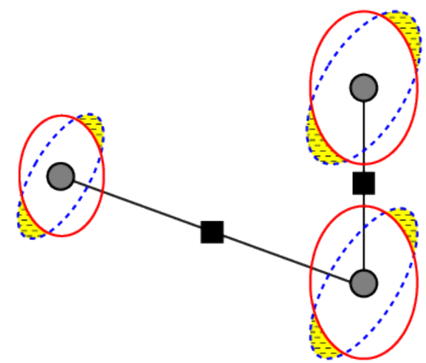




Chow-Liu Tree Approximation

- All proposed methods start with the CLT





$$\mathcal{N}^{-1}(\boldsymbol{\eta}_t, \Lambda_t) \approx \mathcal{N}^{-1}(\tilde{\boldsymbol{\eta}}_t, \tilde{\Lambda}_{\mathrm{CLT}}) = \prod_i p(\mathbf{x}_i | \mathbf{x}_{\mathrm{p}(i)})$$

$$\tilde{\Lambda}_{\mathrm{CLT}} = \sum_i \Psi_i$$

Covariance Intersection

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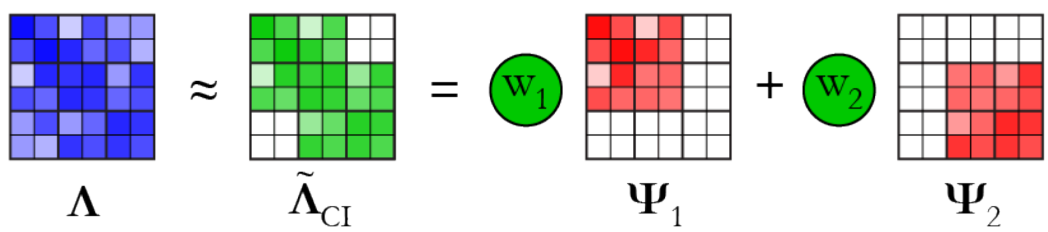
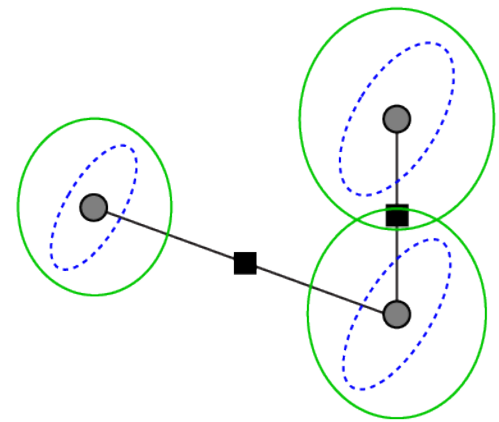
[Julier and Uhlmann, 1997]

- Convex combination of correlated factors





$$\tilde{\Lambda}_{\mathrm{CI}}(\mathbf{w}) = \sum_i w_i \Psi_i$$

$$\begin{aligned} \underset{\mathbf{w}}{\text{minimize}} \quad & f_{KL}(\tilde{\Lambda}_{\mathrm{CI}}(\mathbf{w})) \\ \text{subject to} \quad & \sum_i w_i = 1 \end{aligned}$$

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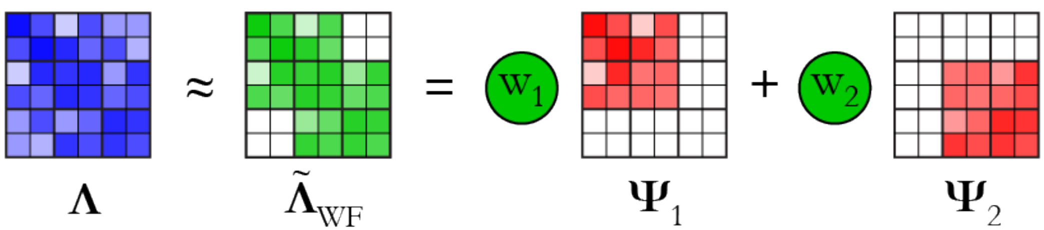
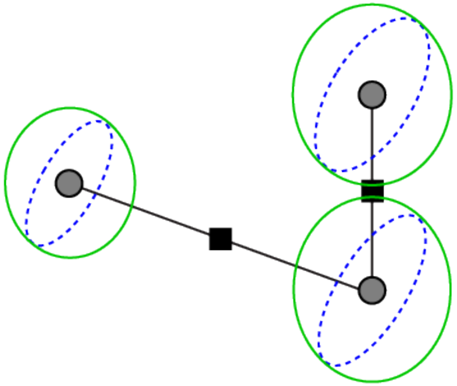
Weighted Factors

- Replace constraint that weights sum to one



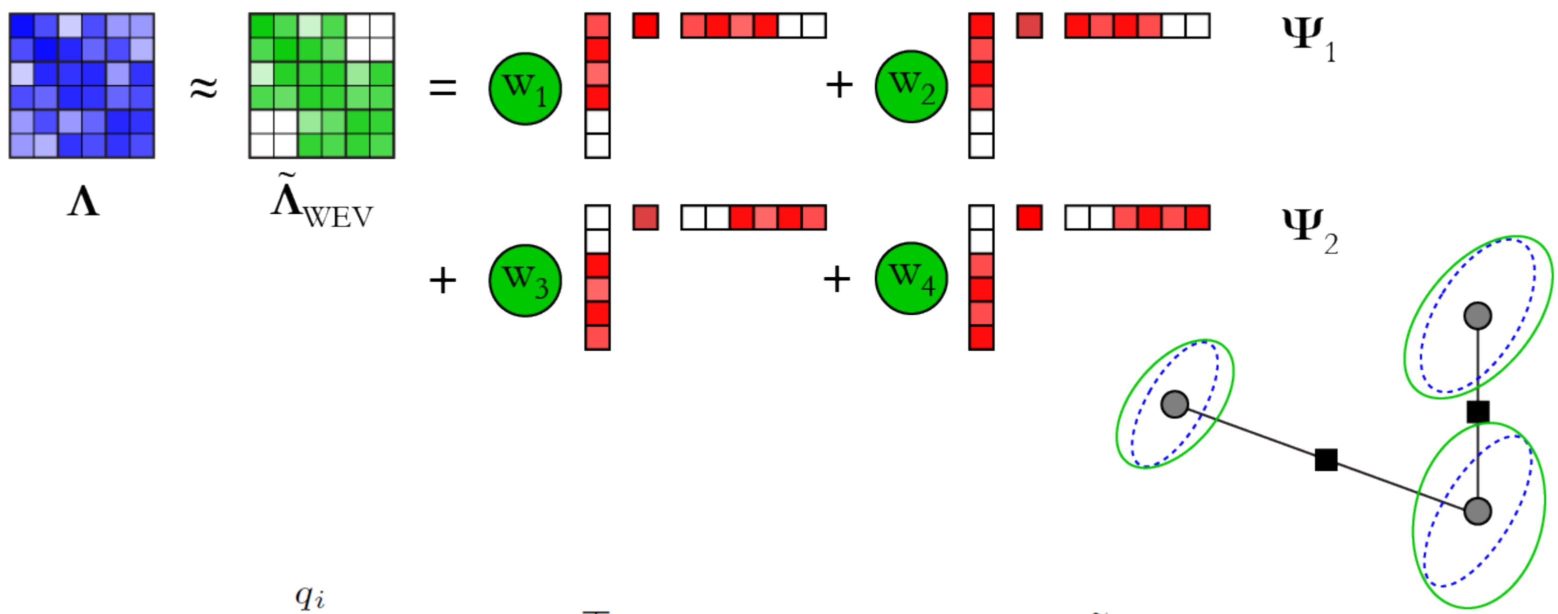


$$\tilde{\Lambda}_{\mathrm{WF}}(\mathbf{w}) = \sum_i w_i \Psi_i$$

$$\begin{aligned} \underset{\mathbf{w}}{\text{minimize}} \quad & f_{KL}(\tilde{\Lambda}_{\mathrm{WF}}(\mathbf{w})) \\ \text{subject to} \quad & 0 \leq w_i \leq 1, \ \forall i \\ & \Lambda_t \geq \tilde{\Lambda}_{\mathrm{WF}}(\mathbf{w}) \end{aligned}$$

Weighted Eigenvalues

- Modify each eigenvalue of each factor

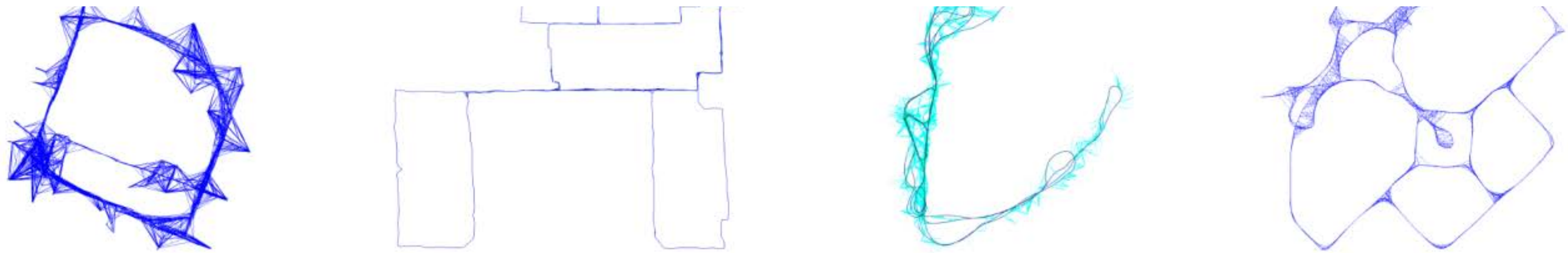


$$\tilde{\Lambda}_{\mathrm{WEV}}(\mathbf{w}) = \sum_{i} \sum_{j=1}^{q_i} w_j^i \lambda_j^i \mathbf{u}_j^i {\mathbf{u}_j^i}^{\top} = \sum_{k} w_k \lambda_k \mathbf{u}_k \mathbf{u}_k^{\top}$$

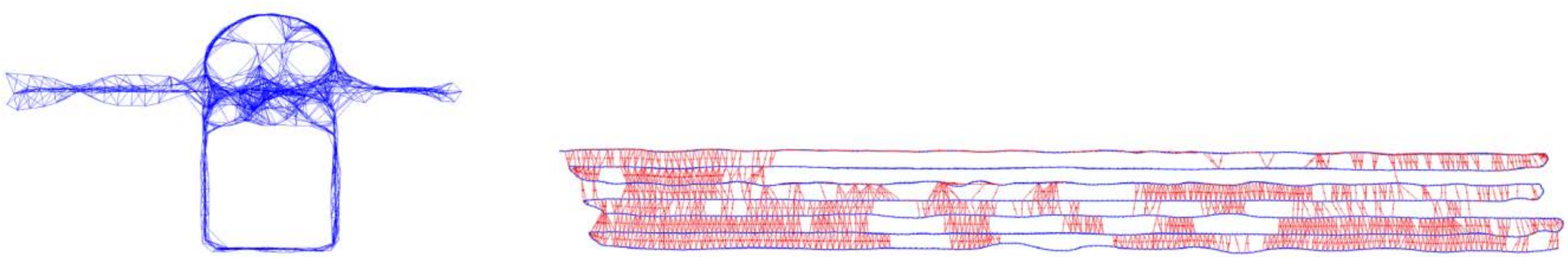
$$\begin{aligned} \underset{\mathbf{w}}{\text{minimize}} \quad & f_{KL}(\tilde{\Lambda}_{\mathrm{WEV}}(\mathbf{w})) \\ \text{subject to} \quad & 0 \leq w_k \leq 1, \ \forall k \\ & \Lambda_t \geq \tilde{\Lambda}_{\mathrm{WEV}}(\mathbf{w}). \end{aligned}$$

Conservative GLC: Experimental Results

Dataset	Node Types	Factor Types	# Nodes	# Factors
Intel Lab	3-DOF pose	3-DOF odom., 3-DOF laser scan-matching	910	4,454
Killian Court	3-DOF pose	3-DOF odom., 3-DOF laser scan-matching	1,941	2,191
Victoria Park	3-DOF pose, 2-DOF lm.	3-DOF odom., 2-DOF landmark observation	7,120	10,609
Duderstadt Center	6-DOF pose	6-DOF odom., 6-DOF laser scan-matching	552	1,774
EECS Building	6-DOF pose	6-DOF odom., 6-DOF laser scan-matching	611	2,134
USS Saratoga	6-DOF pose	6-DOF odom., 5-DOF mono-vis., 1-DOF depth	1,513	5,433



(a) Intel Lab (b) Killian Court (c) Victoria Park (d) Duderstadt Center



(e) EECS Building (f) USS Saratoga

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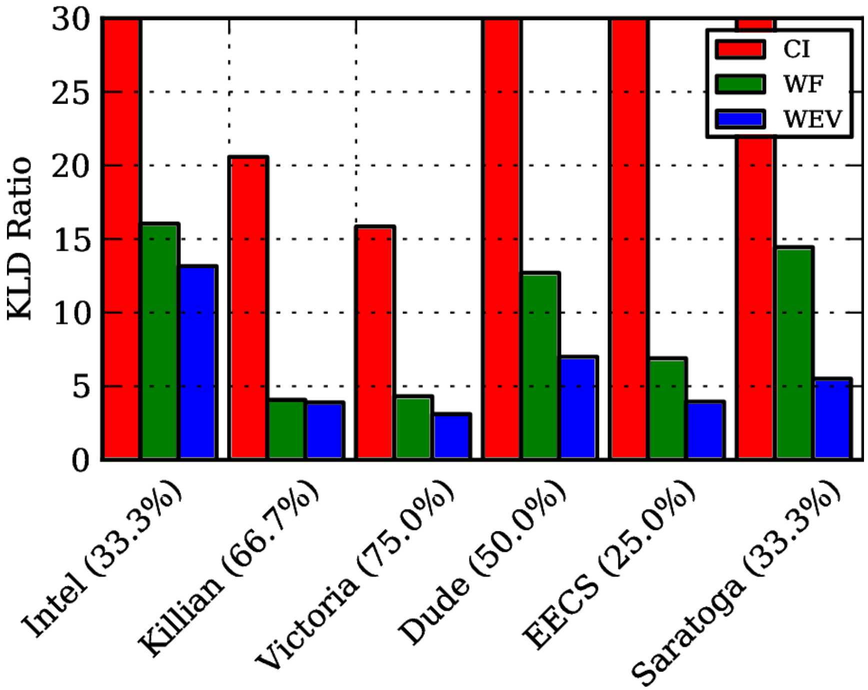
Conservative GLC: Experimental Results



- Remove percentage of evenly spaced nodes from each graph
- CI very conservative
- WF and WEV approach performance of CLT
  - for most graphs
- Room improvement for Intel
  - Higher density of connectivity
  - All factor same strength