

Preprocessing of Propagation Redundant Clauses

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Overview

- ▶ Symmetry breaking in the **Propagation Redundant** (PR) proof system with short clauses without new variables
- ▶ We introduce the pre-processing technique **PRELEARN** that finds **short** PR clauses
- ▶ The clause additions are **verified** using a proof checker
- ▶ The tool significantly improves the SAT solver **KISSAT**'s performance on hard combinatorial problems

SAT Solving

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SAT Solving

- ▶ The **Boolean satisfiability** problem (SAT) asks if a formula in propositional logic is satisfiable
- ▶ A **SAT solver** will produce one of the following:
 - If SAT, a **satisfying assignment** that can be checked by plugging truth values into the formula
 - If UNSAT, a **clausal proof** of unsatisfiability that can be checked by a proof checker

Clausal Proofs

- ▶ A **Clausal Proof** of UNSAT is a sequence of clause additions and deletions that end with the empty clause
- ▶ Each clause addition meets some **redundancy** property, defining the strength of the **proof system**

Clausal Proofs

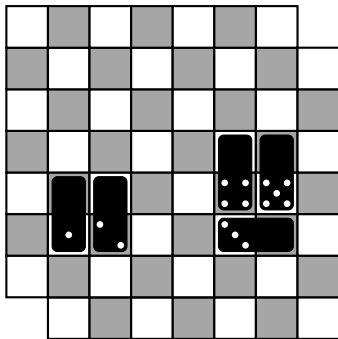
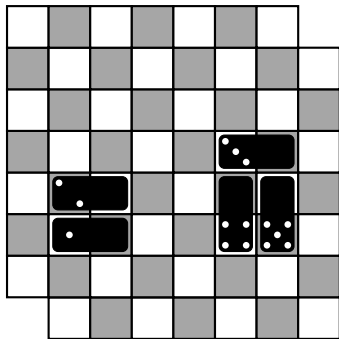
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RUP Reverse Unit Propagation – logically implied
- learned clauses in CDCL

RAT Resolution Asymmetric Tautology – sat. equivalence
- preprocessing

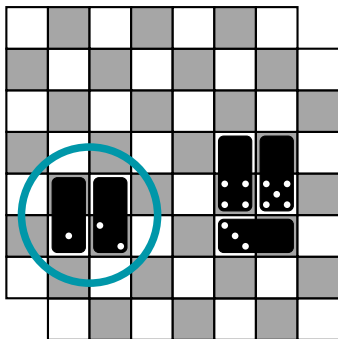
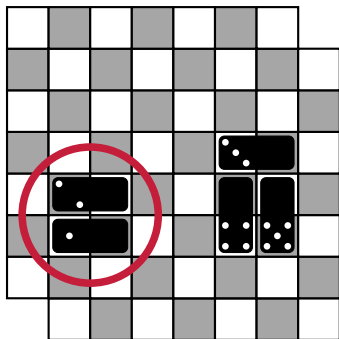
PR Propagation Redundancy – sat. equivalence
- symmetry breaking

Motivating Example



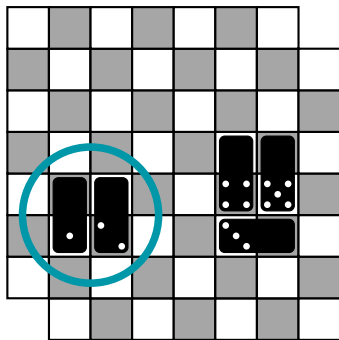
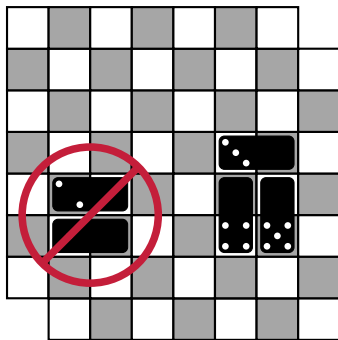
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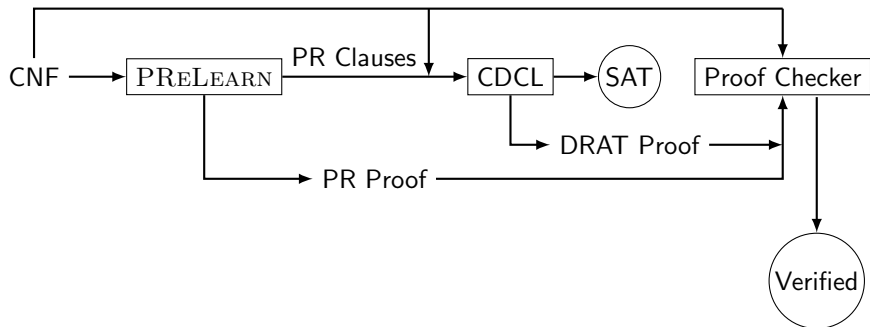
- ▶ **Mutilated Chessboard:** can it be covered by dominoes?
- ▶ Tilings on the left can be blocked with PR clauses, since the ones on the right are **at least as satisfiable**

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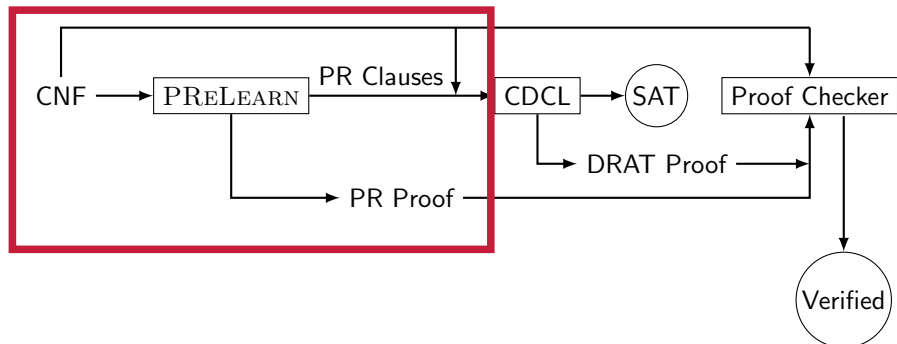


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PRELEARN

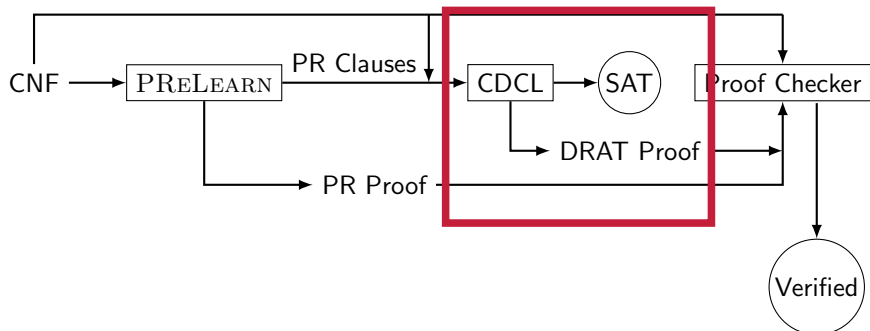


PRELEARN



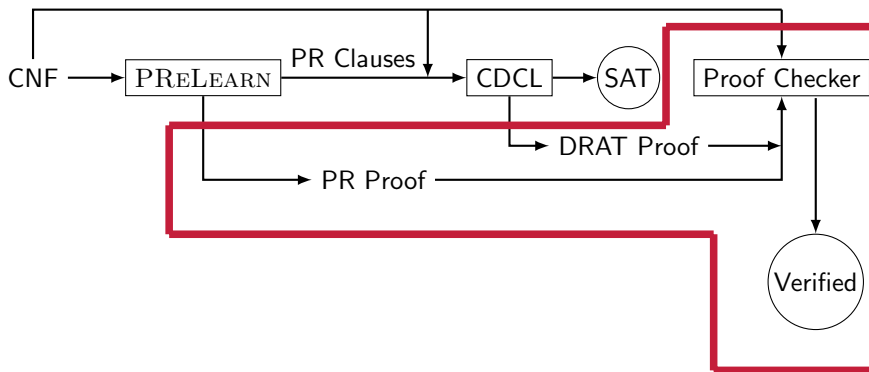
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- ▶ PR proof (PRELEARN) + DRAT proof (CDCL solver)

PR Clauses

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Solution:

- ▶ Generate a formula called the **positive reduct** for a **candidate** PR clause
- ▶ If the **positive reduct** is SAT, the **candidate** is PR and the SAT assignment is a **witness**

SAT Competition Experiment Setup

- ▶ Previous SAT Competition benchmarks grouped by number of clauses 0-10k (323), 10k-50k (348)
- ▶ Using the CDCL solver `KISSAT` on original formulas and formulas + PR clauses
- ▶ `PRELEARN` for 50 iterations with 100 second timeout
- ▶ experiments run with 5,000 second timeout, including `PRELEARN` runtime

SAT Competition Experiments

Learning PR Clauses

- ▶ Found PR clauses in 2/3 of the formulas
- ▶ Most generated reducts are UNSAT (99.5%), i.e., candidates were not PR
- ▶ 2×'s slower per reduct for larger formulas

SAT Competition Experiments

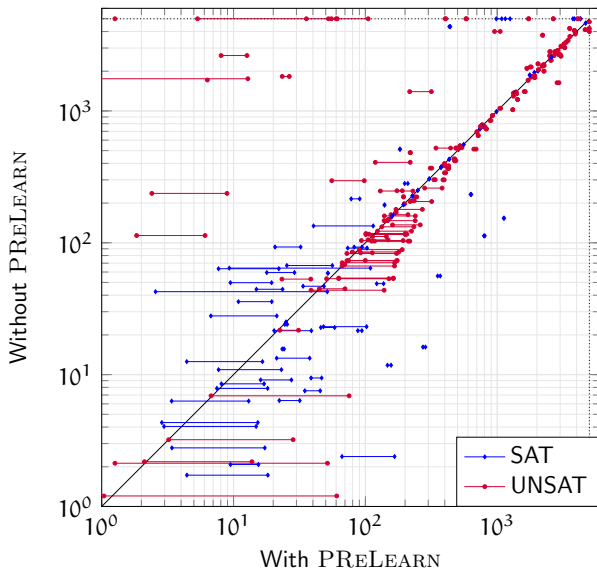
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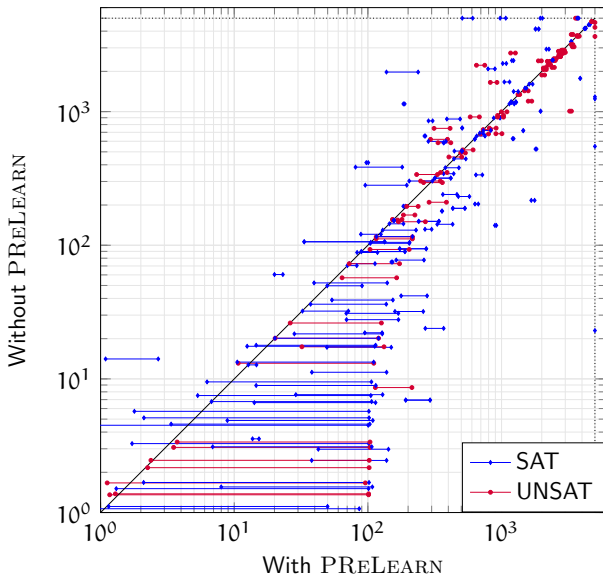
Solving with PR Clauses

- ▶ 19 formulas solved exclusively when using PR clauses
- ▶ Stronger on smaller formulas, SAT and UNSAT

PRELEARN on 0-10k SAT benchmarks



PRELEARN on 10-50k SAT benchmarks



Additional Configurations

- ▶ Expanding set of candidates considered
- ▶ Learn in batches (PR is non-monotonic)
- ▶ Learning larger PR clauses (e.g., ternary clauses)
- ▶ Increased timeout for PRELEARN

<https://github.com/jreeves3/PReLearn>

Conclusion and Future Work

Conclusion

- ▶ PRELEARN highly effective for some problems, but degrades performance on others

Future Work

- ▶ Classification of problems or PR clauses that will be useful
- ▶ Larger PR clauses with more selective heuristics
- ▶ In-processing as units are learned/propagated and irredundant clauses change



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Questions?

SAT Competition Experiments

Set	Benchs	Avg. (s)	Generated Reducts	Sat. Reducts	% Sat.	Failed Lits
0-10k	221/323	22.36	104,850,011	548,417	0.52%	3,416
10k-50k	237/348	71.08	163,014,068	789,281	0.48%	6,290

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	0-10k SAT	0-10k UNSAT	10k-50k SAT	10k-50k UNSAT
Total w/ PRELEARN	84	149	143	89
Total w/o PRELEARN	80	141	143	91
Exclusively w/ PRELEARN	4	10	4	1
Exclusively w/o PRELEARN	0	2	4	3
Improved w/ PRELEARN	20	44	25	13

- ▶ Solved many formulas only with using PRELEARN
- ▶ Stronger on smaller formulas, SAT and UNSAT