Homework 1 — CS 819 C — Spring 2002
January 29, 2002 — due February 5

Problem 1

Give a formal proof of the valid assertion

$$\big((\mathsf{x} \mapsto \mathsf{y} \ * \ \mathsf{x}' \mapsto \mathsf{y}') \ * \ \mathbf{true}\big) \Rightarrow$$
$$\big(((\mathsf{x} \mapsto \mathsf{y} \ * \ \mathbf{true}) \wedge (\mathsf{x}' \mapsto \mathsf{y}' \ * \ \mathbf{true})) \wedge \mathsf{x} \neq \mathsf{x}'\big)$$

from the rules on page 68 of Part 2 of the class notes (as revised January 28), and (some of) the following inference rules for predicate calculus:

$$p \Rightarrow \mathbf{true} \qquad p \Rightarrow p \qquad p \wedge \mathbf{true} \Rightarrow p$$

$$\frac{p \Rightarrow q \qquad q \Rightarrow r}{p \Rightarrow r} \quad \text{(trans impl)}$$

$$\frac{p \Rightarrow q \qquad p \Rightarrow r}{p \Rightarrow q \wedge r} \quad (\wedge\text{-introduction})$$

Your proof will be easier to read if you write it as a sequence of steps (as on the top of page 64 of Part 2), rather than a tree. In the inference rules, you should regard $*$ as left associative, e.g.,

$$e_1 \mapsto e_1' \ * \ e_2 \mapsto e_2' \ * \ \mathbf{true} \Rightarrow e_1 \neq e_2$$

stands for

$$(e_1 \mapsto e_1' \ * \ e_2 \mapsto e_2') \ * \ \mathbf{true} \Rightarrow e_1 \neq e_2.$$

For brevity, you may weaken $\Leftrightarrow$ to $\Rightarrow$ when it is the main operator of an axiom. You may also omit instances of the axiom schema $p \Rightarrow p$ when it is used as a premiss of the monotonicity rule.

Problem 2

None of the following axiom schemata are sound. For each, given an instance which is not valid, along with a description of a state in which the instance is false.

$$
\begin{aligned}
p_1 * p_2 &\not\Rightarrow p_1 \wedge p_2 \\
p_1 \wedge p_2 &\not\Rightarrow p_1 * p_2 \\
(p_1 * p_2) \vee q &\not\Rightarrow (p_1 \vee q) * (p_2 \vee q) \\
(p_1 \vee q) * (p_2 \vee q) &\not\Rightarrow (p_1 * p_2) \vee q \\
(p_1 * q) \wedge (p_2 * q) &\not\Rightarrow (p_1 \wedge p_2) * q \\
(p_1 * p_2) \wedge q &\not\Rightarrow (p_1 \wedge q) * (p_2 \wedge q) \\
(p_1 \wedge q) * (p_2 \wedge q) &\not\Rightarrow (p_1 * p_2) \wedge q \\
\big(\forall x.\ (p_1 * p_2)\big) &\not\Rightarrow (\forall x.\ p_1) * p_2 \quad \text{when } x \text{ not free in } p_2 \\
(p_1 \Rightarrow p_2) &\not\Rightarrow \big((p_1 * q) \Rightarrow (p_2 * q)\big) \\
(p_1 \Rightarrow p_2) &\not\Rightarrow (p_1 \mathrel{-\!\!*} p_2) \\
(p_1 \mathrel{-\!\!*} p_2) &\not\Rightarrow (p_1 \Rightarrow p_2)
\end{aligned}
$$