Assignment 2: Lexing and Parsing

15-411: Compiler Design
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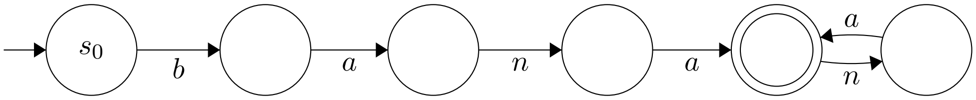
Due Thursday, Sept 28, 2017 (11:59PM)

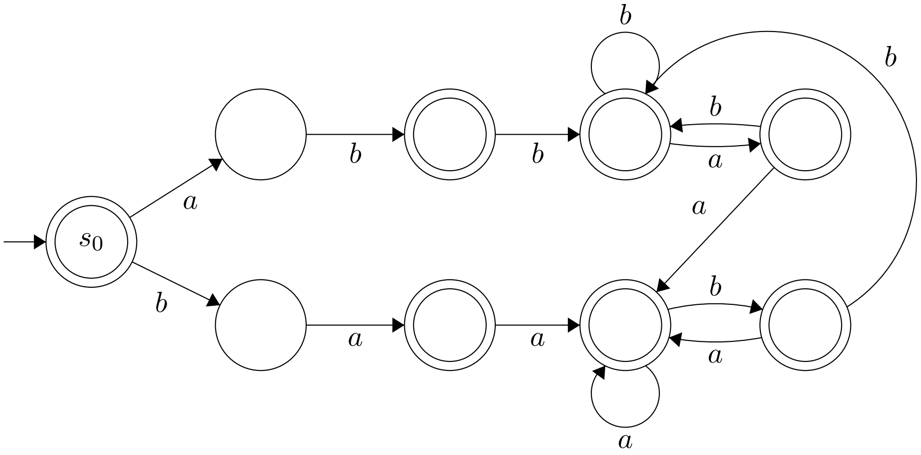
Reminder: Assignments are individual assignments, not done in pairs. The work must be all your own. Hand in your solutions on Gradescope. Please read the late policy for written assignments on the course web page.

Problem 1: Lexing (20 points)

(a) In class, we discussed how lexers can use regular expressions to parse an input corpus into tokens. A common way of implementing a regular expression parser is with deterministic finite automata, or DFAs, which you might have seen in 251 or FLAC[1]. For example, the DFA for the regex `ban(an)*a` would be:



Jonathan has decided to create a new language C_{naught} which is startlingly similar to C_0 but utilizes new and exciting tokens. To lex his identifiers, he hand-crafts the following DFA which accepts a language L over the alphabet $\Sigma = \{a, b\}$.



[1] To learn more about DFAs, you can read about it in the textbook or ask the TAs. Seriously, our office hours aren't very busy. Also check out this neat site: http://hackingoff.com/compilers/regular-expression-to-nfa-dfa

Help Jonathan simplify his language specification by finding a simple regular expression that accepts L. Note that although a DFA is required to have a transition for every character in Σ defined for every state, we omit certain transitions for brevity—these enter a permanent failure state, i.e. a string that enters (like "aa" in the above DFA) it is guaranteed to never be accepted.

(b) Jan stumbles across Jonathan's new language specification and thinks to himself, "Wow, this whole lexing business is far too simple." Reminiscing on his old SIGBOVIK days, Jan makes a new language $C_{\aleph_0}$ which requires that all identifiers must be of the form $a^n b^n c^n$ for $n > 0$. However, Jan quickly finds that his old regular expression-based lexer generator won't properly lex these identifiers. Identify the limitation of Jan's lexer generator and why he cannot decide this language with the regular expression lexer generator.

Problem 2: Grammars (10 points)

In formal language theory, a context-free grammar G is said to be in Chomsky normal form (first described by Noam Chomsky) if all of its production rules are of the form:

$$A \to BC \textbf{ or } A \to a \textbf{ or } S \to \epsilon$$

where A, B, and C are nonterminal symbols, a is a terminal symbol, S is the start symbol, and ϵ denotes the empty string (if it is in the language).

To convert a grammar to Chomsky normal form, a sequence of simple transformations is applied in a certain order; this is described in most textbooks on automata theory. This conversion is called CNF conversion; the conversion algorithm is often used by algorithms as a preprocessing step (including the CYK parsing algorithm).

But why is this useful? The CYK parsing algorithm, used by some parser generators, runs in $\mathcal{O}(n^3)$ time, where n is the number of tokens in the string. This is one of the best parsing algorithms known in terms of worst-case asymptotic complexity; others exist that are better in average running time.

(a) Which language is generated by the grammar G given by the following rules (where S is the start symbol):

$$S \to 0S0 \mid 0B0$$

$$B \to 1B \mid 1$$

(b) Convert the grammar G from part (a) into a grammar G' in Chomsky normal form that generates the same language, that is, $L(G) = L(G')$.

(c) Informally argue why $L(G) = L(G')$.

Problem 3: Parsing (30 points)

After Jan's disastrous foray into lexing, Alice figures she's safe by just writing a context free grammar for his new language, C_0^λ which combines all the usability of lambda calculus with all the safety of C. He specifies it with the following productions (note that x is an identifier token and n is a number token):

$$\begin{array}{lcccl}
\gamma_1 & : & \langle \text{E} \rangle & \rightarrow & \text{n} \\
\gamma_2 & : & \langle \text{E} \rangle & \rightarrow & \text{x} \\
\gamma_3 & : & \langle \text{E} \rangle & \rightarrow & \text{lam x . } \langle \text{E} \rangle \\
\gamma_4 & : & \langle \text{E} \rangle & \rightarrow & \langle \text{E} \rangle \, \langle \text{E} \rangle \\
\gamma_5 & : & \langle \text{E} \rangle & \rightarrow & (\, \langle \text{E} \rangle \,) \\
\gamma_6 & : & \langle \text{E} \rangle & \rightarrow & \langle \text{E} \rangle \oplus \langle \text{E} \rangle
\end{array}$$

(a) Alice gets Anatol's pet bison to review her grammar and uncovers a number of problems. Show two ambiguities in the above grammar by providing for each ambiguity two possible parse trees for the same string.

(b) DeeDee's company Compilers-R-Us is seeking to acquire Alice's revolutionary new language, but the terms of the acquisition require that the grammar is unambiguous. Help Alice achieve her billion dollar buyout and rewrite the grammar so it is unambiguous[2]. For each ambiguity you found in (a), identify which of the two parse trees will be accepted by your new grammar.

(c) Not content to let the TAs have all the fun, you set out to write your own grammar, but run into some familiar pitfalls. Describe an unambiguous grammar G with fewer than 6 productions that contains a reduce/reduce conflict in a shift-reduce parser with lookahead 1. The language $L(G)$ must be context free, but not regular. You may use your parser generator of choice to help.

(d) Prove that the reduce/reduce conflict in (c) exists by giving two conflicting derivations (Compare the example given in the lecture notes.).

[2] Hint: your new grammar may not have the precedence you'd expect from lambda calculus.