PROOF-THEORETIC FOUNDATIONS OF COMPILATION

IN LOGIC PROGRAMMING LANGUAGES

ILIANO CERVESATO

ITT INDUSTRIES / NRL

TACL SEMINAR

DEPT. OF COMPUTER SCIENCE
PRINCETON UNIVERSITY

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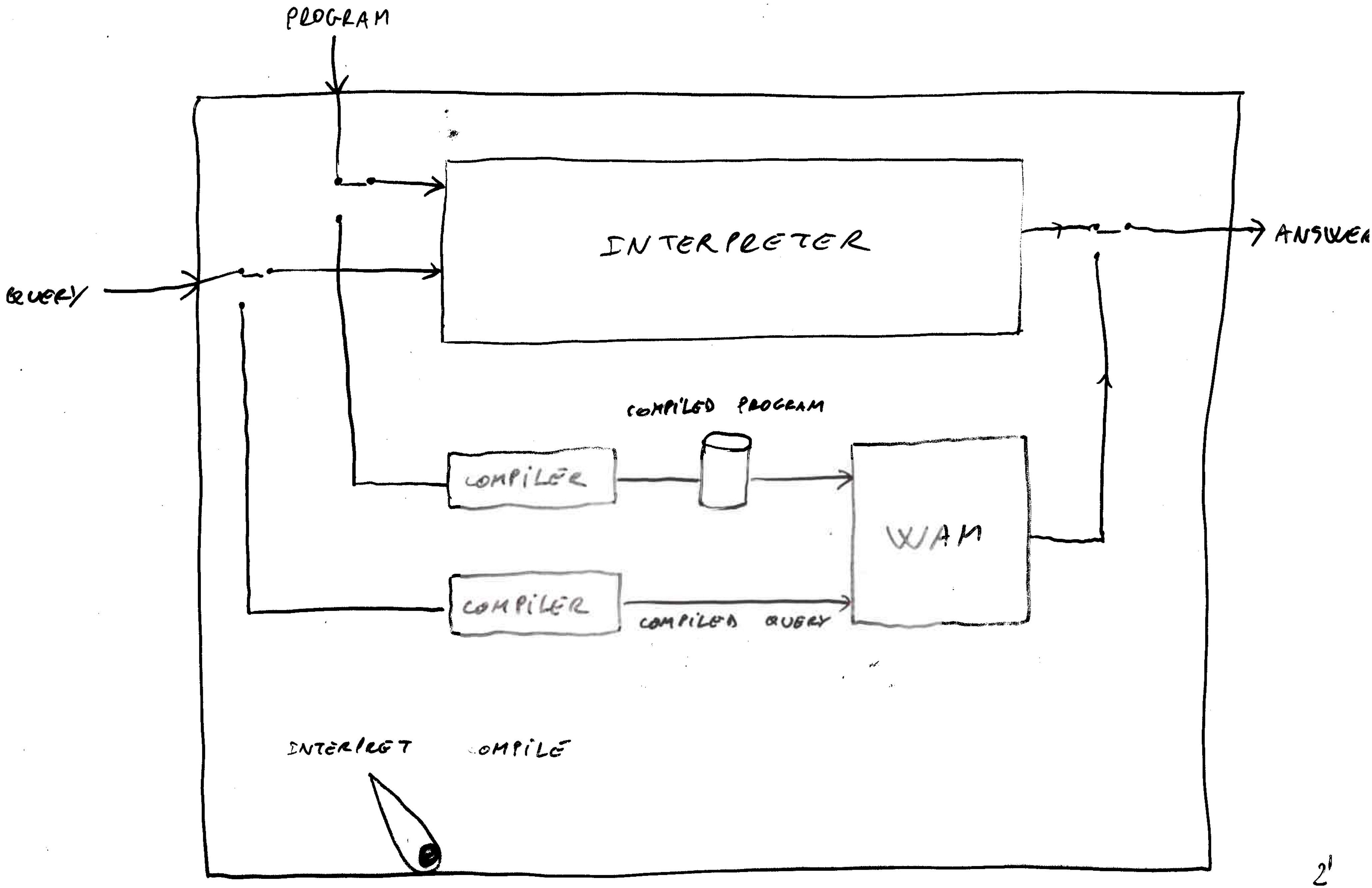
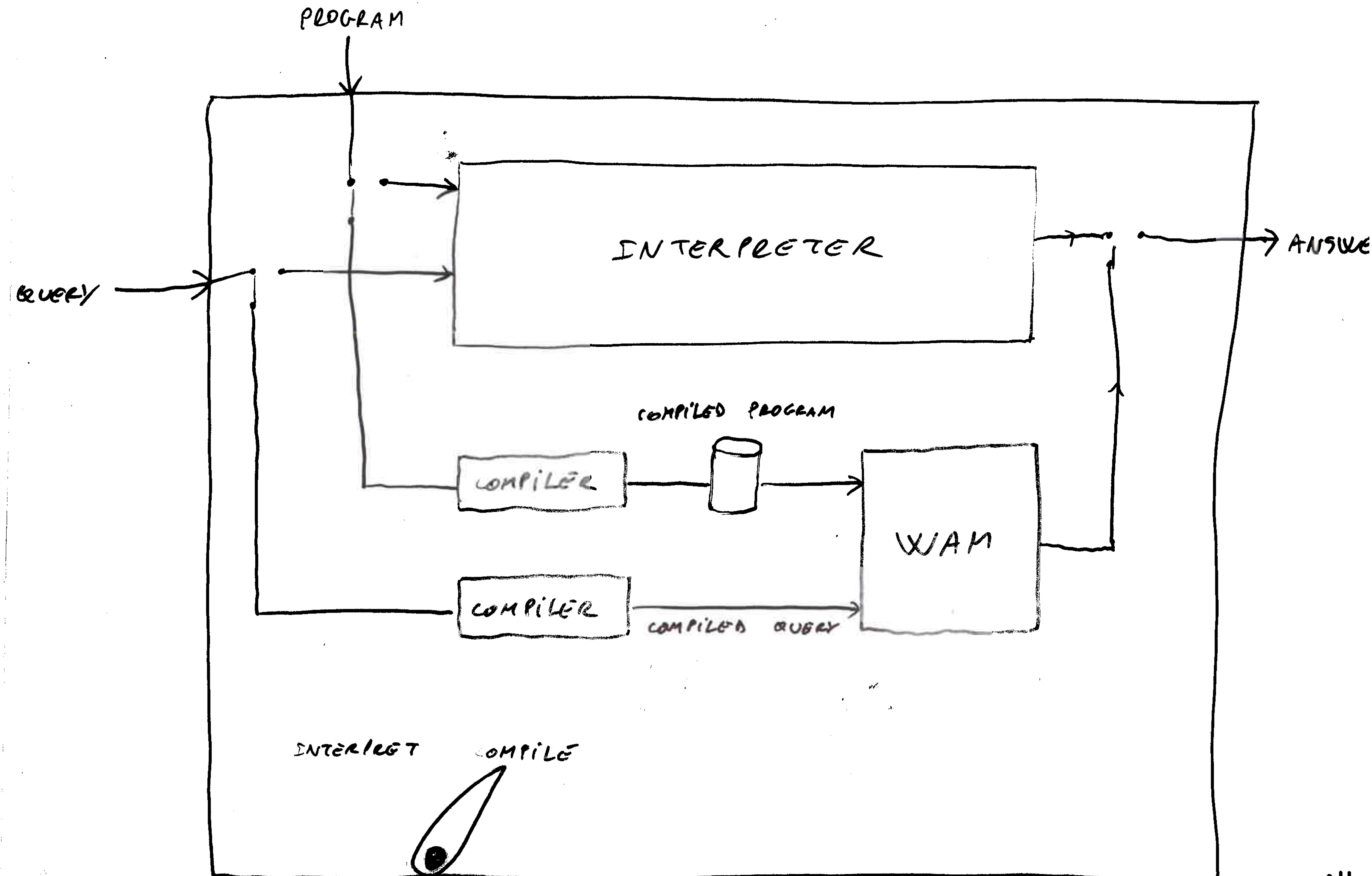
OVERVIEW

- MOTIVATIONS AND BACKGROUND
- ABSTRACT LOGIC PROGRAMMING COMPILATION
- EXAMPLE
- REFINEMENTS

PROLOG ARCHITECTURE



PROLOG ARCHITECTURE

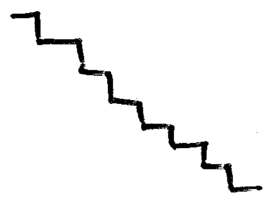


MATHEMATICAL MODEL

- PROGRAM / QUERY : LOGICAL FORMULAS
- ANSWER : DERIVABILITY
- INTERPRETER : UNIFORM PROOF SEARCH

ALPL

LOGICAL RULES



IMPLEMENTABLE SPECIFICATIONS

- COMPILER / WAM : ?

WAM

[WARREN, 83]

- INTERPRETS A SPECIALIZED INSTRUCTION SET FOR PROLOG

+ VERY FAST (~ 40 TIMES)

– COMPLEX

– SPECIALIZED TO PROLOG
  THEN EXTENDED TO
  - CLP($\mathbb{R}$) [JAFFAR & AL, 92]
  - PROTOS-L [BEIERLE & BÖRGER, 92]

  (λPROLOG [NADATHUR & AL, 93])

– <u>NO LOGICAL STATUS</u>
  → WHERE DO THE INSTRUCTIONS COME FROM ?
  → WHAT DOES IT DO ?

CORRECTNESS OF THE WAM

- [RUSSINOFF, 92]
- [BÖRGER & ROSENZWEIG, 95]

- START FROM HIGHLY OPERATIONAL SPEC. OF PROLOG'S SEMANTICS
- COMPLEX
- DO NOT SCALE TO MODERN LOGIC PROG. LANG.

THIS WORK

PROOF - THEORETIC DEFINITION OF COMPILATION FOR ALPL

- ABSTRACT
  NO GORY DETAILS

- LOGIC INDEPENDENT
  APPLIES TO ANY ALPL

- SYSTEMATIC
  EASY PROOFS OF SOUNDNESS/COMPLETENESS

- MODULAR
  MANAGES GORY DETAILS

ALPL

[MILLER & NADATHUR & PFENNING & SCEDROV, 91]

OBJECTIVE:

COMPUTATION = PROOF SEARCH

(SEQUENT) $\Gamma \longrightarrow A$

PROGRAM GOAL

CONNECTIVES IN A : SEARCH DIRECTIVES,

CLAUSES IN Γ : SPEC. OF HOW TO CONTINUE THE SEARCH WHEN THE GOAL IS ATOMIC

MEANS

UNIFORM PROOFS

- GOAL-ORIENTED
- FOCUSED

$$(\mathcal{L}, \longrightarrow)$$

IS AN ALPL IF EVERY PROVABLE SEQUENT HAS A UNIFORM PROOF

EXAMPLE

$\mathcal{L}$: $\quad A ::= a \mid A_1 \supset A_2 \mid \forall x.\, A$

$\longrightarrow$:

$$\frac{\Gamma, A\, \Gamma' \longrightarrow A \gg a}{\Gamma, \underline{A}\, \Gamma' \longrightarrow a}$$

$$\frac{\Gamma, A \longrightarrow B}{\Gamma \longrightarrow A \supset B} \qquad \frac{\Gamma \longrightarrow [c/x]A}{\Gamma \longrightarrow \forall x.\, A}$$

$$\frac{}{\Gamma \longrightarrow a \gg a}$$

$$\frac{\Gamma \longrightarrow A \qquad \Gamma \longrightarrow B \gg a}{\Gamma \longrightarrow A \supset B \gg a} \qquad \frac{\Gamma \longrightarrow [t/x]A \gg a}{\Gamma \longrightarrow \forall x.\, A \gg a}$$

<u>NON-DETERMINISM</u>

<u>NB</u> : CUT IS ADMISSIBLE.

<u>NB</u> : TERM LANGUAGE IS LEFT UNSPECIFIED.

ABSTRACT COMPILATION

UNIFORM PROOF : <u>ALTERNANCE</u> OF

- GOAL DECOMPOSITION
- CLAUSE DECOMPOSITION

$$\begin{tikzcd} \text{SEARCH} \arrow[d, bend right] \\ \text{PREPARATION} \arrow[u, bend right] \end{tikzcd}$$

COMPILATION : <u>FIRST</u> <u>ALL</u> PREPARATION

<u>THEN</u> <u>ALL</u> SEARCH

$$\begin{tikzcd} \text{PREPARATION} \arrow[d] \\ \text{SEARCH} \end{tikzcd}$$

<u>ALPCS</u> FOR $(\mathcal{L}, \rightarrow)$:

$$(\mathcal{L}^c, \gg, \xrightarrow{c})$$

WHERE

$$\begin{tikzcd} \mathcal{L} \arrow[r, "\gg"] \arrow[d, "\rightarrow"'] & \mathcal{L}^c \arrow[dl, "\xrightarrow{c}"] \\ \text{YES/NO} & \end{tikzcd}$$

<u>NOTE</u> : $(\mathcal{L}^c, \xrightarrow{c})$ MUST BE AN ALPC WITH RIGHT RULES ONLY!

DETERMINATION OF $(\mathcal{L}^c, \xrightarrow{c})$

- KEEP EVERY RIGHT RULE OF $(\mathcal{L}, \longrightarrow)$

- ADD OPERATORS THAT BEHAVE ON THE RIGHT THE WAY THE CONNECTIVES OF $\mathcal{L}$ BEHAVE ON THE LEFT

WHERE IS THE LOGIC ? <u>DUALITY</u>

$$\frac{\Gamma \to A \qquad \Gamma \to B \gg a}{\Gamma \to A \supset B \gg a} \quad \rightsquigarrow \quad \frac{\Gamma \to A \qquad \Gamma \to B}{\Gamma \to A \wedge B}$$

$$\frac{\Gamma \to [A/x]A \gg a}{\Gamma \to \forall x.A \gg a} \quad \rightsquigarrow \quad \frac{\Gamma \to [A/x]A}{\Gamma \to \exists x.A}$$

$$\frac{}{\Gamma \to a \gg a} \quad \rightsquigarrow \quad \frac{}{\Gamma \to a \doteq a}$$

<u>NB</u> : COMPILATION MUST BE <u>PARAMETRIC</u> .

EXAMPLE: ABSTRACT MACHINE

$\mathcal{L}^c$:

$$G ::= a \mid (\Lambda\alpha.C) \supset G \mid \forall x.\, G \qquad \text{(GOALS)}$$

$$R ::= a_1 \doteq a_2 \mid R \wedge G \mid \exists x.\, R \qquad \text{(C-GOALS)}$$

$$C ::= a \doteq \alpha \mid C \wedge G \mid \exists x\, C \qquad \text{(CLAUSES)}$$

$$\Psi ::= \cdot \mid \Psi, (\Lambda\alpha.C) \qquad \text{(PROGRAMS)}$$

$\xrightarrow{c}$:

$$\frac{\Psi, \Lambda\alpha.C, \Psi' \to [a/\alpha]C}{\Psi, \Lambda\alpha.C, \Psi' \to a}$$

$$\frac{\Psi, (\Lambda\alpha.C) \to G}{\Psi \to (\Lambda\alpha.C) \supset G} \qquad \frac{\Psi \to [c/x]G}{\Psi \to \forall x.\, G}$$

$$\frac{}{\Psi \to a \doteq a}$$

$$\frac{\Psi \to R \qquad \Psi \to G}{\Psi \to R \wedge G} \qquad \frac{\Psi \to [t/x]R}{\Psi \to \exists x.\, R}$$

NON-DETERMINISM IS PRESERVED.

EXAMPLE : COMPILATION

PROGRAMS

$$\frac{}{\cdot \gg \cdot} \qquad \frac{\Gamma \gg \Psi \qquad A \gg \alpha \setminus C}{\Gamma, A \gg \Psi, (\Lambda \alpha . C)}$$

PROGRAM FORMULAS

$$\frac{}{a \gg \alpha \setminus a \doteq \alpha}$$

$$\frac{B \gg \alpha \setminus C \qquad A \gg G}{A \supset B \gg \alpha \setminus C \wedge G} \qquad \frac{A \gg \alpha \setminus C}{\forall \alpha . A \gg \alpha \setminus \exists \alpha . C}$$

GOAL FORMULAS

$$\frac{}{a \gg a}$$

$$\frac{A \gg \alpha \setminus C \qquad B \gg G}{A \supset B \gg (\Lambda \alpha . C) \supset G} \qquad \frac{A \gg G}{\forall \alpha . A \gg \forall \alpha . G}$$

EXAMPLE

$\forall L.$ APPEND NIL L L.

$\forall K. \forall L. \forall M. \forall X.$

APPEND (X :: K) L (X :: M) :- APPEND K L M

$$\Downarrow$$

$\Lambda \alpha. \exists L.$ (APPEND NIL L L) $\doteq \alpha$

$\Lambda \alpha. \exists K. \exists L. \exists M. \exists X.$

(APPEND (X :: K) L (X :: M)) $= \alpha$

$\wedge$ APPEND K L M

BUT ALSO:

APPEND1 : ALLOCATE L
UNIFY (APPEND NIL L L)

APPEND2 : ALLOCATE K
ALLOCATE L
ALLOCATE M
ALLOCATE X
UNIFY (APPEND (X :: K) L (X :: M))
CALL (APPEND K L M)

EXAMPLE : SOUNDNESS AND COMPLETENESS

SOUNDNESS

i) IF $\Gamma \to A$, $\Gamma \gg \Psi$, AND $A \gg G$, THEN $\Psi \hookrightarrow G$

ii) IF $\Gamma \to A \gg a$, $\Gamma \gg \Psi$, AND $A \gg \alpha \setminus C$

THEN $\Psi \hookrightarrow [a/\alpha]\, G$

PROOF : STRUCTURAL INDUCTION

COMPLETENESS

i) IF $\Psi \hookrightarrow G$, $\Gamma \gg \Psi$, AND $A \gg G$, THEN $\Gamma \to A$

ii) IF $\Psi \to R$, $\Gamma \gg \Psi$, $R = [a/\alpha]\, C$ AND $A \gg \alpha \setminus C$

THEN $\Gamma \to A \gg a$

PROOF : STRUCTURAL INDUCTION

REFINEMENTS

(= WORK IN PROGRESS)

- PREDICATE-GUIDED CLAUSE SELECTION
- UNFOLDING $\doteq$
- LOGICAL MANIPULATIONS

APPEND: $\Lambda\alpha_1 . \Lambda\alpha_2 . \Lambda\alpha_3 .$

$$(\exists L.\ nil \doteq \alpha_1 \wedge L \doteq \alpha_2 \wedge L \doteq \alpha_3)$$
$$\vee\ (\exists K. \exists L. \exists M. \exists x.$$
$$(x :: L) \doteq \alpha_1 \wedge L \doteq \alpha_2 \wedge (x :: M) \doteq \alpha_3$$
$$\wedge\ (\mathrm{APPEND}\ K\ L\ M))$$

- CONNECTION WITH <u>CLARK COMPLETION</u>

CASE STUDIES

- HEREDITARY HARROP FORMULAS
- LINEAR HHF
- λ^{Π}
- $\lambda^{\Pi \multimap \& \top}$
- INTUITIONISTIC S4
- DESCRETE LINEAR TEMPORAL LOGIC